COMPLEXITY ADAPTIVE OFDM SYSTEMS

Jian (Andrew) Zhang, Ying Chen and Dhammika Jayalath

Wireless Signal Processing Program
Naitonal ICT Australia
{Andrew.Zhang, Ying.Chen, Dhammika.Jayalath}@nicta.com.au

ABSTRACT

One disadvantage with current OFDM systems is, the system complexity associated with FFT module, equalier, etc is almost fixed and varies little with information bit rate. In this paper, based on an innovative "Layered" concept in FFT algorithms, we propose a novel OFDM system whose complexity is adaptive to the desired bit rate and system performance by exploiting transmitter diversity. Different diversity options in the transmitter and receiver are discussed, and verified in simulations.

***Index Terms*—** OFDM, FFT

1. INTRODUCTION

OFDM system is an attractive solution for broadband wireless communications because of its anti-multipath capability and high spectrum efficiency [1]. However, one problem with current OFDM systems is, system complexity (FFT module, etc) does not change adaptively with the desired bit rate and performance.

In OFDM, information bit rate is generally changed by using different modulations, coding rates, and/or frequency spreading (Same data assigned to different subcarriers), and/or time spreading (same data over multiple OFDM symbols) [2]. However, the physical-layer data rate is fixed, and system runs at full speed. Both transmitter and receiver require full FFT computation - regardless of multipath conditions and information bit rate. On the contrary, in CDMA systems, spreading gain can be adaptive to data rate, and receiver can choose the number of fingers in a Rake receiver according to the channel condition and desired performance.

In this paper, based on an innovative "Layered" concept in FFT algorithms, we propose a smart OFDM system whose complexity is adaptive to the data rate and the desired system performance. In the proposed system, the receiver has the option of combining different number of multipath, similar to that in CDMA systems.

2. LAYERED FFT STRUCTURE

It is well known that the Divide-and-Conquer approach is the basis of FFT algorithms [3]. Below, we recall the process of the Divide-and-Conquer approach to calculate a N-point DFT of the signal $\mathbf{x} = \{x_i\}, i \in [0, N-1]$.

National ICT Australia is funded by the Australian Government's Department of Communications, Information Technology and the Arts and the Australian Research Council through Backing Australia's Ability and the ICT Centre of Excellence program.

Jian Zhang, Ying Chen and Dhammika Jayalath are also with the Research School of Information Science and Engineering, Australian National University.

Step 1 Stack the input signal $\mathbf{x}$ column-wise into a $P \times Q$ matrix $\mathbf{X} = \{x_{p,q}\}, p \in [0, P-1], q \in [0, Q-1]$ with $x_{p,q} = x_{qP+p}$ and $N = PQ$;

Step 2 Compute the Q-point DFTs for each row of $\mathbf{X}$, and yield a new matrix $\breve{\mathbf{X}} = \{\breve{x}_{l,q}\}, l = 0, 1, \cdots, P-1, q = 0, 1, \cdots, Q-1$ with $\breve{x}_{l,q} = \sum_{m=0}^{Q-1} x_{l,m} W_Q^{mq}, \quad q \in [0, Q-1]$, where $W_Q^{mq} = \exp(-j2\pi mq/Q)$;

Step 3 Multiply $\breve{\mathbf{X}}$ by the phase factors W_N^{lq} and generate a new matrix $\breve{\mathbf{V}} = \{\breve{v}_{l,q}\}$ with $\breve{v}_{l,q} = W_N^{lq} \breve{x}_{l,q}$;

Step 4 Compute the P-point DFTs for each column of $\breve{\mathbf{V}}$;

Step 5 Read the resulting $P \times Q$ matrix $\tilde{\mathbf{X}} = \{\tilde{x}_{p,q}\}, p = 0, 1, \cdots, P-1, q = 0, 1, \cdots, Q-1$ row-wise, and the resulting output is the DFT of $\mathbf{x}$.

