CONVEX RELAXATION BASED LOW-COMPLEXITY OPTIMAL SPECTRUM BALANCING FOR MULTI-USER DSL

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ABSTRACT

In modern DSL networks, crosstalk between different DSL lines in the same cable bundle is a major source of performance degradation. By balancing the transmit power spectra, also referred to as multi-user power control, the impact of crosstalk can be minimized leading to spectacular performance gains. In this paper a novel low-complexity spectrum balancing algorithm is presented. Its performance is compared to optimal spectrum balancing for multiple-user scenarios and it is seen to yield similar results but with a huge reduction in complexity. Moreover, by the use of a Spectrum Management Center and limited message-passing the algorithm can be executed in a distributed fashion, which is a great asset in current DSL networks.

Index Terms— Digital subscriber lines, crosstalk, dynamic spectrum management, MIMO systems, optimization methods

1. INTRODUCTION

The ever increasing demand for higher data rates forces DSL systems to use higher frequencies, up to 30 MHz for VDSL2. At these frequencies, electromagnetic coupling becomes particularly harmfull and causes crosstalk between lines operating in the same cable bundle. This crosstalk, typically 10-20 dB larger than the background noise, is a major source of performance degradation in DSL systems currently under development.

Dynamic Spectrum Management (DSM) refers to a set of solutions to the crosstalk impairment problem. Basically these solutions consist of signal level coordination and/or spectrum level coordination. In this work the focus is on spectrum level coordination also referred to as spectrum balancing or power control. Here the transmit power spectrum of each modem is designed to cause minimal disturbance to other lines, while preserving a high data rate.

Optimal Spectrum Balancing (OSB) [1] [2] is a centralized spectrum balancing algorithm that calculates optimal transmit spectra for a network of interfering DSL lines. By the use of a dual decomposition OSB decouples the spectrum management problem into K independent per-tone optimization problems, where K is the number of active tones in the DSL system. However these per-tone optimization problems are themselves difficult nonconvex problems. An exhaustive search was proposed to find the global optimum. Unfortunately the set size of feasible solutions is exponential in the number of users N, rendering the exhaustive searches computationally intractable for more than five users. In [3] a branch and bound algorithm was presented to optimally solve the nonconvex per-tone optimization problems. This approach reduces the complexity significantly making it possible to simulate up to eight-user scenarios on the same platform. In [4] [5] a near-optimal iterative approach was presented to solve the per-tone optimization problems. Although this algorithm is much faster than the globally optimal spectrum balancing algorithms, it still has a large complexity.

In this paper a novel low-complexity distributed spectrum balancing algorithm is presented based on a convex relaxation. Its performance is compared to OSB for multiple-user scenarios and it is seen to yield similar results with a huge reduction in complexity.

The paper is organized as follows. In section 2 the system model for the crosstalk environment is described. In section 3 the spectrum management problem and the OSB algorithm are reviewed. In section 4 the novel approach is presented. Finally, in section 5 its performance and complexity are compared to OSB algorithms.

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2. SYSTEM MODEL

Most current DSL systems use Discrete Multi-Tone (DMT) modulation. The transmission for a binder of N users can be modeled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \qquad k = 1 \dots K.$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^N]^T$ contains the transmitted signals on tone k for all N users. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n on tone k. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI,.... The vector $\mathbf{y}_k$ contains the received symbols.

The transmit power is denoted as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The vector containing the transmit power of user n on all tones is $\mathbf{s}^n \triangleq [s_1^n, s_2^n, \dots, s_K^n]^T$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading for user n on tone k, given the transmit spectra $\mathbf{s}_k \triangleq [s_k^1, s_k^2, \dots, s_k^N]^T$ of all modems in the system, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m \neq n} |h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \qquad (1)$$

where Γ denotes the SNR-gap to capacity, which is a function of the desired BER, the coding gain and noise margin [6]. The total bit rate for user n and the total power used by user n are

$$R^n = f_s \sum_k b_k^n \text{ and } P^n = \sum_k s_k^n.$$

3. SPECTRUM BALANCING

3.1. The Spectrum Management Problem

The problem of optimally balancing the transmit power spectra is referred to as the spectrum management problem. The objective is to find the optimal transmit spectra for a bundle of interfering DSL lines, maximizing the bit rate of one line subject to bit rate constraints, total power constraints and spectral mask constraints. This can be formulated as follows

