# LATTICE BASED LINEAR PRECODING FOR MIMO BLOCK CODES

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## ABSTRACT

Herein, the design of linear dispersion codes for block based multiple-input multiple-output communication systems is investigated. The receiver as well as the transmitter are assumed to have perfect knowledge of the channel, and the receiver is assumed to employ maximum likelihood detection. We propose to use linear precoding and lattice invariant operations to transform the channel matrix into a lattice with large coding gain. With appropriate approximations, it is shown that this corresponds to selecting lattices with good sphere packing properties. Lattice invariant transformations are then used to minimize the power consumption. An algorithm for this power minimization is presented along with a lower bound on the optimization. Numerical results indicate that there is a potential gain of several dB by using the method compared to channel inversion with adaptive bit loading.

***Index Terms—*** MIMO systems, Communication systems, Signal processing, Fading channels, Channel coding

## 1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems has received much attention in recent years due to the tremendous potential in increased transmission rate [1]. Multiple antenna systems can be combined with temporal block coding, e.g. orthogonal frequency-division multiplexing (OFDM), so that data is multiplexed over space, frequency and time. If the channel is not perfectly known at the transmitter, one typically considers block codes with good diversity properties to protect the data from deep fades. Design of space-time block codes (STBC) with good diversity properties has received much attention, see [2, 3, 4] with references.

In this paper, we assume that the channel is perfectly known at both the receiver and transmitter, hence the main objective is to increase coding gain (to protect data from additive noise) rather than diversity gain. This problem has also received much attention throughout the years, although we believe there are still some open questions especially when it comes to designing practical and efficient block-based signaling schemes. The optimal capacity achieving code was derived in [1]. Unfortunately, the optimal codebooks are infinitely large and virtually impossible to implement in practice. Instead the blocks usually consist of a linear combination of finite sized constellations such as M-QAM (quadrature amplitude modulation) or M-PSK (phase shift keying). In the context of block based MIMO transmission this is referred as linear dispersion codes (LDC) [5]. For completeness, an alternative to linear precoding is non-linear precoding, see [6] with references.

One common strategy is to use the transmitter side channel state information (TX-CSI) to diagonalize the channel into parallel non interfering sub-channels that can be resolved at the receiver using a linear detector. This is the case for OFDM block coding. The data rate in each sub-channel is adapted based on the signal to noise ratio (SNR), commonly using the gap approximation [7]. This diagonalizing strategy has been shown to be suboptimal for certain linear detection algorithms [8, 9], where the error rate is minimized by mixing the sub-streams using discrete fourier transform (DFT) rotations.

Here the precoding problem is analyzed from a different point of view. Instead of using low complexity linear receivers, we assume that the optimal maximum likelihood (ML) detector is employed [10]. An algorithm for designing close to optimal block codes is proposed, and again it is concluded that the diagonalizing precoder is sub-optimal. A bound that specifies the limits of the algorithm and that helps us in the code design is derived. Known results about lattice sphere packing will be used in the design of the code [11].

The set of complex valued $N$ by $M$ matrices is denoted $\mathbb{C}^{N\times M}$, and similarly for real valued matrices $\mathbb{R}^{N\times M}$. The set of integer vectors with dimension $N$ is denoted $\mathbb{Z}^N$. Expectation is denoted E[...]. The vectorization operator on matrices is denoted $\mathrm{vec}(\cdot)$, the determinant is $|\cdot|$, and $\otimes$ is the Kronecker product of matrices. The complex Gaussian distribution is denoted $CN(\cdot,\cdot)$.