Given two N-point signals $\mathbf{x}$, $\mathbf{h}$ and their circular convolution output $\mathbf{y} = \mathbf{x} \otimes \mathbf{h}$, we know that their DFTs have the relationship $\tilde{\mathbf{y}} = \tilde{\mathbf{x}} \odot \tilde{\mathbf{h}}$, where $\otimes$ and $\odot$ denote the circular convolution and dot product of two vectors, respectively. Now let us consider the relationship of these three signals in terms of their intermediate outputs in Step 2 in the Divide-and-Conquer approach.

If we rearrange the frequency domain samples $\tilde{\mathbf{x}}$, $\tilde{\mathbf{h}}$ and $\tilde{\mathbf{y}}$ into $P \times Q$ matrices row-wise according to the reverse process of Step 5 in the Divide-and-Conquer approach, we get $\tilde{\mathbf{X}} = \{\tilde{\mathbf{x}}_q\}$, $\tilde{\mathbf{H}} = \{\tilde{\mathbf{h}}_q\}$, $\tilde{\mathbf{Y}} = \{\tilde{\mathbf{y}}_q\}$, $q = 0, 1, \cdots, Q-1$ with

$$\tilde{\mathbf{x}}_q = (\tilde{x}_{0,q}, \cdots, \tilde{x}_{p,q}, \cdots, \tilde{x}_{P-1,q})^T, \tilde{x}_{p,q} = \tilde{x}_{pQ+q}, \quad (1)$$

$$\tilde{\mathbf{h}}_q = (\tilde{h}_{0,q}, \cdots, \tilde{h}_{p,q}, \cdots, \tilde{h}_{P-1,q})^T, \tilde{h}_{p,q} = \tilde{h}_{pQ+q}, \quad (2)$$

$$\tilde{\mathbf{y}}_q = (\tilde{y}_{0,q}, \cdots, \tilde{y}_{p,q}, \cdots, \tilde{y}_{P-1,q})^T, \tilde{y}_{p,q} = \tilde{y}_{pQ+q}, \quad (3)$$

$p = 0, 1, \cdots, P-1$.

Since $\tilde{\mathbf{y}}_q = \tilde{\mathbf{x}}_q \odot \tilde{\mathbf{h}}_q$, it is straightforward to see that for any $q \in [0, Q-1]$, the vector $\{\breve{y}_{l,q} W_N^{lq}\}$ equals to the length-P circular convolution of the two vectors $\{\breve{x}_{l,q} W_N^{lq}\}$ and $\{\breve{h}_{l,q} W_N^{lq}\}$, where $l = 0, 1, \cdots, P-1$, that is

$$\breve{y}_{l,q} W_N^{lq} = \sum_{m=0}^{P-1} \breve{x}_{m,q} W_N^{mq} \breve{h}_{((l-m))_P,q} W_N^{q((l-m))_P}, \quad (4)$$

where $((l-m))_P$ denotes the index $l-m$ modulo P. Thus for any $q \in [0, Q-1]$, we have

$$\breve{y}_{l,q} = \sum_{m=0}^{P-1} \breve{x}_{m,q} \breve{h}_{((l-m))_P,q} W_N^{q(m-l)} W_N^{q((l-m))_P} \quad (5)$$

$$= \sum_{m=0}^{l} \breve{x}_{m,q} \breve{h}_{(l-m),q} + \sum_{m=l+1}^{P-1} \breve{x}_{m,q} \breve{h}_{(P+l-m),q} W_Q^{q}, \quad (6)$$

where $l = 0, 1, \cdots, P-1$. This establishes the relationship of the intermediate outputs in the Divide-and-Conquer approach. The equation array can also be expressed in matrix form. Denote

$$\breve{\mathbf{y}}_q = (\breve{y}_{0,q}, \breve{y}_{1,q}, \breve{y}_{2,q}, \cdots, \breve{y}_{P-1,q})^T, \tag{7}$$

$$\breve{\mathbf{x}}_q = (\breve{x}_{0,q}, \breve{x}_{1,q}, \breve{x}_{2,q}, \cdots, \breve{x}_{P-1,q})^T, \tag{8}$$

