

A NEW SCHEME TO IDENTIFY GOOD CODES FOR OFDM WITH LOW PMEPR

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ABSTRACT

Golay complementary sequences have been introduced to encode the orthogonal frequency division multiplexing (OFDM) signals. In this paper, we extend Golay complementary sequences to sub-root sequences. Based on sub-root sequences, we propose a new scheme to construct good codes for OFDM with low PMEPR, which has the potential to discover Golay-Davis-Jedwab (GDJ), Non-GDJ complementary codes and Non-Complementary codes. We firstly build the main construction theorem. Then we build short length root codes represented by Boolean functions $\mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$, which are not the form of GDJ complementary sequences, but with low PMEPR. By our theorem, we then extend them to long length root codes with the same PMEPR. In this way, we offer an efficient method to identify a large number of codes for OFDM with low PMEPR.

Index Terms— OFDM, PMEPR, Boolean functions, Golay complementary sequences, LUUT.

1. INTRODUCTION

In multicarrier communications, the orthogonal frequency division multiplexing (OFDM) has been made use widely. However, a major drawback of OFDM signals is the high peak to mean envelope power ratio (PMEPR) of the uncoded OFDM signal. Numbers of PMEPR reduction schemes have been proposed. On the other hand, several coding schemes to reduce the PMEPR of the OFDM waveform have been studied. An idea to use the Golay complementary sequences to encode the OFDM signals with PMEPR of at most 2 has been introduced. However, it only works well for a small number of carriers ($n \leq 32$), and it remains an open problem to discover low PMEPR error-correcting code constructions for a moderately large subcarriers n .

Golay complementary sequences have been introduced to encode the orthogonal frequency division multiplexing (OFDM) signals. In this paper, we extend Golay complementary sequences to sub-root sequences. Based on sub-root sequences, we propose a new scheme to construct good codes for OFDM with low PMEPR, which has the potential to discover Golay-Davis-Jedwab (GDJ), Non-GDJ complementary codes and Non-Complementary codes. We firstly build the main construction theorem. Then we build short length root codes represented by Boolean functions $\mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$, which are not the form of GDJ complementary sequences, but with low PMEPR.

Supported by NSF China #60672067, by NSF Shanghai #062R14041, by Shanghai-Canada NRC #06SN07112, by Cultivation Fund of the Key Scientific and Technical Innovation Project, Ministry of Education of China #706022, and by Program for New Century Excellent Talents in University (NCET).

By our theorem, we then extend them to long length root codes with the same PMEPR. In this way, we offer an efficient method to identify a large number of codes for OFDM with low PMEPR.

2. NOTATION AND PRELIMINARIES

Before proceeding further, let us introduce the OFDM signals, the PMEPR and the related concepts at first. Throughout this paper $\xi = \exp(2\pi j/M)$ denotes a primitive M th root of unity and M is an even positive integer. We also let an M -ary Phase Shift Keying (MPSK) modulated Constellation be denoted as $\xi^{\mathbb{Z}_M} = \{\xi^k : k \in \mathbb{Z}_M\}$, where $\mathbb{Z}_M = \{0, \dots, M-1\}$.

2.1. OFDM and Power Control

Let j be the imaginary unit, i.e., $j^2 = -1$. For an MPSK modulation OFDM, let a codeword $c = (c_0, \dots, c_{n-1})$ with $c_\ell \in \xi^{\mathbb{Z}_M}$, the frequency separation between any two adjacent subcarriers is $\Delta f = 1/T$. Then the n subcarrier complex baseband OFDM signal can be represented as

$$s_c(t) = \sum_{\ell=0}^{n-1} c_\ell e^{j2\pi\ell\Delta f t}, \quad (1)$$

where $0 \leq t < T$. The instantaneous power of the complex envelope $s_c(t)$ is defined by

$$P_c(t) = |s_c(t)|^2. \quad (2)$$

So the peak-to-mean power ratio (PMEPR) of the codeword c is defined by

$$\text{PMEPR}(c) = \frac{1}{n} \sup_{0 \leq t < T} |s_c(t)|^2. \quad (3)$$

Notice that the PMEPR can be as large as n , which occurs, for example, if c is the all-one word. However, it is desirable to use codewords with PMEPR that is substantially lower than n .

