PEP-BOUND ROTATION ANGLE OPTIMIZATION OF 8-TRANSMIT-ANTENNA QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES[†]

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ABSTRACT

We derive a new expression for the rotation angles that maximize the diversity (eigenvalue) product of the 8×8 two-symbol decodable quasi-orthogonal space-time block code (QO-STBC). We show that the previously proposed sum-eigenvalue maximization criterion for the design of rotation angles is not relevant/applicable to the 8-transmit-antenna QO-STBCs and we suggest, instead, minimum eigenvalue maximization. Finally, working directly with the pairwise-error-probability (PEP) upper bound expression, we obtain new true PEP-upper-bound optimal rotation angles. Simulation studies demonstrate and compare the error rate of the three design criteria (diversity product, minimum eigenvalue, PEP upper bound).

Index Terms— Constellation rotation, diversity product, pairwise-error-probability (PEP), quasi-orthogonal space-time block codes (QO-STBC), sum or minimum eigenvalue maximization.

1. INTRODUCTION

Orthogonal space-time block codes (O-STBC) [1] achieve full transmit diversity and allow single-symbol (two real symbols) maximum likelihood (ML) decoding. The drawback of O-STBCs is that full-rate codewords do not exist for more than two transmit antennas. For the case of four transmit antennas, the rate limitation of O-STBCs was overcome by quasi-orthogonal (QO) STBCs at the expense of diversity loss [2]-[4]. Full-rate full-diversity quasi-orthogonal codewords for four transmit antennas were then formed in [5], [6] by retaining the code structure of [2], [3] and modifying the constellation of some of the symbols. ML decoding of the QO-STBCs in [5], [6] requires joint detection of two symbols (four real symbols). Interleaving real and imaginary parts of different symbols enables single-symbol decoding of full-rank, full-diversity QO-STBCs for the four-transmit-antenna case [7], [8], at the expense of some performance loss in comparison with joint two-symbol dectection [5].

The codewords in [1]-[8] partition the symbols into orthogonal sets and ML detection requires only joint decoding of the symbols in each orthogonal set independently. As the complexity of the ML decoder increases exponentially with the number of symbols in each orthogonal set, a trade-off between rate/diversity and decoding requirements is taking shape, especially for large number of transmit antennas. In [9], for 8 transmit antennas QO-STBCs that attain full diversity and full rate were presented that require, however, joint detection of four symbols (eight real symbols). Reduction in complexity was achieved for the 8-antenna case through the process of interleaving the real and imaginary parts of different symbols [10], [11]; the codewords can be partitioned into four orthogonal sets and hence require joint two-symbol decoding only.

All QO-STBCs discussed above achieve full diversity order by employing constellation rotation (CR) that aims to maximize the diversity product. This, in turn, leads to minimization of the upper bound on pairwise-error-probabilities (PEP) at (asymptotically) high signal-to-noise ratios (SNR) [12]. For space-time codes with large diversity order and/or large number of transmit antennas, diversity product maximization may not provide satisfactory PEP bound minimization and error-rate performance over operable SNRs [13].

In this paper, we consider specifically 8-transmit-antenna, full-rate QO-STBCs and analyze their optimization via constellation rotation. We limit our studies to two-symbol decodable 8×8 codes and provide a new, alternative codeword form to the one in [10] to aid our analysis. We examine four different rotation angle optimization criteria: (i) We find a new expression for the rotation angles that maximize the diversity product of the suggested codeword; (ii) we show that sum-eigenvalue maximization as proposed in [13] is irrelevant/non-applicable to the 8-transmit-antenna QO-STBCs and (iii) suggest instead (and solve) minimum-eigenvalue maximization; and, finally, (iv) we use directly the PEP upper bound to obtain new true PEP-upper-bound optimal rotation angles.

The rest of the paper is organized as follows. In Section 2 we present the alternative code structure for the 8×8 QO-STBC. Rotation-angle design criteria are discussed and analyzed in Section 3. Section 4 presents simulation results that demonstrate the error-rate performance of CR modified versions of the codeword according to the examined criteria. A few concluding remarks are drawn in Section 5.

