APPLICATION OF ZERNIKE MOMENTS AS FEATURES IN KNN AND SVM AS SEMI-BLIND DETECTORS FOR STBC MIMO-OFDM SYSTEMS IN IMPULSIVE NOISE ENVIRONMENTS

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ABSTRACT

In Space Time Block Coded (STBC) MIMO-OFDM, most receivers assume the channel noise to be Gaussian, this paper considers the semi-blind detection of symbols in non-Gaussian channels. Our aim is to introduce a new receiver for STBC MIMO-OFDM transmission over Rayleigh fading channels using Zernike moments which apply either k-Nearest Neighbor (KNN) or Support Vector Machine (SVM) for classification. The detrimental effects of multipath on the transmitted symbols are alleviated using invariance to rotation, and scaling properties of the Zernike moments. The performance of the new method is shown in symmetric α-stable (SαS) and Middleton noise environments.

Index Terms— MIMO-OFDM, Zernike moments, support vector machine, k-nearest neighbor, impulsive noise

1. INTRODUCTION

OFDM coupled with MIMO techniques is an emerging technique for reliable and high speed wireless communication over frequency-selective channels. In many communication channels, the observation noise exhibits impulsive characteristics [1, 2, 3]. Probabilistic model of additive noise should from one hand describe the fact that background noise described by normal probability density function (PDF); and from the other hand that, there are moments of time when noise is increased considerably impulsive. The sources of impulsive noise in MIMO-OFDM can be: industrial noise, frequency hopping and ultra-wide band interference, pulsed jamming; therefore intensive investigations are carried out to determine an efficient detector in a non-Gaussian noise, and it is made an urgent task for STBC MIMO-OFDM systems; especially in a very heavy tailed noise environments. However, it has not yet been determined which impulsive noise model best approximates the most important characteristics of naturally occurring impulsive noise. Moreover, the suitable detector and its performance in impulsive noise environment is not specified either. But, the most commonly used models also verified in theory and practice are the Middleton models [1], which is composed of a Rayleigh distribution for the impulsive amplitude and a poisson distribution for the occurrence of the impulses. On the other hand, it has been suggested [3] that the family of α-stable random variable also provides useful models for impulsive phenomena. In [4, 5], receiver design for MIMO-OFDM systems is considered and the noise in such systems is Gaussian. In [6], the receiver design for MIMO systems in a mixture of Gaussian noise and α-stable noise is examined. STBC MIMO-OFDM signal detection in impulsive noise environments is under careful scrutiny and a final decisive detector is not proposed yet. In wireless communications, the channel itself is continually changing due to environmental surroundings, carrier frequency offsets and the relative speed of the transceivers, etc. The current STBC MIMO-OFDM [7] assume that a training symbol is used to obtain the channel state information. The Zernike moments can provide the suitable properties to resolve the multipath effects in presence of impulsive noise. In this paper, we consider MIMO-OFDM systems in an impulsive noise. On the other hand, among all noise models that have been developed for these systems, Middleton noise and SαS noise model [1, 3] are prominent because they have been proven to exist in communications channels in general. In addition, we propose a receiver using the Zernike moments for blind detection. The application of Zernike moments makes the receiver resilient against impulsive noise. The paper is organized as follows: section 2 contains the signal, channel, and noise model. Section 3 focuses on the receiver structure of MIMO-OFDM systems. Section 4 presents the simulation results and at the end, some concluding remarks are provided.

This work is supported by Iran Telecommunication Research Center.

2. SYSTEM MODEL

In this paper, we consider Alamouti MIMO-OFDM system. The bandwidth ($B = (1/T_s)$) is divided into K equally spaced subcarriers at frequencies $k\triangle f, k = 0, 1, \ldots, K-1$ with $\triangle f = B/K$ and T_s as the sampling interval. At the transmitter, information bits are grouped and mapped into complex symbols. According to the Alamouti code, $(X_1(k)\ X_2(k))$ are transmitted from the two antennas simultaneously during the first symbol period ($p = 1$) for each k. During the second symbol period ($p = 2$), $(-X_2^*(k)\ X_1^*(k))$ are transmitted from the two antennas for each k. The IFFT converts each $K \times 1$ complex vector into a time-domain signal and the copy of the last N_g samples are appended as a prefix (cyclic prefix). Thus, the length of an OFDM symbol is $(K + N_g)T_s$. The time-domain transmitted signals from antenna i during the p-th symbol period $x_p^i(n), 0 \leqslant n \leqslant K + N_g - 1, i \in \{1,2\}, p \in \{1,2\}$ are expressed as

$$x_p^i(n) = \sum_{k=0}^{K-1} s_p^i(k) e^{j2\pi k(n-N_g)/K} \tag{1}$$

where $s_p^i(k)$ denotes a complex symbol transmitted from the i-th antenna during the p-th symbol period in an Alamouti codeword over the k-th subchannel. The index for the Alamouti codeword is omitted to keep the notation simple.