2.2. Generalized Boolean Functions and Associated Sequences

A generalized Boolean function f is defined as a mapping $f: \mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$. Such a function can be written uniquely in its algebraic normal form, i.e., f is the sum of weighted monomials $f(x_0, x_1, \dots, x_{m-1}) = \sum_{i=0}^{2^m-1} c_i \prod_{\alpha=0}^{m-1} x_\alpha^{i_\alpha}$, where the weights c_0, \dots, c_{2^m-1} are in \mathbb{Z}_M , and $(i_0, i_1, \dots, i_{m-1})$ is the binary representation of i , $0 \leq i < 2^m$, i.e., $i = \sum_{\alpha=0}^{m-1} i_\alpha 2^\alpha$. The order of the i th monomial is defined to be $\sum_{\alpha=0}^{m-1} i_\alpha$, and the order or algebraic degree of a generalized Boolean function, denoted by $\text{deg}(f)$, is equal to the highest order

of the monomials with a nonzero coefficient in the algebraic normal form of f .

A generalized Boolean function may be equally represented by sequences of length 2^m . Denote $f_i = f(i_0, i_1, \dots, i_{m-1})$, where $(i_0 i_1 \dots i_{m-1})$ is the binary representation of i , $0 \leq i < 2^m$. Define the \mathbb{Z}_M -valued sequence associated with f as $\psi(f) \triangleq (f_0, f_1, \dots, f_{2^m-1})$, and the polyphase sequence associated with f as $\varphi(f) \triangleq (\xi^{f_0}, \xi^{f_1}, \dots, \xi^{f_{2^m-1}})$. Denote $\varphi(f_i) \triangleq \xi^{f_i}$.

We should extend polyphase sequences and the corresponding generalized Boolean functions. Let k be an integer such that $0 < k < m$. Arbitrarily take i_0, i_1, \dots, i_{k-1} such that $0 \leq i_0 < i_1 < \dots < i_{k-1} < m$. Suppose that $j_0, j_1, \dots, j_{m-k-1}$ are the rest indices and $0 \leq j_0 < j_1 < \dots < j_{m-k-1} < m$. Let $\mathbf{x} = (x_{i_0} \dots x_{i_{k-1}})$ and $\mathbf{d} = (d_0 \dots d_{k-1})$ be two binary words of length k . Define the extended complex polyphase sequence $\varphi(f|_{\mathbf{x}=\mathbf{d}})$ of length 2^m as follows. For any $(y_0 y_1 \dots y_{m-k-1})$ in \mathbb{Z}_2^{m-k} , let $i = \sum_{\ell=0}^{m-k-1} y_\ell 2^{j_\ell} + \sum_{\ell=0}^{k-1} d_\ell 2^{i_\ell}$. Define the i th entry of $\varphi(f|_{\mathbf{x}=\mathbf{d}})$ to be ξ^{f_i} and zero otherwise.

2.3. Aperiodic Correlations and Golay Complementary Sequences

Let $\mathbf{A} = (A_0 A_1 \dots A_{n-1})$ and $\mathbf{B} = (B_0 B_1 \dots B_{n-1})$ be two complex vectors of length n , where $A_i, B_i \in \mathbb{C}^n$, $0 \leq i < n-1$ and ℓ be an integer. Define

$$C(\mathbf{A}, \mathbf{B})(\ell) \triangleq \begin{cases} \sum_{i=0}^{n-\ell-1} A_{i+\ell} B_i^*, & \text{if } 0 \leq \ell < n \\ \sum_{i=0}^{n+\ell-1} A_i B_{i-\ell}^*, & \text{if } -n < \ell \leq 0 \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

and $A(\mathbf{B})(\ell) \triangleq C(\mathbf{B}, \mathbf{B})(\ell)$.

From (1), (2) and (4), it is an easy exercise to show that $P_c(t) = |S_c(t)|^2 = \sum_{\ell=1}^{n-1} A(\mathbf{c})(\ell) e^{j2\pi\ell\Delta ft}$
 $= A(\mathbf{c})(0) + 2 \cdot \text{Re} \sum_{\ell=1}^{n-1} A(\mathbf{c})(\ell) e^{j2\pi\ell\Delta ft}$.