## 2. SYSTEM MODEL

Consider a MIMO communication system with $N$ transmitting and $M$ receiving antennas over a flat fading channel. Similar to [5], it is assumed that the channel matrix, $\mathbf{H}$, is constant during at least $L$ channel uses. Using complex notation, the transmitted signal block is $\mathbf{C}\in\mathbb{C}^{N\times L}$, the channel matrix is $\mathbf{H}\in\mathbb{C}^{M\times N}$, and the additive noise block is $\mathbf{V}\in\mathbb{C}^{M\times L}$. The noise is assumed to be zero-mean, complex Gaussian. The received signal block $\mathbf{Y}\in\mathbb{C}^{M\times L}$ is then modeled as

$$\mathbf{Y} = \mathbf{H}\,\mathbf{C} + \mathbf{V}. \tag{1}$$

In this paper it is assumed that the channel matrix is perfectly known to both the transmitter and the receiver. A useful mathematical tool for handling block-based transmission is to vectorize the blocks

$$\mathrm{vec}(\mathbf{Y}) = \mathbf{I}\otimes\mathbf{H}\;\mathrm{vec}(\mathbf{C}) + \mathrm{vec}(\mathbf{V}),$$

and then separate the complex valued system equations to real valued equations of double dimension. Define

$$\mathbf{G} = \begin{bmatrix} \Re(\mathbf{I}\otimes\mathbf{H}) & \Im(\mathbf{I}\otimes\mathbf{H}) \\ -\Im(\mathbf{I}\otimes\mathbf{H}) & \Re(\mathbf{I}\otimes\mathbf{H}) \end{bmatrix},\; \mathbf{y} = \begin{bmatrix} \Re(\mathrm{vec}(\mathbf{Y})) \\ \Im(\mathrm{vec}(\mathbf{Y})) \end{bmatrix},$$

and similarly the vectorizations $\mathbf{C}$ to $\mathbf{c}$, and $\mathbf{V}$ to $\mathbf{v}$. The vectorized system equations are given by

$$\mathbf{y} = \mathbf{G}\,\mathbf{c} + \mathbf{v}. \tag{2}$$

Assume the noise, $\mathbf{V}$, is circularly symmetric complex Gaussian. Assume furthermore (without loss of generality) that the noise has been pre-whitened from the receiver side, so that it is iid and zero mean with variance one. The vectorized real valued noise vector, $\mathbf{v}$, is then Gaussian iid and zero-mean.

Any pulse amplitude modulation (PAM) or QAM modulated linear dispersion code can be assembled using $\mathbf{c} = \mathbf{F}\,\mathbf{x}$, where $\mathbf{F}\in\mathbb{R}^{2NL\times K}$ is a data independent precoding matrix (it is assumed that $2NL \geq K$), and $\mathbf{x}$ is the PAM data vector normalized such that $\mathbf{x}\in\mathbb{Z}^K + 1/2$. The elements of $\mathbf{x}$ are assumed to be independent random variables that represent the data which is to be transmitted. The variance depends on the bit load, i.e. the number of bits an element represents. Let the bit load on each element be $b_1, ..., b_K$, then the covariance matrix is

$$\mathrm{E}\left[\mathbf{x}\mathbf{x}^{\mathrm{T}}\right] \equiv \mathbf{\Sigma} = \mathrm{diag}\Big\{\frac{4^{b_1}-1}{12}, ..., \frac{4^{b_K}-1}{12}\Big\}.$$

Note that the system equation (2) is actually more general than the original system model (1) since $\mathbf{G}$ may include multiple temporal finite impulse response terms to model frequency selective channels. Consequently, the results presented here can be used to design codes for ISI-combating block codes as well as MIMO space-time block codes.

## 3. PROBLEM FORMULATION

The problem we wish to solve in this paper is how to optimally select the bit load $b_1, b_2, ..., b_K$, and the precoder matrix, $\mathbf{F}$, such that the block error rate is minimized under a constraint on the transmitted power. The receiver is assumed to use optimal ML detection. The power constraint may be defined in various ways, herein the average transmitted power is used $P = \mathrm{Tr}\{\mathbf{F}\mathbf{\Sigma}\mathbf{F}^{\mathrm{T}}\}/L \leq P_{max}$. This power constraint is widely adopted in the literature, partly due to its mathematical simplicity.