2. CODE STRUCTURE

Let N_t be the number of transmit antennas, N_r the number of receive antennas, and T the number of time slots over which the code is transmitted. We denote the number of transmitted symbols by K. Herein, we are interested in QO-STBCs with $N_t = T = K = 8$. The symbols $a_k,\ k = 1, \ldots, K$, to be transmitted are formed by mapping the incoming bits onto known constellations, e.g. quadrature-amplitude-modulated (QAM), while their corresponding constellation rotated version $\bar{a}_k,\ k = 1, \ldots, K$, is created by

$$\begin{aligned} \bar{a}_m &= (a_{mR} + i a_{mI}) e^{i\phi},\ m = 1, 2, 5, 6, \\ \bar{a}_n &= (a_{nR} + i a_{nI}) e^{i\theta},\ n = 3, 4, 7, 8, \end{aligned} \tag{1}$$

where a_{kR} and a_{kI} denote the real and imaginary part of the symbol a_k, respectively, and $\phi,\ \theta$, are the rotation angles to be optimized. The symbols $\bar{a}_k$ are interleaved to form $x_k,\ k = 1, \ldots, K$,

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$$
\begin{aligned}
x_1 &= \bar{a}_{1R} + i\bar{a}_{5I}, \quad & x_2 &= \bar{a}_{2R} + i\bar{a}_{6I},\\
x_3 &= \bar{a}_{3R} + i\bar{a}_{7I}, \quad & x_4 &= \bar{a}_{4R} + i\bar{a}_{8I},\\
x_5 &= \bar{a}_{5R} + i\bar{a}_{1I}, \quad & x_6 &= \bar{a}_{6R} + i\bar{a}_{2I},\\
x_7 &= \bar{a}_{7R} + i\bar{a}_{3I}, \quad & x_8 &= \bar{a}_{8R} + i\bar{a}_{4I}.
\end{aligned} \tag{2}
$$

We propose the following new form for the transmitted codeword $\mathbf{X}$:

$$
\mathbf{X} = \begin{bmatrix}
x_1 & x_2 & x_3 & x_4 & & & & \\
-x_2^* & x_1^* & -x_4^* & x_3^* & & \mathbf{0}_{4\times4} & & \\
x_3 & x_4 & x_1 & x_2 & & & & \\
-x_4^* & x_3^* & -x_2^* & x_1^* & & & & \\
 & & & & x_5 & x_6 & x_7 & x_8 \\
 & \mathbf{0}_{4\times4} & & & -x_6^* & x_5^* & -x_8^* & x_7^* \\
 & & & & x_7 & x_8 & x_5 & x_6 \\
 & & & & -x_8^* & x_7^* & -x_6^* & x_5^*
\end{bmatrix}. \tag{3}
$$

Upon reception, joint decoding of the symbol pairs $\{(a_1, a_3), (a_2, a_4), (a_5, a_7), (a_6, a_8)\}$ is ML optimum.

It can be shown that our suggested code structure in (3) is equivalent to the one in [10]. Mathematically, the algebraic simplicity of (3) will enable our theoretical CR analysis that follows. Practically, an intriguing aspect of (3) is that (unlike the codewords in [10], [11]) the codeword may be applied to a 4-transmit-antenna system across eight time slots without any loss in rate (or diversity order under a four-slot block fading assumption).

3. OPTIMIZATION OF ROTATION ANGLES

The probability of receiving the codeword $\widetilde{\mathbf{X}}$ when $\mathbf{X} \neq \widetilde{\mathbf{X}}$ is transmitted is upper bounded by [12]

$$
Pr(\mathbf{X} \rightarrow \widetilde{\mathbf{X}}) \leq \frac{1}{2}\left(\prod_{i=1}^{R}\left\{\frac{1}{1+\frac{A\lambda_i}{4N_t}}\right\}\right)^{N_r} \tag{4}
$$