$$\begin{array}{lll} \max_{\mathbf{s}^1,\ldots,\mathbf{s}^N} R^1 & \text{s.t.} & R^n \geq R^{n,\text{target}}, \ \forall n > 1, \\ & \text{s.t.} & \sum_k s_k^n \leq P^{n,\text{tot}}, \ \forall n, \\ & \text{s.t.} & 0 \leq s_k^n \leq s_k^{n,\text{mask}}, \ \forall n, \forall k, \end{array} \quad (2)$$

where $R^{n,\text{target}}$ denotes the target bit rate for user n, $P^{n,\text{tot}}$ denotes the total power budget for user n and $s_k^{n,\text{mask}}$ denotes the spectral mask for user n on tone k. Note that the total power constraints and target bit rate constraints couple the optimization problem over the tones. The objective function is coupled over the users. This results in a solution set with a dimensionality that is exponential in the number of users and tones, namely $\mathcal{O}(B^{NK})$ where B is the number of possibilities for the bit or power loading for each tone and each user.

3.2. Optimal Spectrum Balancing

In [1] [2] it was shown that the optimal spectrum balancing (OSB) algorithm reduces this dimensionality by a dual decomposition. Using Lagrange multipliers, the constraints causing the coupling over the tones are moved into the cost function:

$$\begin{array}{rl} \mathbf{s}^{1,opt}, \ldots, \mathbf{s}^{N,opt} = & \operatorname{argmax}_{\mathbf{s}^1,\ldots,\mathbf{s}^N} \sum_{n=1}^N \omega_n R^n \\ & + \sum_{n=1}^N \lambda_n \left(P^{n,\text{tot}} - \sum_{k=1}^K s_k^n\right) \end{array} \quad (3)$$

$$\begin{array}{lll} \text{with} & 0 \leq s_k^n \leq s_k^{n,\text{mask}} & , \ \forall n, \forall k, \\ & \lambda_n \geq 0, \omega_n \geq 0 & , \ \forall n. \end{array}$$

In [2] [7] efficient search algorithms were presented to identify the Lagrange multipliers λ_n, ω_n that enforce the constraints. For given Lagrange multipliers, optimization problem (3) is decoupled over the tones, resulting in K independent problems of dimension $\mathcal{O}(B^N)$. Unfortunately these optimization problems are themselves difficult nonconvex problems. For given Lagrange multipliers λ_n, ω_n, each per-tone optimization problem can be formulated as

$$\begin{array}{c} s_k^{1,opt}, \ldots, s_k^{N,opt} = \operatorname{argmin}_{s_k^1,\ldots,s_k^N} -\sum_{n=1}^N \omega_n f_s b_k^n + \sum_{n=1}^N \lambda_n s_k^n \\ \text{subject to} \quad 0 \leq s_k^n \leq s_k^{n,\text{mask}} \quad n = 1 \ldots N. \end{array} \quad (4)$$

Note that the sign of the objective function is changed and the maximization is changed into a minimization for convenience.
The originally proposed method to solve this per-tone optimization problem was an exhaustive search [1]. As the dimensionality of the solution set is still exponential in the number of users, this becomes computationally intractable for more than five users.
In [3] a branch and bound algorithm was presented to reduce this complexity significantly without sacrificing optimality. In spite of this complexity reduction the algorithm still has a huge computational complexity (e.g. one week for computing a seven-user scenario). Unfortunately, binders typically consist of 20-100 users. Therefore there is a strong need for low-complexity spectrum balancing methods still producing optimal transmit power spectra.

4. LOW-COMPLEXITY DISTRIBUTED SPECTRUM BALANCING ALGORITHM

In this section a novel low-complexity spectrum balancing algorithm is presented. Its performance is similar to the optimal branch and bound algorithm [3] but it reduces the simulation time, e.g. from one week to a few seconds for a seven-user scenario. Moreover it is explained how this algorithm can be executed in a distributed fashion by the use of a Spectrum Management Center (SMC) and limited message-passing. A similar idea has been proposed recently [8] based on a relaxation of the nonconvex per-tone optimization problem (4). Our approach is based on a different convex relaxation, leading to a more direct and conceptually simple procedure.