Using ML, the probability of detection error, $P_e$, (disregarding possible outer error protecting codes) is difficult to evaluate in general. It can however be upper bounded using the well known union bound

$$P_e \leq \bar{P}_e \equiv \frac{1}{2^{RL}}\sum_i\sum_{j\neq i}\mathrm{PEP}_{i,j}, \tag{3}$$

that consists of a sum of pairwise error probabilities defined as

$$\mathrm{PEP}_{i,j} = Q\bigg(\frac{|\mathbf{GF}(\mathbf{x}_j-\mathbf{x}_i)|}{\sqrt{2}}\bigg).$$

For a definition of the $Q$-function see for example [4]. Due to the steep decent of $Q$-function, only a few terms in the union bound contribute to $\bar{P}_e$, and minimizing the maximum PEP will, for moderately large SNR, effectively minimize the union bound. The $Q$-function is furthermore a strictly decreasing function so we would like to design a codebook with data rate $R$ and where the minimum distance

$$d^2_{\text{min}} = \min_{i,j\neq i}(\mathbf{x}_i-\mathbf{x}_j)^{\mathrm{H}}\mathbf{F}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{F}(\mathbf{x}_i-\mathbf{x}_j),$$

is as large as possible. Because $\mathbf{x}_i - \mathbf{x}_j$ belongs to the $\mathbb{Z}^K$-lattice we know that the minimum distance is always greater than or equal to the minimum distance of the lattice with generator matrix $\mathbf{M} = \mathbf{GF}$. Let us denote the minimum Euclidian distance of the lattice $\mu(\mathbf{M})$. In the general case $d_{\text{min}} \geq \mu(\mathbf{M})$, however for high data rates and/or 'nice' lattice structures we may expect $\mu(\mathbf{M}) = d_{\text{min}}$. This approximation has a number of advantages, first of all the minimum distance decouples from the actual bit load since $\mu(\mathbf{M})$ does not directly depend on $\mathbf{\Sigma}$. Secondly, the minimum distance becomes invariant to certain lattice transformations. Let $\mathbf{U}$ be an arbitrary real valued unitary matrix and let $\mathbf{B}$ be a unimodular integer valued matrix with determinant $|\mathbf{B}| \pm 1$, then we have $\mu(\mathbf{UMB}) = \mu(\mathbf{M})$. This means there are three different properties that can be adjusted to make the code more power efficient without changing the minimum distance. The power that is saved can then be utilized to increase the minimum distance.

There is a slight problem with this approach that one needs to be aware of. A lattice point has more than one nearest neighbor at a distance $\mu(\mathbf{M})$ and this fact may have a negative impact on the performance. Define the number of nearest neighbors of a lattice as $N(\mathbf{M})$. Again, assuming only the closest lattice points affect the union bound (3), it can be approximated as

$$\bar{P}_e \approx N(\mathbf{M})\,Q\bigg(\frac{\mu(\mathbf{M})}{\sqrt{2}}\bigg).$$

For low dimensional or unstructured lattices, the number of nearest neighbors does not have a substantial effect on the performance. However, for structured, dense, high dimensional lattices this number can actually dominate the error performance.