where A is the received signal energy, R is the rank of $(\mathbf{X} - \widetilde{\mathbf{X}})$, and $\lambda_i,\ i = 1, \ldots, R$, are the eigenvalues of $(\mathbf{X} - \widetilde{\mathbf{X}})^H(\mathbf{X} - \widetilde{\mathbf{X}})$. The aim is to obtain rotation angles ϕ, θ in (1) which minimize (4) over all codeword pairs. To proceed, we require the eigenvalues λ_i of $(\mathbf{X}-\widetilde{\mathbf{X}})^H(\mathbf{X}-\widetilde{\mathbf{X}})$. Set $\Delta\mathbf{X} \triangleq \mathbf{X}-\widetilde{\mathbf{X}}$ and $\Delta\mathbf{x}_k \triangleq \mathbf{x}_k - \widetilde{\mathbf{x}}_k$, $k = 1, \ldots, 8$; then,

$$
\Delta\mathbf{X}^H\Delta\mathbf{X} = \begin{bmatrix}
\Delta a\mathbf{I}_2 & \Delta b\mathbf{I}_2 & \multicolumn{2}{c}{\mathbf{0}} \\
\Delta b\mathbf{I}_2 & \Delta a\mathbf{I}_2 & & \\
\multicolumn{2}{c}{\mathbf{0}} & \Delta c\mathbf{I}_2 & \Delta d\mathbf{I}_2 \\
 & & \Delta d\mathbf{I}_2 & \Delta c\mathbf{I}_2
\end{bmatrix} \tag{5}
$$

where

$$
\begin{aligned}
\Delta a &\triangleq \textstyle\sum_{k=1}^{K} |\Delta x_k|^2,\\
\Delta b &\triangleq \textstyle\sum_{k=1}^{K/2} 2Re\{\Delta x_k^* \Delta x_{k+K/2}\},\\
\Delta c &\triangleq \textstyle\sum_{k=K+1}^{2K} |\Delta x_k|^2,\\
\Delta d &\triangleq \textstyle\sum_{k=K+1}^{K+K/2} 2Re\{\Delta x_k^* \Delta x_{k+K/2}\}, \quad K = 4.
\end{aligned} \tag{6}
$$

From (5) we observe that the eigenvalues of $\Delta\mathbf{X}^H\Delta\mathbf{X}$ are $\{(\Delta a - \Delta b), (\Delta a + \Delta b), (\Delta c - \Delta d), (\Delta c + \Delta d)\}$ and exist with multiplicity of two. Expanding and simplifying, the eigenvalues are as shown in (7) on top of the following page. As each eigenvalue is a summation of squares, the set of minimum eigenvalues over all possible codewords pairs is

$$
\begin{aligned}
\big\{&(\Delta\bar{a}_{1R} - \Delta\bar{a}_{3R})^2, (\Delta\bar{a}_{1R} + \Delta\bar{a}_{3R})^2,\\
&(\Delta\bar{a}_{1I} - \Delta\bar{a}_{3I})^2, (\Delta\bar{a}_{1I} + \Delta\bar{a}_{3I})^2\big\}
\end{aligned} \tag{8}
$$

and represents the worst case scenario for the upper bound in (4). As long as all eigenvalues in (8) are non-zero, full diversity order is achieved.

We now consider four different criteria to optimize the rotation angles and show how ϕ, θ can be obtained for each criterion using (8).

3.1. Diversity product maximization

At high SNR values assuming full transmit diversity and $N_r = 1$, (4) can be *approximated* by

$$
Pr(\mathbf{X} \rightarrow \widetilde{\mathbf{X}}) \leq \frac{1}{2}\left(\prod_{i=1}^{N_t}\lambda_i\right)^{-1}\left(-\frac{A}{4N_t}\right)^{-N_t}. \tag{9}
$$

Worst-case minimization of the bound in (9) is equivalent to maximization of the minimum product of the eigenvalues (determinant of $\Delta\mathbf{X}^H\Delta\mathbf{X}$) over all possible codeword pairs, which in turn is commonly represented by the diversity product ζ,

$$
\zeta = \frac{1}{2\sqrt{N_t}} min_{\mathbf{X}\neq\tilde{\mathbf{X}}} \left|det\left[\Delta\mathbf{X}\Delta\mathbf{X}^H\right]\right|^{1/(2T)}. \tag{10}
$$