The signals from the two transmit antennas go through independent channels. The wireless channel can be described as L resolved multipath components $\ell \in \{0, 1, \ldots, L-1\}$, each characterized by

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an amplitude $h_p^{i,j}(n,\ell)$ and a delay ℓT_s, where $h_p^{i,j}(n,\ell)$ stands for the ℓ-th resolved multipath component amplitude between the i-th transmit antenna and the j-th receive antenna at time n(sample index) during the p-th symbol period. The maximum delay spread of the two channels is assumed to be the same and equal to $(L-1)T_s$. The received signals during an Alamouti codeword period are

$$r_p^j(n) = \sum_{i=1}^{2}\sum_{\ell=0}^{L-1} h_p^{i,j}(n,\ell)x_p^i(n-\ell) + \eta_p^j(n), \quad p \in \{1,2\} \tag{2}$$

where $\eta_p(n)$ is a circularly symmetric zero-mean white complex Gaussian random process. It can be observed that the received signals are a superposition of signals from the two transmit antennas. If the CP length N_g is longer than $L-1$, the received signals (2) (after removing the prefix) can be considered as the circular convolution result of the transmitted signals (1) and the channel. Consequently, the demodulated signals in the frequency domain via the FFT are expressed as

$$u_p^j(k) \triangleq (\boldsymbol{u}_p^j)_k = \sum_{i=1}^{2}\sum_{m=0}^{K-1} s_p^i(m)a_p^{i,j}(k,m) + v_p^j(k)$$
$$\boldsymbol{u}_p^j \triangleq [u_p^j(0), u_p^j(1), \ldots, u_p^j(K-1)]^T \tag{3}$$

where

$$a_p^{i,j}(k,m) \triangleq \sum_{\ell=0}^{L-1} H_{p,\ell}^{i,j}(k-m)e^{-j2\pi m\ell/K} \tag{4}$$

$$H_{p,\ell}^{i,j}(k-m) \triangleq \frac{1}{K}\sum_{n=0}^{K-1} h_p^{i,j}(n,\ell)e^{-j2\pi(k-m)n/K}. \tag{5}$$

The notation $H_{p,\ell}^{i,j}(k-m)$ represents the FFT of the ℓ-th multipath component between the i-th transmit antenna and the j-th receive antenna during the p-th symbol period. The overall goal of this paper is to find an effective strategy to detect $\boldsymbol{s}_p^i = [s_p^i(0), s_p^i(1), \ldots, s_p^i(K-1)]^T$ in presence of complex impulsive noise $\boldsymbol{v}_p^j = [v_p^j(0), v_p^j(1), \ldots, v_p^j(K-1)]^T$.

2.1. Noise Models

By in large, almost all STBC MIMO-OFDM systems designs and analysis's are developed for Gaussian noise $\boldsymbol{v}_p^j$, however, the communication system observations and analyzes provide accurate examples where this assumption is invalidated [1], the non-Gaussianity could lead to performance degradation worse than Gaussian ones, hence a non-Gaussian noise model is required for a realistic approach. The noise between transmit-receive antenna pair can be either SαS impulsive noise and or middleton's class noises, Gaussianity is a subcategory both of these models. In the case of SαS noise, the elements of $\boldsymbol{v}_p^j$ are modeled as independently and identically distributed complex SαS random variables. that is $\forall\ v_p^j(k)$, $v_p^j(k) = \Re(v_p^j(k)) + j\Im(v_p^j(k))$, in which $\Re(v_p^j(k))$, $\Im(v_p^j(k))$ obey the bivariate joint SαS distribution. $\alpha \in (0,2]$ is called *characteristic exponent*, which controls the heaviness of the tails of the stable density and hence implies the impulsiveness of the respective SαS noise [3]. The *dispersion* parameter plays an analogous role to the variance of Gaussian distribution. Non-Gaussian impulsive noise is known to be one of the major sources of errors in digital transmission systems. Therefore, a more realistic noise model might be an additive mixture of Gaussian thermal noise and non-Gaussian impulsive noise. One of the models that has been proposed to meet these requirements is the general model derived by Middleton. According to the relation between the durations of the noise impulses and the spectral bandwidth of the receiver, Middleton derived three general classes of the impulsive noise: class A, B, and C [1]. In this paper, we use a class-A impulsive noise model because it is known to fit closely a variety of non-Gaussian noise and also it is an analytically tractable model of Gaussian/non-Gaussian noise. Moreover, through some experimental measurements, the actual impulsive noise supports the Middleton canonical class-A model. The class-A impulsive noise model of Middleton is a generalized model of the Gaussian noise combined with a non-Gaussian impulsive noise. Further details of this model are found in [1, 8]. The importance of such model lies in their application to the fundamental signal processing problems of telecommunications in-the-large, which includes radio, radar, sonar, etc.