## 4. LATTICE BASED PRECODING ALGORITHM

If the dimension of the channel Gram matrix, $\mathbf{G}^{\mathrm{T}}\mathbf{G}$, is larger than the number of spatial sub-channels, $K$, then it is always optimal to let the precoder project on to the $K$'th strongest singular values of $\mathbf{G}^{\mathrm{T}}\mathbf{G}$ (this minimizes the transmitted power for any fixed lattice, $\mathbf{M}$). If the singular value decomposition (SVD) of the channel matrix is $\mathbf{G} = \mathbf{U_G}\mathbf{\Lambda_G}\mathbf{V}^{\mathrm{T}}_{\mathbf{G}}$, the subspace containing the $K$ strongest singular values is defined as $\tilde{\mathbf{U}}_{\mathbf{G}}\tilde{\mathbf{\Lambda}}_{\mathbf{G}}\tilde{\mathbf{V}}^{\mathrm{T}}_{\mathbf{G}}$, where $\tilde{\mathbf{\Lambda}}_{\mathbf{G}}$ is $K \times K$, and $\tilde{\mathbf{U}}_{\mathbf{G}}$, $\tilde{\mathbf{V}}_{\mathbf{G}}$ are the corresponding orthonormal column vectors. The precoding matrix can then be projected to form a $K \times K$ matrix as $\tilde{\mathbf{F}} = \tilde{\mathbf{V}}^{\mathrm{T}}_{\mathbf{G}}\mathbf{F}$. Using such a precoder, and a lattice with generator matrix $\mathbf{M}$, we can calculate an equivalent lattice with a $K \times K$ generator matrix on the form $\mathbf{UMB} = \tilde{\mathbf{\Lambda}}_{\mathbf{G}}\tilde{\mathbf{F}}$. The generator matrix $\mathbf{M}$ will be referred as the base lattice to separate it from the generator matrix $\mathbf{UMB}$ that is used for transmission. The transmitted power is

$$P = \mathrm{Tr}\big\{\tilde{\mathbf{\Lambda}}^{-2}_{\mathbf{G}}\mathbf{UMB}\mathbf{\Sigma}\mathbf{B}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\big\}/L.$$

The goal now is to minimize the power by selecting a suitable bit load (implicitly $\mathbf{\Sigma}$), a unimodular integer matrix $\mathbf{B}$ and an orthonormal matrix $\mathbf{U}$. The problem is difficult since it involves joint optimization of discrete and real valued variables. Herein a sub-optimal algorithm for this optimization is proposed. The idea is to optimize each parameter at a time, and then iterate until the solution converge. Due to the discrete nature of $\mathbf{B}$ and the bit load, convergence is reached in finite time.

### 4.1. Optimization of U

Assuming the singular values in $\tilde{\mathbf{\Lambda}}_{\mathbf{G}}$ have been ordered in decreasing order along the diagonal, the optimization of $\mathbf{U}$ simply boils down to solving a SVD. Define the SVD of

$$\mathbf{MB\Sigma B}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}} = \mathbf{V\Lambda V}^{\mathrm{T}},$$

where the singular values in $\mathbf{\Lambda}$ are decreasing along the diagonal, then the optimal $\mathbf{U} = \mathbf{V}^{\mathrm{T}}$. The proof follows from majorization theory (see [12] chapter 3).

### 4.2. Optimization of the bit load

Redistributing the bit load, $b_1, ..., b_K$, so that the power is minimized while the data rate is fixed can be optimally solved using the following algorithm. First, define the basis power vector as

$$\mathbf{d} = \mathrm{diag}\{\mathbf{B}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\tilde{\mathbf{\Lambda}}^{-2}_{\mathbf{G}}\mathbf{UMB}\}.$$

The vector specifies the power of the basis vectors of the precoding matrix, and basis vectors with high power cost should intuitively have a low bit load. Assume that the bit loading has been initialized, and let the current bit load be $b_1, ..., b_K$, then the power consumption is

$$P = \sum_i \frac{d_i(4^{b_i}-1)}{12\,L},$$

where $d_i$ is the $i$'th element of $\mathbf{d}$. If one bit is moved from substream $n$ to $m$, the change in power consumption will be

$$P_{\text{new}} - P_{\text{old}} = -\frac{d_n 4^{b_n}}{16\,L} + \frac{d_m 4^{b_m}}{4\,L}.$$

So, in order to reduce the power consumption, $n$ and $m$ have to be selected such that $d_n 4^{b_n} \geq 4\,d_m 4^{b_m}$. Using this result we propose the following algorithm for bit loading

1. Let $n$ be the index of the maximum element of $d_1 4^{b_1}, ..., d_K 4^{b_K}$ with non-zero bit load. Then let $m$ be the index of the minimum element.

2. Then check whether $4\,d_n 4^{b_n} > d_m 4^{b_m}$. If so, move one bit from $b_n$ to $b_m$. Go back to 1. Otherwise, the bit load has been optimized and the algorithm can terminate.