The derivation of our algorithm starts with rewriting the objective function of (4) in the following form using (1)

$$\begin{array}{l} L = \underbrace{-\sum_{n=1}^N \omega_n f_s \log_2\left(\sum_{m=1}^N |\tilde{h}_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n\right) + \lambda_n s_k^n}_{A} \\ \quad \underbrace{+\sum_{n=1}^N \omega_n f_s \log_2\left(\sum_{m\neq n} |\tilde{h}_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n\right)}_{B} \end{array} \quad (5)$$

$$\text{with } \tilde{h}_k^{n,m} \begin{cases} = \Gamma h_k^{n,m} & \text{, if } n \neq m \\ = h_k^{n,m} & \text{, if } n = m \end{cases}$$

which consists of a convex part A and a concave part B. This objective function is a difference of convex (d.c.) functions which is known to correspond to a hard optimization problem [9]. The crucial step is now to relax the nonconvex part B by hyperplane overestimators, leading to the following relaxed objective

$$\begin{array}{l} L_{rel} = A + \sum_{n=1}^N \omega_n f_s \sum_{m\neq n} a_k^{m,n} s_k^m + c_k^n \\ \text{where } \sum_{m\neq n} a_k^{m,n} s_k^m + c_k^n \\ \qquad \geq \log_2\left(\sum_{m\neq n} |\tilde{h}_k^{n,m}|^2 s_k^m + \sigma_k^n\right) \quad , \forall n \\ \text{with equality in the approximation point } \mathbf{s}_k(\text{ap}). \end{array} \quad (6)$$

The overestimators are readily obtained by solving a linear system of N equations in N unknowns. The obtained relaxed objective function is a convex function. The constraints are also convex leading to a convex optimization problem which can be solved efficiently. The solution of this convex relaxation forms an upper bound for the global minimum. Using the obtained upper bound as a new point of approximation (see algorithm 1) it can be proven that the sequence of relaxations produces a monotonically decreasing objective value and will always converge. The proof is trivial and omitted due to space limitations. Upon convergence it can be proven that the obtained solution is a local optimum. Although there is no theoretical proof for global optimality, simulation results are very promising showing global optimality for very different multi-user scenarios. A final remark on algorithm 1 is that the convex relaxed problem does not need to be fully minimized, an improved objective value is sufficient.

Algorithm 1 Iterative linear approximation approach (tone k)

choose initial approx. point $\mathbf{s}_k(\text{ap})$;
repeat
  calculate approx. at $\mathbf{s}_k(\text{ap})$: $a_k^{n,m}, c_k^n, \forall n, \forall m \neq n$;
  $\mathbf{s}_k$ = solve convex relaxed problem with objective L_{rel} (6);
  $\mathbf{s}_k(\text{ap}) = \mathbf{s}_k$;
until convergence

As an alternative to solving the relaxed convex problem with objective L_{rel} (6) by means of standard convex software, we also propose a distributed solution. For given Lagrange multipliers $\lambda_1, \ldots, \lambda_n$, (6) can be solved by finding its stationary points, where

$$\frac{\partial L_{rel}}{\partial s_k^n} = \sum_m \frac{\omega_m f_s |\tilde{h}_k^{m,n}|^2 / \ln(2)}{\sum_p |\tilde{h}_k^{m,p}|^2 s_k^p + \Gamma\sigma_k^m} - \lambda_n - \sum_{m\neq n} a_k^{n,m} = 0. \quad (7)$$

This then leads to the following fixed point equation

$$s_k^n = \left(\frac{\omega_n f_s / \ln(2)}{\underbrace{\lambda_n + \sum_{m\neq n} a_k^{n,m} - \sum_{m\neq n} \frac{\omega_m f_s |\tilde{h}_k^{m,n}|^2 / \ln(2)}{\sum_p |\tilde{h}_k^{m,p}|^2 s_k^p + \Gamma\sigma_k^m}}_{D_k^n}} \right) - \frac{\sum_{m\neq n} |\tilde{h}_k^{n,m}|^2 s_k^m + \Gamma\sigma_k^n}{|\tilde{h}_k^{n,n}|^2} \triangleq g_k^n(s_k^n). \quad (8)$$