3. RECEIVER STRUCTURE

Multipath communication channels have destructive effects on data. As a result, they cause undesired variations of phase angle and amplitude of data. Zernike and pseudo-Zernike moments are the best in terms of information redundancy and noise sensitivity [9]. The Zernike moments achieve the desirable invariance to unknown phase angle, and can be made scale and translation invariant as well; by first scale and translation normalizing the original symbols such that they are of the same dimension and their centroids are positioned at the origin. These moments are obtained once the orthogonality and the invariance properties are considered simultaneously [9]. Another feature of interest about these moments is the dependence on SNR. The classification method is again based on computing Zernike moments of the output of the demodulator ($\hat{Z}_{pq}$) and comparing these moments with the predetermined moments Z_{pq} for each class using the k-Nearest Neighbor (KNN) classifier [10] or Support Vector Machine (SVM) classifier [11]. The first classifier is suitable for data streams. When a new sample arrives, KNN finds the k-neighbors nearest to the new sample from the training space based on some suitable similarity or distance metric. KNN classifier depends on finding the feature vector of minimum distance to the unknown feature vector. In KNN, k is a predetermined value. The k-nearest neighbors to the unknown feature vector; transmitted symbol, are found, and the vote of each class is assigned to the number of neighbors they have. The classification result is the class which has the maximum vote of k-nearest neighbors. The plurality class among the nearest neighbors is the class label of the new sample. A common similarity function is based on the *Euclidian distance* between two data tuples. For two tuples, $\boldsymbol{\zeta} = \langle \zeta_1, \zeta_2, \ldots, \zeta_{n-1}\rangle$ and $\boldsymbol{\xi} = \langle \xi_1, \xi_2, \ldots, \xi_{n-1}\rangle$ (excluding the class labels), the *Euclidian* similarity function is

$$d_2(\boldsymbol{\zeta}, \boldsymbol{\xi}) = \sqrt{\sum_{i=1}^{n-1}(\zeta_i - \xi_i)^2}. \tag{6}$$

The second classifier is primarily a method that performs classification tasks by constructing hyperplanes in a multidimensional space that separates cases of different class labels. By in large, the joint resulting effect of multipath and noise can transform the transmitted signal such that a linear transformation of the received signal cannot be good classifier. Extracting features from possible transmitted signals can be utilized to define vectors in the feature space, then hyperplane that divides clusters of vector could be found, so that data with one category of the target variable are on one side of the plane and the data with another category are on the other side of the plane. Besides of separating the data into different categories, the objective of SVM is to find an optimal hyperplane that correctly classifies the data as much as possible and separates the data as far as possible. The Alamouti receiver resembles a pattern recognition problem, by using Zernike moments as the feature vectors. The magnitude of the computed Zernike moments are rotation invariant [9], this is specially important since the channel can rotate the transmitted symbols. Therefore, the phase of transmitted and received symbols are unaffected via the Zernike moments. This rotation invariance property takes care of the random phase shift imposed on the incoming signals through the multipath channel. On the other hand, scale invariance property inherited in Zernike moments would compensate for the random attenuation imposed on the transmitted data. In absence of the noise term $\boldsymbol{v}_p^j$ in (3) we have perfect detection.