$$\breve{\mathbf{H}}_q = \begin{pmatrix} \breve{h}_{0,q} & W_Q^q \breve{h}_{P-1,q} & \cdots & W_Q^q \breve{h}_{1,q} \\ \breve{h}_{1,q} & \breve{h}_{0,q} & \cdots & W_Q^q \breve{h}_{2,q} \\ \breve{h}_{2,q} & \breve{h}_{1,q} & \cdots & W_Q^q \breve{h}_{3,q} \\ \vdots & \vdots & \ddots & \vdots \\ \breve{h}_{P-1,q} & \breve{h}_{P-2,q} & \cdots & \breve{h}_{0,q} \end{pmatrix}, \tag{9}$$

we have

$$\breve{\mathbf{y}}_q = \breve{\mathbf{H}}_q \breve{\mathbf{x}}_q, \tag{10}$$

for $q = 0, 1, \cdots, Q-1$.

The Divide-and-Conquer approach and the new relationship given in (10) motivates the new concept of *Layered FFT Structure*. In principle, we can design systems by exploiting the data in any step in the Divide-and-Conquer approach. Particularly, if we design OFDM systems by letting the inputs be the intermediate outputs $\breve{\mathbf{x}}_q$ as shown in (10), we can achieve multiple advantages, including system complexity adaptive to the desired data rate and system performance.

3. EXEMPLIFIED SYSTEM STRUCTURE

The proposed complexity adaptive OFDM system will be exemplified with $P = 2$ and $Q = N/2$. Extension to any P equal to a power of 2 is straightforward.

The structure of the proposed system is shown in Fig. 1. The N input symbols are divided into two streams, represented by $\breve{\mathbf{s}}_0$ and $\breve{\mathbf{s}}_1$, each containing $N/2$ symbols. An $N/2$-point IFFT is then applied for each of them, yielding two time domain vectors $\mathbf{s}_0 = (s_{0,0}, s_{0,1}, \cdots, s_{0,N/2-1})$ and $\mathbf{s}_1 = (s_{1,0}, s_{1,1}, \cdots, s_{1,N/2-1})$. These outputs are then interleaved and parallel-to-serial converted to generate $\mathbf{x} = (s_{0,0}, s_{1,0}, s_{0,1}, s_{1,1}, \cdots, s_{0,N/2-1}, s_{1,N/2-1})$. A cyclic-prefix (CP) or Zero-padded-prefix (ZP) is then appended to $\mathbf{x}$. The output is defined as a *frame*. Comparing it with the Divide-and-Conquer algorithm, we can see that this process actually corresponds to Step 1 and 2 in the latter in an reverse order. Thus $\breve{\mathbf{s}}_0$, $\breve{\mathbf{s}}_1$ and $\breve{\mathbf{x}}_q$, as defined in (8), are associated by

$$\breve{\mathbf{x}}_q = (\breve{s}_{0,q}, \breve{s}_{1,q})^T. \tag{11}$$

According to (10), if we know $\breve{\mathbf{y}}_q$, $\breve{\mathbf{x}}_q$ can then be estimated based on previously estimated channel matrix. The computation of $\breve{\mathbf{y}}_q$ is straightforward as $\breve{\mathbf{y}}_q$ is just the Step 2 output of the received time domain signal $\mathbf{y}$ in the Divide-and-Conquer approach. As shown in Fig. 1, after removing CP or processing ZP, the received signal $\mathbf{y} = (y_0, y_1, \cdots, y_{N-1})$ is divided into two streams $\mathbf{y}_0 = (y_0, y_2, \cdots, y_{N-2})$ and $\mathbf{y}_1 = (y_1, y_3, \cdots, y_{N-1})$. An $N/2$-point FFT is then applied to each of them. Then $\breve{\mathbf{y}}_q = (\breve{y}_{0,q}, \breve{y}_{1,q})^T$ is formed by collecting the q-th coefficient from each FFT output.

For $P = 2$, (10) becomes

$$\breve{\mathbf{y}}_q = \begin{pmatrix} \breve{h}_{0,q} & W_Q^q \breve{h}_{1,q} \\ \breve{h}_{1,q} & \breve{h}_{0,q} \end{pmatrix} \breve{\mathbf{x}}_q + \breve{\mathbf{n}}_q, \tag{12}$$

where $\breve{\mathbf{n}}_q$ is the noise vector.