By iteratively updating the transmit powers s_k^n using (8) i.e. $[s_k^n(t+1) = g_k^n(s_k^n)]$ where t is the iteration number, convergence to the stationary point can be achieved. The reason is that the derivative of $g_k^n(s_k^n)$ is typically much smaller than one for all points $\mathbf{s}_k^n$. In order to keep within the spectral mask constraints the spectra have to be bounded, which leads to the following updates

$$s_k^n(t+1) = \max(0, \min(g_k^n(s_k^n(t)), s_k^{n,\text{mask}})). \quad (9)$$

In [2] [7] efficient update formulas are presented to search for the Lagrange multipliers λ_n, ω_n that enforce the constraints. These formulas are in the following gradient descent form

$$\lambda_n(t+1) = \left[\lambda_n(t) - \mu(P^{n,\text{tot}} - \sum_k s_k^n)\right]^+ \quad (10)$$

where t is the iteration number and μ is a step size parameter [2]. Because all the variables of formula (10) are locally known for each user n, the update of the Lagrange multipliers λ_n can be done locally. A final remark is that formula (9) again does not need to be fully optimized before the Lagrange multipliers can be updated.
In order to perform the updates each user needs to have information D_k^n of formula (8). The SMC can construct and deliver this information based on messages $M_k^n (= \sum_{m\neq n} |\tilde{h}_k^{n,m}|^2 + \Gamma\sigma_n)$ and s_k^n transmitted by the users to the SMC. Moreover it is assumed that the SMC has full channel knowledge which is a reasonable assumption. This leads to the message-passing system described in algorithm 2 in order to execute the spectrum balancing method in a distributed way. This basic system is adopted from [8], but now based on formulae (8)-(9).

5. SIMULATION RESULTS

This section presents simulation results of the proposed distributed spectrum balancing algorithm. Its performance and complexity are compared to OSB algorithms.
The ADSL Downstream (DS) scenario is shown in figure 1. The simulations are performed for a two-user case ($N = 2$) up to a seven-user case ($N = 7$). The four-user scenario, for example, consists of active modems 1,2,3,4 where modems 5,6,7 are inactive. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum transmit power is 20.4 dBm [10]. The SNR gap Γ is 12.9 dB, corresponding to a coding gain of 3 dB, a noise margin of 6 dB and a target symbol error probability of 10^{-7}. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. The simulations are performed in Matlab on a dual Opteron 250 with 4 GB RAM and a 2.4 GHz processor. Figure 2 shows the resulting bit loadings for the four-user scenario. The discrete curves are the resulting bit loadings of the four users for the optimal branch and bound algorithm [3]. The continuous curves are the resulting bit loadings of the four users for our novel approach. It can be seen that these curves are similar. In fact because of the continuous character of our approach, the obtained performance is even better than the performance of the optimal discrete branch and bound solution. Table 1 shows the simulation times for the scenarios with up to seven users. For the four-user case it can be seen that an exhaustive search would require 8 hours

Algorithm 2 Distributed message-passing protocol

User n algorithm:
loop
  Receive message D_k^n from SMC
  repeat
    Update s_k^n using (9)
    Update λ_n using (10)
  until total power constraints satisfied
  Transmit M_k^n,s_k^n to SMC
end loop
SMC algorithm:
loop
  Receive messages M_k^n,s_k^n from users
  Calculate messages D_k^n and send to each user n.
end loop