3.1. Zernike Moments

The Zernike polynomials were first proposed in 1934 by Zernike. Zernike moments have proven to have rotation invariant property [9], which can be useful in classification of space-time codes. Complex Zernike moments are constructed using a set of complex polynomials which form a complete orthogonal basis set defined on the unit disc $(x^2+y^2) \leqslant 1$. They are expressed as Z_{pq}, two dimensional Zernike moment, for Alamouti receiver using (3):

$$Z_{pq} = \frac{p+1}{\pi} \int_x \int_y \boldsymbol{u}(x,y) V_{pq}^*(x,y) dx\, dy \tag{7}$$

where $p = 0,1,2,\ldots,\infty$ defines the order, $0 \leqslant |q| \leqslant p$ and $p-|q|$ is even. In our circumstance, we use the sampled version of (7) in maximum likelihood sense

$$\hat{Z}_{pq} = \frac{p+1}{\pi} \sum_x \sum_y \boldsymbol{u}(x,y) V_{pq}^*(x,y). \tag{8}$$

The Zernike polynomials $V_{pq}(x,y)$ expressed in polar coordinates are:

$$V_{pq}(r,\theta) = R_{pq}(r) e^{jq\theta} \tag{9}$$

where (r,θ) are defined over the unit disc, $\sqrt{j} = -1$ and the orthogonal radial polynomial, $R_{pq}(r)$, is given as follows:

$$R_{pq}(r) = \sum_{s=0}^{\frac{p-|q|}{2}} (-1)^s \frac{(p-s)!}{s!(\frac{p+|q|}{2}-s)!(\frac{p-|q|}{2}-s)!} r^{p-2s} \tag{10}$$

$\boldsymbol{u}(x,y)$ in (8) is the value of the output of demodulator for computing $\hat{Z}_{pq}$, and it is the value of constellation point for computing Z_{pq}. To help reduce computation complexity, it may prove useful to express the Zernike moments in terms of Cartesian moments [12]. This removes the need for the polar mapping of the data, while also removing the dependence on the trigonometric functions. Alternatively, expressing Cartesian moments in this way would aid the selection of less correlated descriptors. This conversion can be achieved by slightly rearranging the Zernike moment equation. By substituting $k = p - 2s$ and rearranging $R_{pq}(r)$, we have:

$$R_{pq}(r) = \sum_{k=q}^{p} B_{pqk} r^k, \quad (p-k) \text{ is even}, \ q \geqslant 0 \tag{11}$$

where

$$B_{pqk} = \frac{(-1)^{(p-k)/2}(\frac{p+k}{2})!}{(\frac{p-k}{2})!(\frac{k+q}{2})!(\frac{k-q}{2})!} \tag{12}$$

Using this manipulated form of the radial polynomials produces Zernike moment definitions (in continuous form) of:

$$Z_{pq} = \frac{p+1}{\pi} \sum_{k=q}^{p} B_{pqk} \int_0^{2\pi} \int_0^1 r^k e^{-jq\theta} \boldsymbol{u}(r,\theta)\, r\, dr\, d\theta \tag{13}$$

which when translated to Cartesian coordinates is:

$$Z_{pq} = \frac{p+1}{\pi} \sum_{k=q}^{p} B_{pqk} \int_x \int_y (x-jy)^q (x^2+y^2)^{(k-q)/2} \boldsymbol{u}(x,y)\, dx\, dy \tag{14}$$

Translation and scale invariants of Zernike moments can normally be achieved by moving the centroid of the received MIMO-OFDM data to the origin of unit circle and by enlarging or reducing the mass of received data from (3) to the predetermined level respectively [9]. The Zernike moments with higher order save more detail features of the transmitted signal and therefore an effective index to combat noise and multipath. However, higher order increases computational cost. However, this conventional normalization technique can introduce errors and hence does not produce the desired invariances. In classifying the transmitted symbols, size of the received data may vary and this has an impact on the moment calculations. By assuming that the transmitted symbols are scaled uniformly with an equifactor of β from two antennas, the scaled Zernike moments; used in (6) for predetermined Z_{pq} in transmitter, and for classification $\hat{Z}_{pq}$ in the receiver, are

$$\tilde{Z}_{pq} = \frac{p+1}{\pi} \sum_{k=q}^{p} B_{pqk} \int_0^{2\pi} \int_0^1 \tilde{r}^k e^{-jq\theta} \boldsymbol{u}(\tilde{r},\theta) \tilde{r} d\tilde{r}\, d\theta,\ \tilde{r} = \beta r, \tag{15}$$

β amounts to the equivalent attenuation imposed onto the transmitted symbols via multipath fading channel. In other words, β is the uniform scale factor that scaled the original symbol constellation to the noise and multipath infected constellation illustrated through (3).