A $\xi^{\mathbb{Z}_M}$ -sequence a of length n is called a Golay complementary sequence [6] if there is a $\xi^{\mathbb{Z}_M}$ -sequence b of length n such that

$$A(\mathbf{a})(\ell) + A(\mathbf{b})(\ell) = n\delta(\ell), \quad (5)$$

where the Dirac function $\delta(\ell)$ is defined by $\delta(0) = 1$ and $\delta(\ell) = 0$ for $\ell \neq 0$. It is easy to see that $P_a(t) + P_b(t) = 2n$. Thus $\text{PMEPR}(a) \leq 2$ if a is a Golay complementary sequence.

3. NEW CONSTRUCTION SCHEME

In this section we will give the main results of our theorem, which are mainly based on the Linear Unimodular Unitary Transforms (LUUTs), including one- and multi-dimensional generalized IFFTs. Due to limited space, the proofs are omit. From here on, let $\eta = e^{j2\pi\Delta ft}$.

3.1. LUUTs and Their Relation with OFDM Power Control

In [4] and [5], Parker and Tellambura give some idea about the Power Control Problem with Linear Unimodular Unitary Transforms (LUUTs), here we will use the similar concepts.

Definition 1 Let a complex vector $\mathbf{Z} = (z_0, z_1, \dots, z_{n-1})$, we call \mathbf{Z} unimodular if any element in the sequence has magnitude 1, i.e., $z_i \in \mathbb{C}^n$ and $|z_i| = 1$, $0 \leq i < n-1$. For any given complex sequence $\mathbf{A} = (A_0 A_1 \dots A_{n-1})$, where $A_i \in \mathbb{C}^n$, $0 \leq i < n-1$, we define the One-dimensional LUUT and N -dimensional LUUT of \mathbf{A} respectively by $\mathbf{A} \odot \mathbf{Z} = \sum_{i=0}^{n-1} A_i \cdot Z_i$, and $\mathbf{A} \odot \mathbf{Z}_1 \odot \mathbf{Z}_2 \dots \odot \mathbf{Z}_N = \sum_{i=0}^{n-1} A_i \cdot Z_{1,i} \cdot Z_{2,i} \dots \cdot Z_{N,i}$, where $\mathbf{Z}, \mathbf{Z}_1, \dots, \mathbf{Z}_N$ are all unimodular.

Let $L(z) = (z^0, z^1, \dots, z^{n-1})$, equation (1) is equivalent to $s_c(t) = c \odot L(z)|_{z=\eta}$. It is easy to see that the OFDM modulation scheme is just a special case of LUUTs.

It is well known that if two $\xi^{\mathbb{Z}_M}$ -sequence a and b of length n form a Golay complementary pair, then $|a \odot L(z)|^2 + |b \odot L(z)|^2 = 2n$. Now we come to extend this result to the case of higher dimensional LUUTs, which will be used in the sequel.

Proposition 1 If two $\xi^{\mathbb{Z}_M}$ -sequences a and b of length n form a Golay complementary pair, then $|a \odot L(z_1) \dots \odot L(z_N)|^2 + |b \odot L(z_1) \dots \odot L(z_N)|^2 = 2n$.

Moreover, this proposition also holds for general complex vectors, which is summarized as follows.

Corollary 1 If two $\xi^{\mathbb{Z}_M}$ -sequences a and b of length n form a Golay complementary pair, then $|a \odot Z_1 \dots \odot Z_N|^2 + |b \odot Z_1 \dots \odot Z_N|^2 = 2n$, where Z_i , $1 \leq i \leq N$, are complex vectors and $Z_{i,1} = 1$.

3.2. New Construction Model

Now we will give our new construction model. First we introduce the definition of the root pair.

Definition 2 Let $f(x_1, x_2, \dots, x_m)$ and $g(x_1, x_2, \dots, x_m)$ be two Boolean functions over $\mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$. Denote $N = 2^m$. If for a given real number α , there is a complex vector $\mathbf{Z} = (z_0, z_1, \dots, z_{N-1}) \in \mathbb{C}^n$ such that

$$|\varphi(f) \odot L(\eta)|^2 + |\varphi(g) \odot \mathbf{Z}|^2 \leq \alpha N, \quad (6)$$

then $[f, g, \alpha, m]$ is called a root pair.