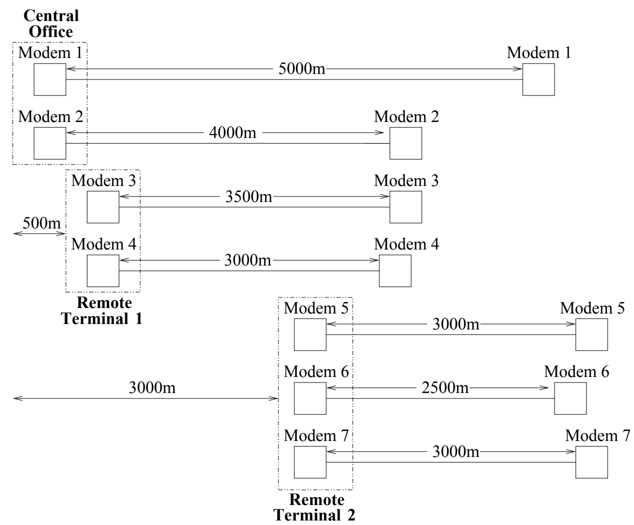

Fig. 1. Asymmetric N-user ADSL scenario

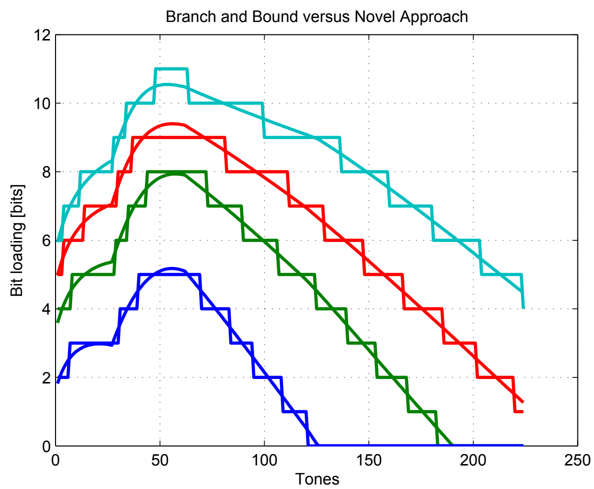

Fig. 2. Downstream bit loadings 4-user scenario: Branch and bound (discrete) versus novel approach (continuous)

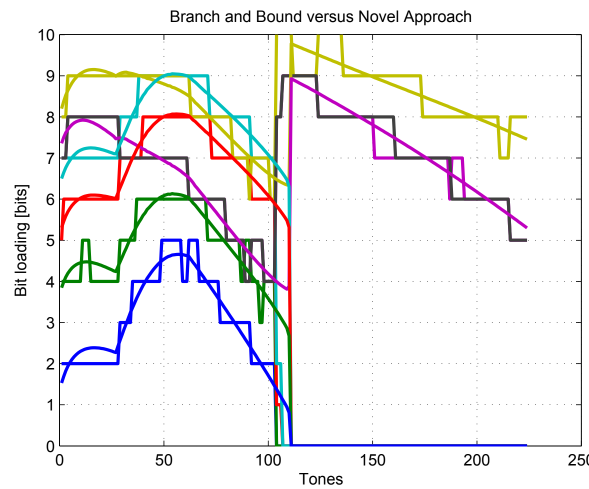

Fig. 3. Downstream bit loadings 7-user scenario: Branch and bound (discrete) versus novel approach (continuous)

Table 1. Comparison of simulation times for different spectrum balancing algorithms executed on the same platform

Users	Exhaust. Search	Branch & Bound	Our Approach
2	100 s	30 s	0.06 s
3	30 min	3 min	0.08 s
4	8 h	20 min	0.21 s
5	6 d	3 h	1.16 s
6	110 d	1 d	8.78 s
7	4.85 y	7 d	10.50 s

of simulation time. Using the branch and bound this is reduced to 20 minutes whereas it only requires 0.215 seconds to calculate the transmit spectra with the novel approach.

Figure 3 shows the resulting bit loadings for the seven-user scenario. The resulting bit loadings are similar for the optimal discrete branch and bound solution. Table 1 shows enormous complexity reductions for this seven-user scenario. An exhaustive search would require 4.85 years of simulation time. The branch and bound approach would require 1 week whereas the novel proposed method only requires 10.5 seconds with similar resulting bit loadings.

6. CONCLUSION

In this paper a novel low-complexity spectrum balancing algorithm is presented. The algorithm is based on a relaxation of the non-convex per-tone optimization problem obtained with the OSB procedure. By the use of a Spectrum Management Center and limited message-passing it is shown that the algorithm can be executed in a distributed fashion, which is a great asset in current DSL networks. Its performance is compared to OSB algorithms for scenarios with up to seven users and it is seen to yield similar results. The simulation times are reduced, e.g. from a week down to only a few seconds for a seven-user scenario.

7. REFERENCES

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