It can be shown that this algorithm always finds the global optimum. The proof is omitted here and we would like to refer to the coming journal article on this topic.

### 4.3. Optimization of $\mathbf{B}$

Optimization of $\mathbf{B}$ is somewhat more difficult to do optimally. However, there exist efficient algorithms for basis reduction that will at least reduce the transmitted power. One such algorithm that has a polynomial complexity is the Lenstra, Lenstra and Lovasz (LLL) algorithm [13]. Here, the LLL algorithm is utilized to reduce the basis of $\mathbf{A} = \tilde{\mathbf{\Lambda}}_{\mathbf{G}}^{-1}\mathbf{UM}$. The result is a matrix $\mathbf{B}$ that has determinant $|\mathbf{B}| \pm 1$ and that reduces the lengths of the basis vectors and consequently the total power consumption will be reduced (at least not increase).

### 4.4. Combined optimization

The ordering of the three partial optimization steps above may have an impact on the final result. Without going too deep into this question, the ordering that appears to give the best result in most cases is to first initialize $\mathbf{B} = \mathbf{I}$, distribute the bit load as evenly as possible, and perform an initial optimization of $\mathbf{U}$. The algorithm should then iteratively optimize the matrices in the following order $\mathbf{B}$, $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{U}$. The iteration should stop when the algorithm converges. Typically no more than two iterations are needed.

## 5. SELECTING THE BASE LATTICE

Minimizing the transmitted power for a base lattice, $\mathbf{M}$, is clearly a step towards minimum distance maximization. However, the choice of optimal base lattice is yet to be determined. In this section, a fundamental lower bound to how much the power can be decreased using the algorithm in Section 4 is derived, and the task reduces to use the lattice that has the lowest fundamental lower bound. The relation between arithmetic and geometric means gives us the following bound

$$PL \geq K\big|\tilde{\mathbf{\Lambda}}_{\mathbf{G}}^{-2}\mathbf{UMB\Sigma B}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\big|^{1/K} = K\left(\frac{|\mathbf{MM}^{\mathrm{T}}||\mathbf{\Sigma}|}{|\tilde{\mathbf{\Lambda}}_{\mathbf{G}}|^2}\right)^{1/K} \geq \frac{K(4^{\frac{RL}{K}}-1)}{12}\cdot\frac{|\mathbf{MM}^{\mathrm{T}}|^{1/K}}{|\tilde{\mathbf{\Lambda}}_{\mathbf{G}}|^{2/K}}, \quad (4)$$

where the last inequality is due to the fact that $|\mathbf{\Sigma}|$ is minimized for a certain bit load if all bits are equally distributed. Note that the bound (4) is truly a lower bound to the power minimization algorithm since the right hand side is independent of $\mathbf{B}$, $\mathbf{\Sigma}$, and $\mathbf{U}$. The determinant $|\mathbf{MM}^{\mathrm{T}}|$ is the squared volume of a Voronoi cell of the lattice and it is related to the minimum distance of the lattice by the packing gain, $\sigma(\mathbf{M})$, of the lattice, defined here as $\mu^2(\mathbf{M}) = |\mathbf{MM}^{\mathrm{T}}|^{1/K}\sigma^2(\mathbf{M})$. To conclude, there is a fundamental lower bound to how much the transmitted power can be reduced by utilizing invariance properties of a certain lattice type

$$P \geq \frac{K(4^{RL/K}-1)}{12\,L}\cdot\frac{d_{\text{min}}^2}{\sigma^2(\mathbf{M})|\tilde{\mathbf{\Lambda}}_{\mathbf{G}}|^{2/K}}. \quad (5)$$