Diversity product maximization was used as the rotation angle design criterion in [10], [11]. For the codeword in (3), the minimum determinant of $\Delta\mathbf{X}^H\Delta\mathbf{X}$ over all codeword pairs is

$$
\begin{aligned}
&\min\,(det(\Delta\mathbf{X}^H\Delta\mathbf{X})) = \left[(\Delta\bar{a}_{1R}^2 - \Delta\bar{a}_{3R}^2)(\Delta\bar{a}_{1I}^2 - \Delta\bar{a}_{3I}^2)\right]^4\\
&\quad = [(\Delta a_{1R}cos(\phi) - \Delta a_{1I}sin(\phi))^2 - (\Delta a_{3R}cos(\theta) - \Delta a_{3I}sin(\theta))^2]^4\\
&\cdot [(\Delta a_{1R}sin(\phi) + \Delta a_{1I}cos(\phi))^2 - (\Delta a_{3R}sin(\theta) + \Delta a_{3I}cos(\theta))^2]^4
\end{aligned} \tag{11}
$$

and ϕ, θ should be chosen to maximize (11). We evaluate and note that the codeword in (3) (and the proposed codewords in [10], [11]) achieve diversity product of 0.1735 with 4-QAM constellation and 0.1030 with the non-rectangular 8-QAM constellation for the 8-transmit-antenna case.

3.2. Minimum sum-of-eigenvalues maximization

In [13], it was proposed that for high diversity order systems (number of transmit antennas greater than four) the minimum trace of $\Delta\mathbf{X}^H\Delta\mathbf{X}$ be maximized over all codeword pairs. From our expression (7), we observe that

$$
\begin{aligned}
\min\,(Tr(\Delta\mathbf{X}^H\Delta\mathbf{X})) &= 2[(\Delta\bar{a}_{1R} - \Delta\bar{a}_{3R})^2 + (\Delta\bar{a}_{1R} + \Delta\bar{a}_{3R})^2\\
&\quad + (\Delta\bar{a}_{1I} - \Delta\bar{a}_{3I})^2 + (\Delta\bar{a}_{1I} + \Delta\bar{a}_{3I})^2]\\
&= 4[\Delta\bar{a}_{1R}^2 + \Delta\bar{a}_{3R}^2 + \Delta\bar{a}_{1I}^2 + \Delta\bar{a}_{3I}^2]\\
&= 4[\Delta a_{1R}^2 + \Delta a_{3R}^2 + \Delta a_{1I}^2 + \Delta a_{3I}^2]
\end{aligned} \tag{12}
$$

and is independent of ϕ, θ.

Hence, for the proposed codeword and that of [10], [11], the sum-of-eigenvalues criterion in [13] is not relevant.

3.3. Minimum eigenvalue maximization

If $r < N_t$ eigenvalues of $\Delta\mathbf{X}^H\Delta\mathbf{X}$ are significantly less than 1, then even for large SNR values, $1 + \frac{A\lambda_i}{4N_t} \simeq 1$ and (4) is *approximated* by

$$
Pr(\mathbf{X} \rightarrow \widetilde{\mathbf{X}}) \leq \frac{1}{2}\left(\prod_{i=1}^{N_t-r}\lambda_i\right)^{-1}\left(-\frac{A}{4N_t}\right)^{-(N_t-r)}. \tag{13}
$$

$$
\begin{aligned}
\{ \; & (\Delta\bar{a}_{1R}-\Delta\bar{a}_{3R})^2+(\Delta\bar{a}_{2R}-\Delta\bar{a}_{4R})^2+(\Delta\bar{a}_{5I}-\Delta\bar{a}_{7I})^2+(\Delta\bar{a}_{6I}-\Delta\bar{a}_{8I})^2, \\
& (\Delta\bar{a}_{1R}+\Delta\bar{a}_{3R})^2+(\Delta\bar{a}_{2R}+\Delta\bar{a}_{4R})^2+(\Delta\bar{a}_{5I}+\Delta\bar{a}_{7I})^2+(\Delta\bar{a}_{6I}+\Delta\bar{a}_{8I})^2, \\
& (\Delta\bar{a}_{5R}-\Delta\bar{a}_{7R})^2+(\Delta\bar{a}_{6R}-\Delta\bar{a}_{8R})^2+(\Delta\bar{a}_{1I}-\Delta\bar{a}_{3I})^2+(\Delta\bar{a}_{2I}-\Delta\bar{a}_{4I})^2, \\
& (\Delta\bar{a}_{5R}+\Delta\bar{a}_{7R})^2+(\Delta\bar{a}_{6R}+\Delta\bar{a}_{8R})^2+(\Delta\bar{a}_{1I}+\Delta\bar{a}_{3I})^2+(\Delta\bar{a}_{2I}+\Delta\bar{a}_{4I})^2 \; \} .
\end{aligned} \tag{7}
$$