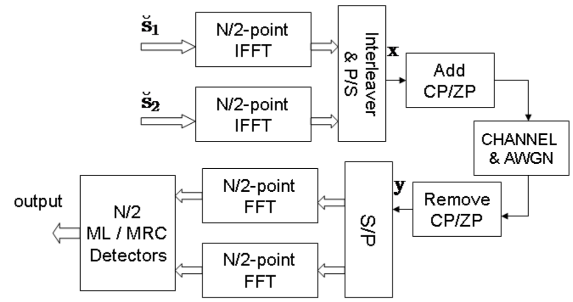

Fig. 1. Simplified system structure of the proposed OFDM system.

The $\breve{\mathbf{x}}_q$ in (12) can be solved by various algorithms, for example, zero-forcing (ZF), iterative receiver and maximal likelihood detector. The system structure as demonstrated in Fig. 1 also has some attractive properties for a full speed transmission, for example, slightly reduced Peak-to-average power ratio (PAPR) and more flexibility in the receiver design. Similar systems are also proposed and discussed in [4] and [5]. However, the focus of this paper is on the design of complexity and performance adaptive OFDM systems by exploiting transmitter diversity (Tx-diversity) based on the proposed structure. In such systems, when the full data rate is determined, several basic sub- data rates are achieved by transmitter diversity.

Now if we let $\breve{\mathbf{x}}_q$ convey only one symbol, that is, $\breve{s}_{1,q} = f(\breve{s}_{0,q})$ and $f(s)$ is a determined function of s, frequency diversity can be achieved. The receiver can have variable complexity according to the desired performance.

3.1. Receiver Options

When $\breve{s}_{1,q} = f(\breve{s}_{0,q})$, three types of diversity schemes are feasible: Maximal ratio combining (MRC), Equal gain Combining (EGC) and Selective Combining (SLC). A simplified system with these diversity options is shown in Fig. 2.

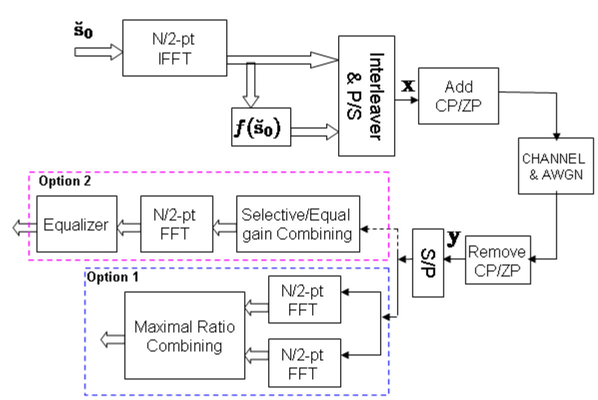

Fig. 2. System structure with reduced complexity and different diversity options.

3.1.1. MRC

MRC maximizes the signal to noise ratio of each symbol. It computes two estimates of $\breve{s}_{0,q}$ from $\breve{y}_{0,q}$ and $\breve{y}_{1,q}$, respectively, then uses two optimal weights to combine them and outputs the final estimate. When $f(\breve{s}_{0,q})$ is a linear function of $\breve{s}_{0,q}$, for example, $\breve{s}_{1,q} = c\breve{s}_{0,q}$ and c is a constant, MRC estimator becomes

$$\breve{s}_{0,q} = \frac{(\breve{h}_{0,q} + c\,W_Q^q \breve{h}_{1,q})^* \breve{y}_{0,q} + (\breve{h}_{1,q} + c\,\breve{h}_{0,q})^* \breve{y}_{1,q}}{|\breve{h}_{0,q} + c\,W_Q^q \breve{h}_{1,q}|^2 + |\breve{h}_{1,q} + c\,\breve{h}_{0,q}|^2}, \quad (13)$$

where * denotes the conjugation operator.

The MRC estimator involves 2 $N/2$-point FFT and $3N/2$ multiplications for one frame.