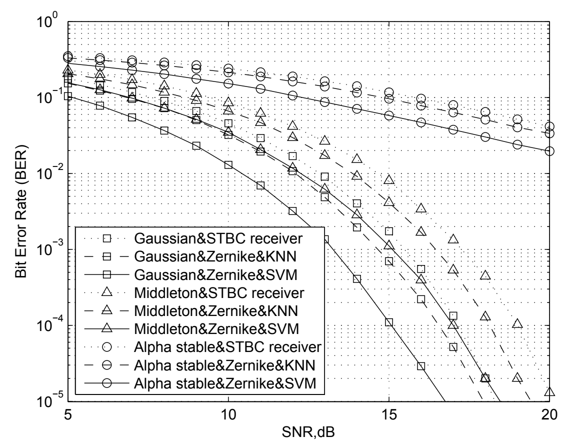

Fig. 1. Comparison between the new method and the method based on STBC orthogonality [13] in various noise.

Next we perform some simulations to validate the effectiveness of the Zernike moments for detection of transmitted symbols.

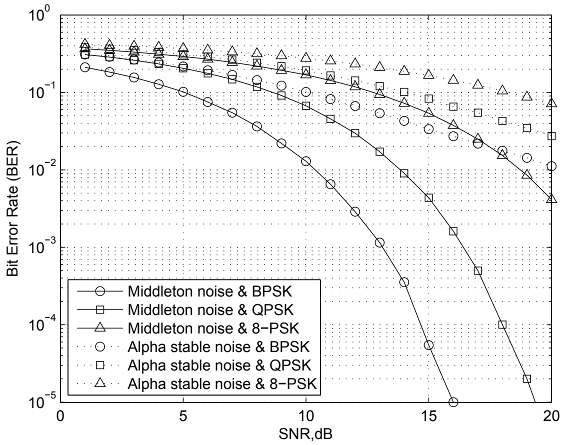

Fig. 2. BER performance of STBC MIMO-OFDM system with various modulation using Zernike moments.

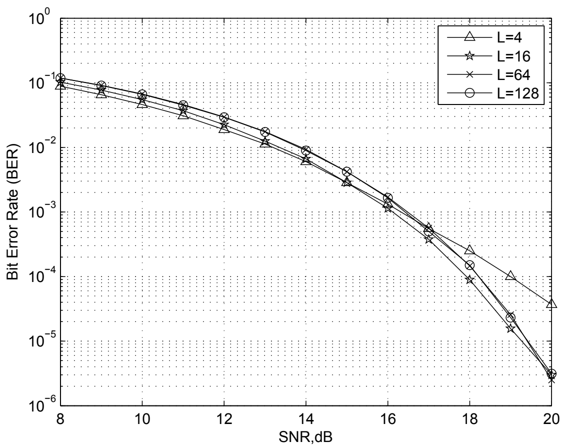

Fig. 3. The effect of multipaths in BER performance of STBC MIMO-OFDM system using Zernike moments.

4. SIMULATION RESULTS

We simulate a 2×2 MIMO-OFDM system under Rayleigh flat fading channel. The coding scheme is based on Alamouti method. The number of multipaths is $L = 64$, with equal power strength. The packet size is five OFDM symbol, consisting of $K = 64$ subcarriers. The *characteristic exponent* of the complex SαS impulsive noise is set to 1.5 and its *dispersion* is equal to the variance of the Gaussian noise. The *impulsive index* [1, 8] in middleton noise is $A = 0.3$. Fig. 1 is the comparison between BER performance of our approach and the method based on STBC orthogonality in various noise[13]. In this simulation, QPSK is used as the subcarrier modulation and Gaussian kernel function [14] is used in Support Vector Machine (SVM) classifier. As it is clear, fig. 1 shows better performance of our approach. Fig. 2 shows BER performance for various subcarrier modulation that uses k-Nearest Neighbor classifier. The effect of multipaths in BER performance of this system for QPSK subcarrier modulation in Middleton noise is shown in fig. 3. The performance of the system is delicately tangible while the number of multipaths is going up, this figure specially signifies the robustness of utilizing Zernike moments in highly multipath environment.

5. CONCLUSION

In this paper we have considered the receiver design of MIMO-OFDM systems in impulsive noise modeled as a symmetric α-stable (SαS) noise and Middleton noise. We propose a receiver that uses Zernike moments for symbol detection. Zernike moments which are based on orthogonal polynomials in two dimensions are invariant to rotation, scaling, and translation and they are able to challenge destructive nature of channel. Simulations show the proposed receiver can achieve better performance than STBC receiver.

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