The system seems to lose diversity; for the codeword in (3) and the codewords in [10], [11], the occurrence of the eigenvalues in pairs causes loss of diversity in steps of two. In such circumstances, it appears reasonable to consider rotation angle choices that maximize the minimum possible eigenvalue over all pairs of codewords. For our code structure in (3), the rotation angles that maximize the minimum eigenvalue are

$$
\begin{aligned}
&(\phi,\theta) = \underset{\phi,\theta}{argmax} \min \left\{ (\Delta\bar{a}_{1R}-\Delta\bar{a}_{3R})^2, \right. \\
&\left. (\Delta\bar{a}_{1R}+\Delta\bar{a}_{3R})^2, (\Delta\bar{a}_{1I}-\Delta\bar{a}_{3I})^2, (\Delta\bar{a}_{1I}+\Delta\bar{a}_{3I})^2 \right\} .
\end{aligned} \tag{14}
$$

3.4. PEP-bound Minimization

We now prove that for all STBCs that employ CR and have no more than two unique eigenvalues of $\Delta\mathbf{X}^H\Delta\mathbf{X}$ over all possible codeword pairs, maximization of the diversity product is equivalent to minimization of the upper bound on PEP for all SNRs. Minimization of the bound in (4) is equivalent to maximizing $\prod_{i=1}^{R}\left\{1+\frac{A\lambda_i}{4N_t}\right\}$. If λ_1 and λ_2 represent the 2 unique eigenvalues of $\Delta\mathbf{X}^H\Delta\mathbf{X}$, then

$$
\begin{aligned}
&argmax \prod_{i=1}^{R}\left(1+\frac{A\lambda_i}{4N_t}\right) = argmax \left[\left(1+\frac{A\lambda_1}{4N_t}\right)\left(1+\frac{A\lambda_2}{4N_t}\right)\right]^p \\
&= argmax \left[1+\frac{A}{4N_t}(\lambda_1+\lambda_2)+\frac{A^2}{16N_t^2}(\lambda_1\lambda_2)\right]^p
\end{aligned} \tag{15}
$$

where p denotes the multiplicity of the eigenvalues. Since $\frac{1}{p}(\lambda_1+\lambda_2) = tr(\Delta\mathbf{X}^H\Delta\mathbf{X}) = \|\Delta\mathbf{X}\|_F^2$ is independent of rotation angles[1] we need to maximize only the product of the eigenvalues to minimize the bound in (4). In the case of a single unique eigenvalue (as in O-STBCs for example), the STBC is independent of the rotation angle.

While the case of two unique eigenvalues applies to the 4×4 QO-STBC codes proposed in [5], [7], [8] and their choice of rotation angle is PEP-bound optimal, for the 8×8 codewords four unique eigenvalues exist and maximizing the eigenvalue (diversity) product over all possible codeword pairs does not necessarily minimize the maximum bound in (4).

We now directly find the rotation angles that minimize the maximum (worst case) PEP upper bound. Substitution of (8) in (4) gives us the worst case scenario for all codeword pairs. We need to optimize ϕ,θ such that

$$
\begin{aligned}
&(\phi,\theta) = \underset{\phi,\theta}{argmax} \prod_{i=1}^{4}\left[\left(1+\frac{A\lambda_i}{4N_t}\right)\right]^2, \\
&\lambda_i = \left\{(\Delta\bar{a}_{1R}-\Delta\bar{a}_{3R})^2, (\Delta\bar{a}_{1R}+\Delta\bar{a}_{3R})^2, \right. \\
&\qquad \left. (\Delta\bar{a}_{1I}-\Delta\bar{a}_{3I})^2, (\Delta\bar{a}_{1I}+\Delta\bar{a}_{3I})^2\right\}, \;\; i=1,2,3,4.
\end{aligned} \tag{16}
$$

Suitable values for A are such that $A\lambda_i > 1 \; \forall i = 1,2,3,4$.