3.1.2. EGC

EGC uses two identical weights to combine the two estimates. For $\breve{s}_{1,q} = c\breve{s}_{0,q}$, EGC estimator becomes

$$\breve{s}_{0,q} = \frac{\breve{y}_{0,q} + \breve{y}_{1,q}}{(1+c)\breve{h}_{0,q} + (1 + c\,W_Q^q)\breve{h}_{1,q}}. \quad (14)$$

The summation of $\breve{y}_{0,q}$ and $\breve{y}_{1,q}$ can be shifted to the place before the $N/2$-point FFT operation because FFT is a linear operation and $\breve{y}_{0,q} + \breve{y}_{1,q} = \mathbf{F}_{N/2}(q,:)(\mathbf{y}_0 + \mathbf{y}_1)^T$. Thus only one $N/2$-point FFT is required in EGC.

In terms of performance, EGC is inferior to MRC , and theoretically, 3dB better than SLC.

3.1.3. SLC

SC simply chooses one between $\mathbf{y}_0$ and $\mathbf{y}_1$ to do estimation in one frame. The choice usually depends on the value of channel coefficients. Thus SC enables the receiver to operate at half sampling rate, and requires one $N/2$-point FFT.

3.2. Transmitter Inputs

When designing the function $f(s)$, several factors need to be considered, including transmitter and/or receiver complexity reduction, PAPR reduction, and receiver performance. For transmitter complexity reduction, a common rule is that $f(s)$ is chosen such that only one $N/2$-point complex IFFT needs to be implemented in the transmitter. Below, we give some examples and highlight that inputs have impact on the diversity performance.

3.2.1. Inputs Derived from general OFDM

From the structure of $\breve{\mathbf{H}}_q$ in (9), we can see that, $\breve{\mathbf{H}}_q$ is actually a circulant matrix and can be diagonalized as

$$\mathbf{D}_q = \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{H}}_q \Phi_q \mathbf{F}_P^H, \quad (15)$$

where Φ_q is a diagonal matrix

$$\Phi_q = \mathrm{diag}(1, W_N^{-q}, W_N^{-2q}, \cdots, W_N^{-(P-1)q}), \quad (16)$$

$\mathbf{F}_P$ is the P-point DFT matrix, and H denotes the Hermitian conjugate.

Left multiplying $\mathbf{F}_P \Phi_q^{-1}$ to $\breve{\mathbf{y}}_q$, we get

$$\begin{aligned} \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{y}}_q &= \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{H}}_q \breve{\mathbf{x}}_q \\ &= \mathbf{D}_q \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{x}}_q. \end{aligned} \quad (17)$$

From (17), we can derive the following relationship according to the Divide-and-Conquer approach

$$\tilde{\mathbf{y}}_q = \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{y}}_q; \quad (18)$$

$$\tilde{\mathbf{x}}_q = \mathbf{F}_P \Phi_q^{-1} \breve{\mathbf{x}}_q; \quad (19)$$

$$\mathrm{diag}(\tilde{\mathbf{h}}_q) = \mathbf{D}_q. \quad (20)$$

Using (19), any input method for frequency diversity in general OFDM systems can be converted to the new system proposed in this paper. Take the frequency diversity approach in Multiband OFDM [2] as an example, where each input and its conjugate are spaced at $N/2$ subcarriers, $\tilde{x}_q = \tilde{x}_(N/2+q)^*$. According to (19), the input in the proposed system becomes

$$\begin{aligned} \begin{pmatrix} \breve{s}_{0,q} \\ \breve{s}_{1,q} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & W_N^{-q} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \tilde{x}_q \\ \tilde{x}_q^* \end{pmatrix} \\ &= \begin{pmatrix} \mathrm{Re}(\tilde{x}_q) \\ j\,W_N^{-q}\,\mathrm{Im}(\tilde{x}_q) \end{pmatrix}, \end{aligned} \quad (21)$$

where $q = 0, 1, \cdots, N/2-1$, Re and Im denotes to take the real and imaginary part, respectively.