The lower bound is tight if the singular values of $\tilde{\mathbf{\Lambda}}_{\mathbf{G}}^{-2}\mathbf{UMB\Sigma B}^{\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}$ are all equal and the bit loads at the same time are as equally distributed as possible. If we can assume that the power may be minimized sufficiently close to the fundamental lower bound for most 'nice' lattice generator matrices, then the question of maximizing the minimum distance boils down to finding a lattice with the best possible sphere packing (i.e. lattices with large $\sigma^2(\mathbf{M})$). This is a mathematical problem that has no known optimal solutions for most dimensions, however many lattices with good sphere packing properties have been found [11] and we can simply pick the best known lattice for each dimensional size of interest. In Tab. 1 the packing gain and the number of nearest neighbors are listed for some well known lattices that are all available in [11]. The optimal dimension of the lattice can

| Lattice | Dimensions | $\sigma^2(\mathbf{M})$ [dB] | $N(\mathbf{M})$ |
|---|---|---|---|
| $\mathbb{Z}^K$ | $K$ | 0.000 | $2 \cdot K$ |
| $E_8$ | 8 | 3.010 | 240 |
| $K_{12}$ | 12 | 3.635 | 756 |
| $\Lambda_{16}$ | 16 | 4.515 | 4320 |

**Table 1**. Examples of packing gains, $\sigma^2(\mathbf{M})$, and the number of nearest neighbors, $N(\mathbf{M})$,for different lattices with dense packings.

be approximated by comparing the fundamental lower bound for certain values of $K$. When doing so, it is important to note that the diagonal matrix $\tilde{\mathbf{\Lambda}}_{\mathbf{G}}$ also depends on $K$. Possible sources of errors in this analysis are the bound that is approximative, and the impact of nearest neighbors that quickly grows with the dimension size. In the numerical examples below, a maximum lattice dimension is specified and used unless a lower dimension would perform better according to the bound (5).

is used to check whether a lower dimension would perform better, in which case the lower dimension is used. In this way one can also be ensured that the complexity of the ML decoder at the receiver is at an acceptable level.

## 6. NUMERICAL RESULTS

The proposed scheme can provide a packing gain of several dB's for sufficiently large SNR's and sufficiently large lattice dimensions. Packing gain is however not the only factor influencing the performance. Specifically, the number of nearest neighbors can have a dominant negative effect for low to moderately low SNR's. It remains to be seen how much of the packing gain that can be realized under realistic transmission conditions. In this section we will try to shed some light on these questions by using numerical examples.

There are many potential applications for the proposed scheme, the examples herein are however limited to narrow band MIMO systems of various dimensions, where the channel matrix consists of independent Rayleigh fading matrix elements. The channel matrices are generated as $\mathrm{vec}(\mathbf{H}) \sim CN(\mathbf{0}, \mathbf{I})$. This channel model is widely used for modelling MIMO channels with rich scattering, such as non line of sight indoor channels. Due to the fact that the TX-CSI is perfect, the definition of the SNR is not trivial. Here, it is simply defined as the transmitted power, $P$.

The optimization algorithm will be applied to the $\mathbb{Z}^K$-lattice, equivalent to channel inversion with adaptive bit loading, as well as the lattice with the best known sphere packing. In addition to this, the so called blind transmission will be simulated for comparison. It is a relatively simple transmission scheme that does not utilize any TX-CSI. The bit load is as evenly distributed as possible and the precoding matrix is given by $\mathbf{F} = \mathbf{I}_{2NL}$.

Fig. 1 shows a comparison of the block error rate performance of a six by six MIMO system as a function of SNR for the $\mathbb{Z}^{12}$ and $K_{12}$ lattices. The maximum dimension of the lattices is 12, although lower dimensions are used whenever it improves the fundamental lower bound (5). The data rate is 24 bits per channel use. We observe that the packing gain of the $K_{12}$ lattice ensures a gain over the $\mathbb{Z}^{12}$ lattice of approximately 2 dB in the high SNR region. The loss compared to the gain in Tab. 1 can be explained by the higher