4. SIMULATION STUDIES

We now evaluate the performance of the QO-STBC in (3) under minimum eigenvalue CR optimization by (14), diversity product CR optimization by (11), and the proposed direct maximum PEP-bound CR optimization by (16). In Fig. 1, we plot the block-error-rate versus SNR when the symbols are chosen from a 4-QAM constellation. For direct PEP-bound optimization of ϕ,θ, we use a fixed energy value A that corresponds to received SNR of 20dB. We observe that the rotation angles that maximize the diversity product and the rotation angles that minimize the maximum PEP bound provide the best results with the latter having indeed better performance. The exact angle values are shown in Table I (along with the resulting diversity product and minimum eigenvalue).

In Fig. 2, we repeat the studies of Fig. 1 for symbols chosen from a 8-QAM non-rectangular constellation [5]. To obtain the PEP-bound optimal ϕ,θ values we fix A to the value that corresponds to received SNR of 30dB. Again all calculated values are given in Table I. For reference purposes, we include in our Fig. 2 comparisons the 8×8 single-symbol decodable STBC in [7]. Since the code in [7] contains only two unique eigenvalues the rotation angle of $tan^{-1}(1/2)$ is PEP-bound optimal for that code; since its rate is $3/4$, we select symbols from a 16-QAM constellation to ensure equal spectral efficiency for all codewords under comparison. The PEP-bound optimized codeword in (3) offers a gain of about 1 dB over the single-symbol decodable STBC in [7]. The minimum eigenvalue optimized version performs almost similarly well. As argued in Section 3.3, due to decreased values of the minimum eigenvalues as compared to the 4-QAM scenario, the maximum diversity (eigenvalue) product optimized system seems to lose diversity over the operable SNR range. Similar performance loss was also observed in [10] when the rotation angles were chosen to maximize the diversity product.

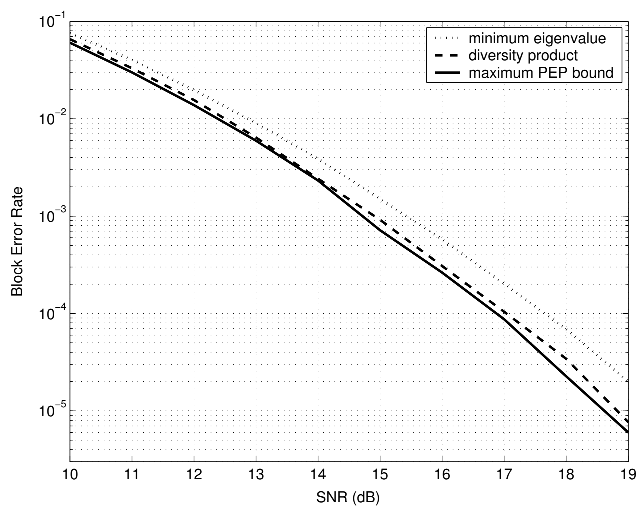

Fig. 1. Block-error-rate versus SNR for 4-QAM constellation.

5. CONCLUSIONS

We gave an alternative representation of the 8×8 two-symbol decodable quasi-orthogonal space-time block code (QO-STBC) and found

[1]Constellation rotation or interleaving does not change the transmitted energy of the STBC codeword.

TABLE I
OPTIMAL ANGLES

QAM	Criterion	(ϕ, θ)	Diversity Product	Min. Eigenvalue
4	Diversity Product	$(37.9, 21.4)$	0.1747	0.0093
4	Min. Eigenvalue	$(30.9, 13.3)$	0.1623	0.0524
4	Max. PEP Bound	$(28.5, 40)$	0.1352	0.0112
8	Diversity Product	$(tan^{-1}(2)/2, tan^{-1}(1/2)/2)$	0.1071	2.35×10^{-4}
8	Min. Eigenvalue	$(3, tan^{-1}(2))$	0.0732	0.0022
8	Max. PEP Bound	$(7.2, 25.1)$	0.0792	2.7×10^{-4}