With this input option, the transmitter can be implemented with 2 real $N/2$-point IFFTs, however, the diversity scheme derived from (12) is not applicable directly. Instead, (17) needs to be formed from the received signal, yielding

$$\begin{pmatrix} 1 & W_N^{-q} \\ 1 & -W_N^{-q} \end{pmatrix} \tilde{\mathbf{y}}_q = \begin{pmatrix} \breve{h}_{0,q} + W_Q^q \breve{h}_{1,q} & 0 \\ 0 & \breve{h}_{0,q} - W_Q^q \breve{h}_{1,q} \end{pmatrix} \begin{pmatrix} \tilde{x}_q \\ \tilde{x}_q^* \end{pmatrix}. \quad (22)$$

This requires N extra complex multiplications in the receiver. However, the diversity performance is superior as to be seen below.

3.2.2. Linear function $f(s) = c\,s$

A simple and efficient input configuration is to let $f(s)$ be a linear function, $\breve{s}_{1,q} = c\,\breve{s}_{0,q}$. Different values of c have different impact on the system. Two examples are $c = 0$ and $c = 1$. In both cases, only one complex $N/2$-point IFFT is needed in the transmitter, and the diversity schemes discussed in Section 3.1 apply directly. However, performance with MRC diversity is different.

When c is a constant, the performance of MRC combining depends on the denominator $\gamma = |\breve{h}_{0,q} + c\,W_Q^q \breve{h}_{1,q}|^2 + |\breve{h}_{1,q} + c\,\breve{h}_{0,q}|^2$ in (13). Fig. 3 gives geometrical illustration of MRC combining for for $c = 0$ and $c = 1$. Intuitively, we can see that when $c = 1$, γ varies largely with q and the angle between the two channel coefficients $\breve{h}_{0,q}$ and $\breve{h}_{1,q}$. This is not the property that MRC combining prefers. Comparatively, when $c = 0$, γ remains stable and only depends on the magnitude of the channel coefficients. Actually, from the "Parallelogram law", we can get

$$\begin{aligned} |\breve{h}_{0,q}|^2 + |\breve{h}_{1,q}|^2 &= |\breve{h}_{0,q}|^2 + |W_Q^q \breve{h}_{1,q}|^2 \\ &= |\breve{h}_{0,q} + W_Q^q \breve{h}_{1,q}|^2 + |\breve{h}_{0,q} - W_Q^q \breve{h}_{1,q}|^2, \end{aligned} \quad (23)$$

which shows the diversity gain with $c = 0$ equals to that obtained in (22). However, this input design increases the transmitter's PAPR, compared to $c = 1$.

The value of c can also be chosen by optimizing power loading between $\breve{\mathbf{s}}_0$ and $\breve{\mathbf{s}}_1$ when channel is known.

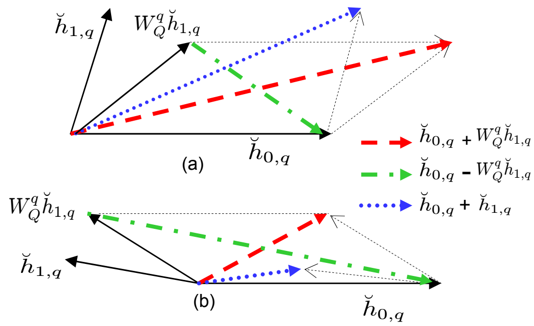

Fig. 3. Geometrical illustration of MRC combining for different inputs.

3.3. Extensions and Summations

According to the desired data rate, the systems in Fig. 1 and Fig. 2 can be extended to $P = 4$, and theoretically, any value equivalent to a power of 2. Other base sub- data rates can then be formed. Thus we can realize complexity adaptive OFDM systems where the lower the data rate, the lower the complexity and the larger the diversity. A complete data-rate set can be formed as follows.

- Determine the system structure in the full data rate case;
- Form several base sub- data rates by applying the transmitter diversity scheme discussed above;
- Form Intermediate data rates by changing coding rate and/or modulation based on the base sub- data rates.

One concern with this system is how to justify its performance, compared with changing modulations and codings. Intuitively, performance difference should depend on SNR and channel fading because they deal with different problems. Modulation deals with AWGN, Coding deals with AWGN and fading, and frequency diversity scheme mainly deals with fading. Thus for deep fading channels, this scheme may surplus the coding or modulation reduction approach (in higher SNR case). This is verified by the following simulation results.