3.3. Some Comments On the New Construction Model

The motivation for us to propose the above construction model is that although some complementary scheme have provided good solution for the identification of good codes for OFDM with low PMEPR, the number of the identified codes is limited. This is because that some codewords with low PMEPR is not complementary. If we want to get more codes with low PMEPR, we need not only to extend the searching boundary from GDJ sequences to Non-GDJ sequences, but also from Complementary sequences to Non-Complementary sequences. As shown above, the sequences we need for OFDM is just the ones identified by the Boolean function f , whereas g just acts as an assistant to the identification. So we can expect a larger number of sequences with low PMEPR.

On the other hand, adjusting the variable α , we can get a good trade-offs between PMEPR and code rate. Although this new model provides some methods to search more possible good codes for OFDM with low PMEPR as well as high code rate, it is hard to give an explicit construction method except with the aid of computer search for the general case. In the following, we focus on the case $\mathbf{Z} = L(\eta)$. We call it a sub-root pair particularly. Furthermore, by the extending method will be discussed later, we can even get the new long length root pair from a short length root pair with the same PMEPR.

3.4. New Construction Scheme for the case $\mathbf{Z} = L(\eta)$

Firstly, we introduce some propositions without proofs, which will be used later.

Proposition 2 Let a sub-root pair be $[f, g, \alpha, m]$, i.e., $|\varphi(f) \odot L(\eta)|^2 + |\varphi(g) \odot L(\eta)|^2 \leq \alpha N$. Then for an arbitrary positive integer n , we have $|\varphi(f) \odot L(\eta^n)|^2 + |\varphi(g) \odot L(\eta^n)|^2 \leq \alpha N$.

Proposition 3 Let $f(x_1, x_2, \dots, x_m)$ be a Boolean function over $\mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$. Denote $\hat{f} = -f(\widehat{x_1}, \widehat{x_2}, \dots, \widehat{x_m})$, where $(\widehat{x_1}, \widehat{x_2}, \dots, \widehat{x_m})$ is the baseminusones complement of (x_1, x_2, \dots, x_m) . Then $|\varphi(f) \odot L(\eta)|^2 = |\varphi(\hat{f}) \odot L(\eta)|^2$.

In the following let $a(x_1, x_2, \dots, x_{m_1})$ and $b(x_1, x_2, \dots, x_{m_1})$ be two Boolean functions over $\mathbb{Z}_2^{m_1} \rightarrow \mathbb{Z}_M$. Let $c(x_1, x_2, \dots, x_{m_2})$ and $d(x_1, x_2, \dots, x_{m_2})$ be Golay Pair of length 2^{m_2} over \mathbb{Z}_M , i.e., $\varphi(c) - \varphi(d)$ is lifting of a binary sequence to $\mathbb{Z}_{\frac{M}{2}}$ [1], [2]. We also denote $X_1 = (x_1, \dots, x_{m_1})$ and $X_2 = (x_{m_1+1}, \dots, x_{m_1+m_2})$.

Proposition 4 Let a be defined as above, then we have $\varphi(\hat{a}) \odot L(z) = z^{2^{m_1}-1}(\varphi(a) \odot L(z))^*$.

In the following, we will give some schemes to build long length sub-root pairs from short length sub-root pairs.

Theorem 1 Suppose that $[a, b, \alpha, m_1]$ is a sub-root pair. Let

$$\begin{aligned} f(X_1, X_2) &= \begin{cases} a(X_1) + c(X_2), & \text{if } c(X_2) = d(X_2) \\ b(X_1) + c(X_2), & \text{if } c(X_2) \neq d(X_2) \end{cases} \\ g(X_1, X_2) &= \begin{cases} a(X_1) + \widehat{d}(X_2), & \text{if } c(X_2) \neq d(X_2) \\ b(X_1) + \widehat{d}(X_2), & \text{if } c(X_2) = d(X_2) \end{cases} \end{aligned}$$

Then $[f, g, \alpha, m_1 + m_2]$ is a sub-root pair.