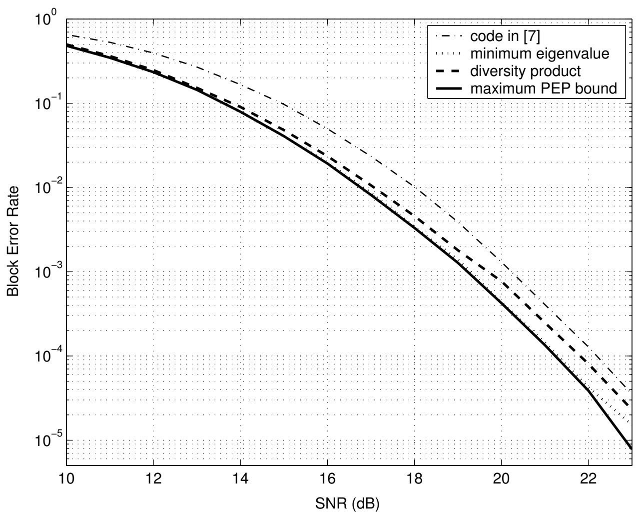

Fig. 2. Block-error-rate versus SNR for non-rectangular 8-QAM constellation (16-QAM for the rate $3/4$ code in [7]).

three different sets of rotation angle values: maximum diversity-product optimal, maximum minimum-eigenvalue optimal, and minimum maximum-pairwise-error-probability-bound (PEP) optimal. As a side result, we showed that maximization of the sum of eigenvalues is an irrelevant/non-applicable criterion for the 8-transmit-antenna QO-STBCs.

The rotation angle design criterion that this work is eventually promoting is the direct minimization of the maximum value of the PEP upper bound. As we showed for any STBC, diversity product maximization succeeds in minimizing the maximum PEP-upper-bound value only if two unique eigenvalues alone appear over all codeword pairs (not the case of course, for the examined 8×8 QO-STBC). For larger symbol constellations with decreased Euclidean distance, minimum eigenvalue maximization may be an effective simple alternative.

6. REFERENCES

[1] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, May 1999.

[2] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal non-orthogonality rate 1 space-time block code for 3+ Tx antennas," in *Proc. IEEE 6th Int. Symp. Spread-Spectrum Techniques and Applications (ISSSTA 2000)*, Parsippany, NJ, Sept. 2000, pp. 429-432.

[3] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, pp. 1-4, Jan. 2001.

[4] C. B. Papadias and G. J. Foschini, "Capacity-approaching space-time codes for systems employing four transmitter antennas," *IEEE Trans. Inform. Theory*, vol. 49, pp. 726-732, Mar. 2003.

[5] W. Su and X. G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inform. Theory*, vol. 50, pp. 2331-2347, Oct. 2004.

[6] N. Sharma and C. B. Papadias, "Improved quasi-orthogonal codes through constellation rotation," *IEEE Trans. Commun.*, vol. 51, pp. 332-335, Mar. 2003.

[7] Z. A. Khan, B. S. Rajan, and M. H. Lee, "Rectangular co-ordinate interleaved orthogonal designs," in *Proc. IEEE GLOBECOM*, San Francisco, CA, Dec. 2003, vol. 4, pp. 2004-2009.

[8] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Quasi-orthogonal STBC with minimum decoding complexity," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2089-2094, Sept. 2005.

[9] N. Sharma and C. B. Papadias, "Full rate full diversity linear quasi-orthogonal space-time codes for any transmit antennas," *EURASIP Journal on Applied Signal Proc.*, pp. 1246-1256, 2004.

[10] Z. A. Khan, M. H. Lee and, B. S. Rajan, "A rate-one full-diversity quasi-orthogonal design for eight Tx antennas," *Internal Report, TR-PME-2002-15*, available at http://pal.ece.iisc.ernet.in/PAM/tech_rep02.html.

[11] C. Yuen, Y. L. Guan, and T. T. Tjhung, "A class of four-group quasi-orthogonal STBC achieving full rate and full diversity for any number of antennas," in *Proc. IEEE Intl. Symp. on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Berlin, Germany, Sept. 2005, vol. 3, pp. 1683-1687.

[12] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.

[13] J. Yuan, Z. Chen, B. Vucetic, and W. Firmanto, "Performance and design of space-time coding in fading channels," *IEEE Trans. Commun.*, vol. 51, pp. 1991-1996, Dec. 2003.