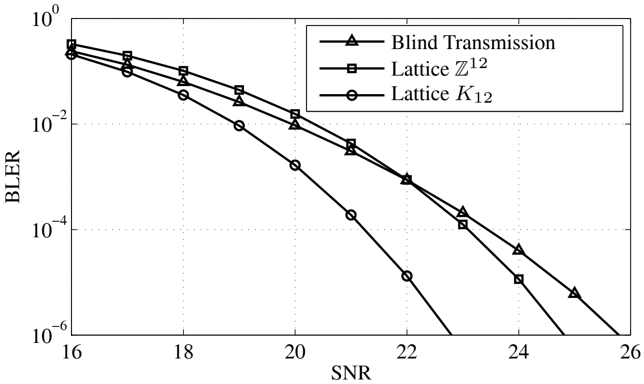


**Fig. 1**. Block error rate comparison between the $\mathbb{Z}^{12}$ lattice, the $K_{12}$ lattice, and 12-dimensional V-BLAST. The channel is a $6 \times 6$ MIMO Rayleigh fading channel, and the data rate is 24 bits/use.

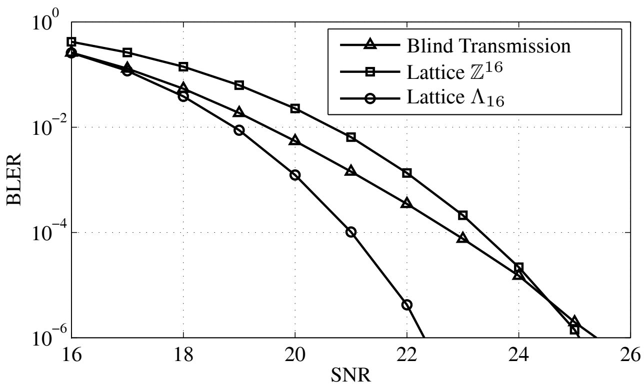


**Fig. 2**. Block error rate comparison between the $\mathbb{Z}^{16}$ lattice, the $\Lambda_{16}$ lattice, and 16-dimensional V-BLAST. The channel is a $8 \times 8$ MIMO Rayleigh fading channel, and the data rate is 32 bits/use.

number of nearest neighbors of the $K_{12}$ lattice. Interestingly, blind transmission works quite well for low SNRs. This can again be explained by the lower amount of nearest neighbors due to the irregularity of the code. The conclusion is that if one would like to utilize TX-CSI to reduce the BLER, one needs to use a lattice with a good tradeoff between sphere packing gain and number of nearest neighbors. Traditional channel inversion with adaptive bit loading is not sufficient in this case. This fact is even more evident for systems with higher dimensions. In Fig. 2 an eight by eight MIMO channel is simulated. The data rate is 32 bits per channel use. In this case the blind transmission scheme outperforms $\mathbb{Z}^{16}$ lattice precoding. Note however that in the more realistic scenario with correlation between channel elements one can expect blind transmission to suffer greater loss in performance than the $\mathbb{Z}^{16}$ precoder. The densest lattice for this scenario is the $\Lambda_{16}$ lattice. The gain is on the order of 2 - 3 dB which is smaller than the packing gain would imply. This can be explained by the larger number of nearest neighbors of the $\Lambda_{16}$ lattice compared to the $\mathbb{Z}^{16}$ lattice. As the dimension grows larger, the loss due to nearest neighbors 'eats up' the gain due to the increased packing density. Hence, there is a tradeoff that has to be considered when selecting the lattice.

## 7. CONCLUSIONS

The problem of designing linear dispersion codes for block based MIMO communication systems has been investigated. The receiver as well as the transmitter have perfect knowledge of the channel, and the receiver employs ML detection. The main idea is to use linear precoding and lattice invariant operations to transform the channel matrix into a lattice with good sphere packing properties. We demonstrated how to minimize the transmit power for an arbitrary lattice. An algorithm and a optimization bound for this power minimization was presented. The bound motivated the use of lattices with dense sphere packing, although it was also concluded that the number of nearest neighbors affects the performance. Numerical results indicate that there is a potential gain of several dB by using the method compared to channel inversion with adaptive bit loading.

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