Theorem 2 Suppose that $[a, b, \alpha, m_1]$ is a sub-root pair. Let

$$\begin{aligned} f(X_2, X_1) &= \begin{cases} a(X_1) + c(X_2), & \text{if } c(X_2) = d(X_2) \\ b(X_1) + c(X_2), & \text{if } c(X_2) \neq d(X_2) \end{cases} \\ g(X_2, X_1) &= \begin{cases} a(X_1) + \widehat{d}(X_2), & \text{if } c(X_2) \neq d(X_2) \\ b(X_1) + \widehat{d}(X_2), & \text{if } c(X_2) = d(X_2) \end{cases} \end{aligned}$$

Then $[f, g, \alpha, m_1 + m_2]$ is a sub-root pair.

3.5. Further Extension of the above model

Let's use the tensor operator \otimes [3], [4] to describe the above model. Consider the construction of f in Theorem 1 and Theorem 2. Let \vec{s} be a $\xi^{\mathbb{Z}_M}$ -sequence which takes $\varphi(a)$ or $\varphi(b)$. Consider the two matrices $[\vec{s} \otimes \varphi(c_0), \vec{s} \otimes \varphi(c_1), \dots, \vec{s} \otimes \varphi(c_{2^{m_2}-1})]$ and

$$\begin{bmatrix} \vec{s} \otimes \varphi(c_0) \\ \vec{s} \otimes \varphi(c_1) \\ \dots \\ \vec{s} \otimes \varphi(c_{2^{m_2}-1}) \end{bmatrix}. \text{ These are two schemes based on a row vector}$$

and a column vector of size 2^{m_2} . We list the entries of the matrices column by column, from left to right, and in each column, we list downwards. Then the resulting sequence is $\varphi(f)$. This is actually a scheme using concatenation and interleaving. In order to get more concatenation and interleaving freedom, we can list the elements decided by $\varphi(c)$ in a $2^{m_2} \times 2^{m_3}$ matrix as follows:

$$\begin{bmatrix} \vec{s} \otimes \varphi(c_{0,0}) \dots \vec{s} \otimes \varphi(c_{0,2^{m_3}-1}) \\ \vec{s} \otimes \varphi(c_{1,0}) \dots \vec{s} \otimes \varphi(c_{1,2^{m_3}-1}) \\ \vdots \\ \vec{s} \otimes \varphi(c_{2^{m_2}-1,0}) \dots \vec{s} \otimes \varphi(c_{2^{m_2}-1,2^{m_3}-1}) \end{bmatrix}$$

Using the similar scheme, let $Y = (y_1, y_2, \dots, y_{m_1})$ index the vector \vec{s} . Let $X_1 = (x_1, x_2, \dots, x_{m_2})$ and $X_2 = (x_1, x_2, \dots, x_{m_3})$ be the row and column indices of the $2^{m_2} \times 2^{m_3}$ matrix respectively. Then we have the follows.

Theorem 3 Let Y, X_1 and X_2 be defined as above. Suppose that $[a, b, \alpha, m_1]$ is a sub-root pair. Let c and d be two Boolean function representations of Golay complementary sequences of length

$2^{m_2+m_3}$. Then $[f, g, \alpha, m_1 + m_2 + m_3]$ is a sub-root pair, where

$$\begin{aligned} f(X_1, Y, X_2) &= \begin{cases} a(Y) + c(X_1, X_2), & \text{if } c(X_1, X_2) = d(X_1, X_2) \\ b(Y) + c(X_1, X_2), & \text{if } c(X_1, X_2) \neq d(X_1, X_2) \end{cases} \\ g(X_1, Y, X_2) &= \begin{cases} a(Y) + \widehat{d}(X_1, X_2), & \text{if } c(X_1, X_2) \neq d(X_1, X_2) \\ b(Y) + \widehat{d}(X_1, X_2), & \text{if } c(X_1, X_2) = d(X_1, X_2) \end{cases} \end{aligned}$$

Now consider the Golay complementary pair c and d of the form in [5], i.e.,

$$\begin{cases} c = \frac{M}{2} \sum_{k=1}^{m_2+m_3-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m_2+m_3} e_k x_k + e_0 \\ d = c + e_0 + \frac{M}{2} x_{\pi(1)} \end{cases}$$

where π is a permutation of $\{1, 2, \dots, m_2+m_3\}$ and $e'_0, e_0, e_1, \dots, e_{m_2+m_3} \in \mathbb{Z}_M$. Let $e'_0 = e_0$. Applying Theorem 3, we have the following results.