4. SIMULATIONS

We show some simulation results with $N = 256$ and $P = 2$ in this section. Diversity schemes with $c = 0$ and $c = 1$ are simulated, and compared with the frequency diversity scheme in multiband OFDM. The CM2 channel model from IEEE802.15.3a is adopted. The data is encoded with $1/2$-rate convolutional code and decoded with the Viterbi algorithm. In SLC, output is chosen from the branch which has a larger summation of channel power per package.

Due to the page limit, we can only present one figure here, showing the bit error rate (BER) performance for 16QAM without diversity and 64QAM with diversity. In Fig. 4, SOFDM denotes the proposed system shown in Fig.1, G-OFDM denotes the general OFDM system. The two dashed curves represent the performance for no-Tx-diversity, both SOFDM and G-OFDM are equalized with ZF equalizer. There is about 1dB difference between SOFDM and G-OFDM due to the noise enhancement and error propagation in SOFDM associated with simple ZF equalizer. Other solid curves illustrate the BER for Tx-diversity with $P = 2$ and $c = 1$. As discussed in Section 3.2, the BER performance of G-OFDM is about 1dB better than SOFDM, using their respective inputs. However, SOFDM provides reduced complexity and flexible receiver options.

Comparing the performance between Tx-diversity and no-diversity, we can observe that the diversity scheme in SOFDM achieves better performance than the no-diversity scheme, while the former's data rate is 1.5 times of the latter's. The performance difference increases with the SNR, which justifies the intuitive explanation of the roles of modulation, coding and diversity in combating noise and fading.

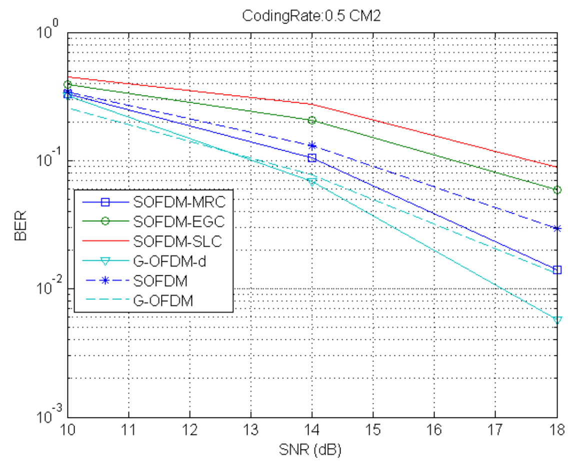

Fig. 4. Coded BER Performance for 16QAM without diversity and 64QAM with diversity.

5. CONCLUSIONS

In this paper, we proposed complexity adaptive OFDM systems where system complexity is adaptive to the data rate and desired performance by exploiting Tx-diversity. Inputs can be designed to meet different requirements. Simulation results show that in a fading environment, the proposed Tx-diversity scheme could achieve better performance than the no-Tx-diversity scheme with lower order modulation at similar data rates. .

6. REFERENCES

[1] J. A. C. Bingham, “Multicarrier modulation for data transmission: An idea whose time has come,” *IEEE Commun. Mag.*, p. 514, May 1990.

[2] MBOA-SIG, “Multiband OFDM Physical Layer Proposal for IEEE 802.15 Task Group 3a,” *MultiBand OFDM Alliance SIG.*, Sep. 2004.

[3] John G. Prokis and Dimitris G. Manolaki, *Digital Signal Processing: Principles, Algorithms, and Applications, 3rd Edition*, Prentice Hall PTR, 1996.

[4] V. G. S. Prasad and K. V. S. Hari, “Interleaved orthogonal frequency division multiplexing (iofdm) system,” *IEEE Trans. Signal Processing*, vol. 52 (6), pp. 1711–1721, June 2004.

[5] Xiang-Gen Xia, “Precoded and vector ofdm robust to channel spectral nulls and with reduced cyclic prefix length in single transmit antenna systems,” *IEEE Trans. Commun.*, vol. 49(8), pp. 1363–1374, Aug. 2001.