Corollary 2 Suppose that $[a, b, \alpha, m_1]$ is a sub-root pair. Let c and d be two Boolean function representations of Golay complementary sequences of length $2^{m_2+m_3}$. Then $[f, g, \alpha, m_1 + m_2 + m_3]$ is a sub-root pair, where

$$\begin{cases} f(X_1, Y, X_2) = a(Y)(1 - x_{\pi(1)}) + b(Y)(x_{\pi(1)}) + c(X_1, X_2) \\ g = f + \frac{M}{2} x_{\pi(m_2+m_3)} + e \end{cases}$$

Naturally, we may list the entries of the matrices based on $\varphi(c)$ in a higher dimension case, e.g., 3-dimension. Then we have

$$[\varphi(c_{0,0,0}), \dots, \varphi(c_{0,0,2^{m_2}-1})], [\varphi(c_{0,1,0}), \dots, \varphi(c_{0,1,2^{m_2}-1})], \dots, [\varphi(c_{2^{m_4}-1,2^{m_3}-1,0}), \dots, \varphi(c_{2^{m_4}-1,2^{m_3}-1,2^{m_2}-1})]$$

In these higher dimension schemes, we may use more binary index vectors together to index f and g . But as shown in Theorem 3, the index of a or b , i.e., Y can only separate the index of f and g into two parts. So it is easy to know that these schemes will not produce more concatenation and interleaving. Hence, how can we get more sequences? In fact, Proposition 3 gives us some insight, and we have the follows:

Corollary 3 Corollary 2 also holds for $f(X_1, Y, X_2) = a(Y)(1 - x_{\pi(\ell)})(1 - x_{\pi(\ell-1)}) + b(Y)(1 - x_{\pi(\ell)})(x_{\pi(\ell-1)}) + \widehat{b}(Y)(x_{\pi(\ell)})(1 - x_{\pi(\ell-1)}) + \widehat{a}(Y)(x_{\pi(\ell)})(x_{\pi(\ell-1)}) + c(X_1, X_2)$, where ℓ is an integer and $2 \leq \ell \leq m_2 + m_3$.

4. A NEW SCHEME TO IDENTIFY GOOD CODES FOR OFDM WITH LOW PMEPR

In this section we provide a scheme to identify good codes for OFDM with low PMEPR. First, we use the exhaustive computer search to get all of the good codes represented by the Boolean functions $f : \mathbb{Z}_2^2 \rightarrow \mathbb{Z}_M$ with PMEPR less than or equal to α . We call them the primitive root. Then we can use them to identify more long length good codes for OFDM. We get the number of sub-root pairs as in the table below. We also list the sub-root $[a, b, 4, 2]$ with 16 representatives as in Table 1.

Z_M	$[a, b, 2, 2]$	$[a, b, 4, 2]$	$[a, b, 2, 3]$	$[a, b, 4, 3]$
Z_2	1	16	0	2456
Z_4	2	876	4	
Z_8	4	54749		

By Corollary 2-3, we can identify a class of codes lie in the third, fourth and even higher order Reed-Muller codes. When using root $[a, b, \alpha, 2]$, f and g have cubic components $(b_3 - a_3)y_1 y_2 x_{\pi(1)}$ by Corollary 2 and $(b_3 - a_3)y_1 y_2 x_{\pi(\ell-1)} - (b_3 + a_3)y_1 y_2 x_{\pi(\ell)} + (2a_1 + a_3 - 2b_1 - b_3)y_1 x_{\pi(\ell-1)} x_{\pi(\ell)} + (2a_2 + a_3 - 2b_2 - b_3)y_2 x_{\pi(\ell-1)} x_{\pi(\ell)}$

Table 1. The sub-root $[a, b, 4, 2]$ with 16 representatives.

a			b			Max $\{ a \odot L(\eta) ^2 + b \odot L(\eta) ^2\}$
a_1	a_2	a_3	b_1	b_2	b_3	
0	0	0	0	1	0	4
0	0	0	1	0	0	4
0	0	0	1	1	0	4
0	0	1	0	1	1	2
0	0	1	1	0	1	2
0	0	1	1	1	0	3
0	0	1	1	1	1	3.414
0	1	0	0	1	1	3
0	1	0	1	0	0	4
0	1	0	1	0	1	3
0	1	0	1	1	0	4
0	1	1	1	0	1	3.414
0	1	1	1	1	1	2
1	0	0	1	1	0	4
1	0	1	1	1	1	2
1	1	0	1	1	1	3

by Corollary 3. So for $[a, b, \alpha, 3]$, f and g have third, fourth and fifth order components. Select a sub-root pair from the six row of Table 1, i.e., $a = y_1 y_2$ and $b = y_1 + y_2$. Then the constructed f and g are third order Non-DJ Reed-Muller codes with the same PMEPR as a and b , i.e., 3. By numerical results, we can get 192 and 98 different Non-GDJ codes from this root based on four ordered pairs, i.e., $\langle a, b \rangle$, $\langle b, a \rangle$, $\langle \hat{a}, \hat{b} \rangle$ and $\langle \hat{b}, \hat{a} \rangle$ by Corollary 2 and 3 respectively. We have shown their OFDM power spectrum in Fig. 1 and 2. It is straightforward to show that the long length Non-GDJ f and g have the same PMEPR as a and b , hence our scheme is an efficient method to identify good codes for OFDM with low PMEPR.

5. CONCLUSION AND OPEN PROBLEMS

Although our construction scheme has partly answer the questions in [5], it is still very difficult to algebraically count the number of sequences sprawned by above methods, not only because they may degrade to GDJ, but also because there are may intersections and repeated enumerations. Unfortunately, we can not give a compute formula to count the numbers, which is left as an open problem. On the other hand, in order to get more good codes, we need to compute the root-pairs represented by a and b over $\mathbb{Z}_2^m \rightarrow \mathbb{Z}_M$ directly for larger m and M .

6. REFERENCES

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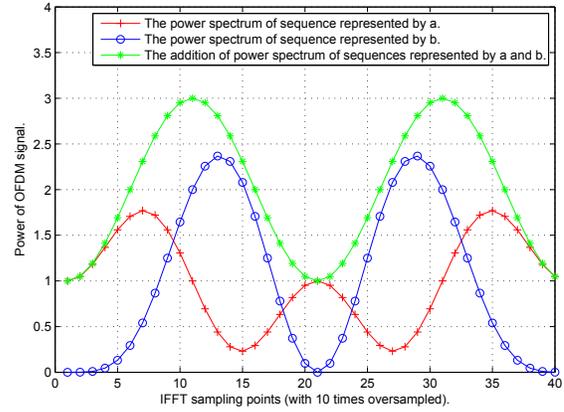


Fig. 1. The power spectrum of $a = y_1 y_2$, $b = y_1 + y_2$ and the addition of power spectrum of a and b .

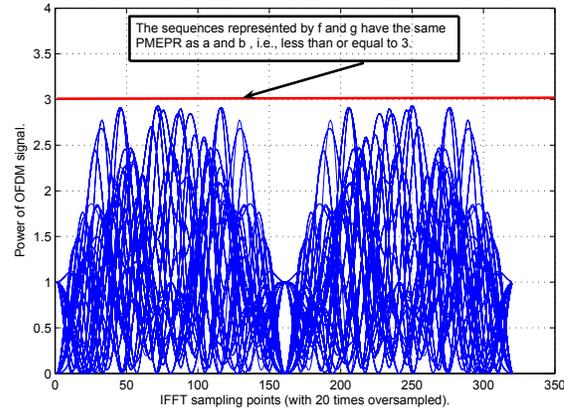


Fig. 2. The Power Spectrum of 192 and 98 Non-GDJ f and g .

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