# SOME RESULTS ON THE BLIND IDENTIFIABILITY OF ORTHOGONAL SPACE-TIME BLOCK CODES FROM SECOND ORDER STATISTICS

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# ABSTRACT

In this paper, the conditions for blind identifiability from second order statistics (SOS) of multiple-input multiple-output (MIMO) channels under orthogonal space-time block coded (OSTBC) transmissions are studied. The main contribution of the paper is to show that, assuming more than one receive antenna, any OSTBC with a transmission rate higher than a given threshold, which is inversely proportional to the number of transmit antennas, permits the blind identification of the MIMO channel from SOS. Additionally, some previous identifiability results have been extended. The implications of these theoretical results include the explanation of previous simulation examples found in the literature and, from a practical point of view, they show that the only non-identifiable OSTBC codes with practical interest are the Alamouti codes and the real square orthogonal design with four transmit antennas. Further discussion and empirical analysis are also provided.

*Index Terms*— Orthogonal space-time block codes (OSTBC), multiple-input multiple-output (MIMO) communications, blind identifiability, second-order statistics.

#### 1. INTRODUCTION

In the recent years, space-time block coding (STBC) has emerged as one of the most promising techniques to exploit spatial diversity in multiple-input multiple-output (MIMO) systems. Among the space-time coding schemes, the orthogonal space-time block coding (OSTBC) is one of the most attractive because it is able to provide full diversity gain with very simple encoding and decoding. The special structure of OSTBCs implies that the optimal maximum likelihood (ML) decoder is a simple linear receiver, which can be seen as a matched filter (MF), followed by a symbol-by-symbol detector. When the channel state information (CSI) is not available at the receiver, training approaches can be used to obtain an estimate of the channel. However, the price to be paid is reduced bandwidth efficiency, and even inaccurate channel estimates due to the effect of the noise and the limited number of training symbols. Popular approaches to avoid the reduction on the bandwidth efficiency include the so-called differential space-time coding schemes [1], which incur a penalty in performance of 3-dB.

Recently, several methods for blind channel estimation or blind decoding have been proposed. These methods can be divided in two groups depending on whether they exploit the higher-order statistics (HOS) or the second-order statistics (SOS) of the signals. The main advantage of SOS-based methods [2–5] is their low complexity, but

it is well known that there are several OSTBCs (including the Alamouti code [6]) which do not permit the blind channel identification. There are some partial identifiability results in the literature [5,7,8], but, to the best of our knowledge, the identifiability conditions still remain unclear. The main goal of this paper is to fill this gap by presenting, in a unified manner, some new results regarding the blind identifiability conditions of OSTBCs.

The main contribution of this work is based on the definition of identifiable and non-identifiable OSTBCs, and consists on the claim that any OSTBC transmitting at a rate higher than a given threshold, which is inversely proportional to the number of transmit antennas, permits the blind channel identification for any number of receive antennas  $n_R > 1$ . Additionally, we have found that any OSTBC transmitting an odd number of real symbols permits the blind identification of the MIMO channel regardless of the number of receive antennas, which extends to complex OSTBCs the first result in [5]. Moreover, it has been proven that any real OSTBC with an odd number of transmit antennas is identifiable, which explains some of the numerical examples in [7].

The implications of these results include the explanation of the simulation examples in [4]; the generalization of the identifiability conditions in [5] and [7], and the proof that the only non-identifiable OSTBCs with practical interest are the Alamouti code and the real code with  $n_T = 4$  transmit antennas and transmission rate R = 1.

# 2. SOME BACKGROUND ON OSTBCS

## 2.1. Notation

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., **X**, with elements  $x_{i,j}$ ; bold-faced lower case letters for column vector, e.g., **x**, and light-faced lower case letters for scalar quantities. The superscripts  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and Hermitian, respectively. The real and imaginary parts will be denoted as  $\Re(\cdot)$  and  $\Im(\cdot)$ , and superscript  $(\hat{\cdot})$  will denote estimated matrices, vectors or scalars. The trace, range (or column space) and Frobenius norm of matrix **A** will be denoted as  $\operatorname{Tr}(\mathbf{A})$ , range(**A**) and  $\|\mathbf{A}\|$ , respectively. Finally, **0** will denote the zero matrix,  $\mathbf{I}_p$  will denote the identity matrix of size p (although the subindex will be omitted if it is not necessary), and  $\lceil q \rceil$  will denote the smallest integer greater or equal than q.

A flat fading MIMO system with  $n_T$  transmit and  $n_R$  receive antennas is assumed during the paper. The  $n_T \times n_R$  complex channel matrix is  $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_{n_R}]$ , where  $\mathbf{h}_j = [h_{1,j}, \ldots, h_{n_T,j}]^T$  contains the channel response associated with the *j*-th receive antenna, and  $h_{i,j}$  denotes the channel response between the *i*-th transmit and the *j*-th receive antennas. The complex noise at the receive antennas is considered both spatially and temporally white with variance  $\sigma^2$ .

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#### 2.2. Data Model for STBCs

Let us consider a space-time block code (STBC) transmitting M symbols during L slots and using  $n_T$  antennas at the transmitter side. The transmission rate is defined as R = M/L. Let us also define the vector  $\mathbf{s}[n] = [s_1[n], \ldots, s_{M'}[n]]^T$  containing the M' real information symbols transmitted in the *n*-th block, where M' = M for real STBCs and M' = 2M for complex STBCs. For a STBC, the *n*-th block of data can be expressed as

$$\mathbf{S}[n] = \sum_{k=1}^{M'} \mathbf{C}_k s_k[n],$$

where  $\mathbf{C}_k \in \mathbb{C}^{L \times n_T}$ , k = 1, ..., M', are the STBC code matrices. In the case of real STBCs, the transmitted matrix  $\mathbf{S}[n]$  and the code matrices  $\mathbf{C}_k$  are real.

The combined effect of the STBC and the *j*-th channel can be represented by the  $L \times 1$  complex vectors  $\mathbf{w}_k(\mathbf{h}_j) = \mathbf{C}_k \mathbf{h}_j$ , and defining the real vectors  $\tilde{\mathbf{y}}_j[n] = [\Re(\mathbf{y}_j[n])^T, \Im(\mathbf{y}_j[n])^T]^T$  and  $\tilde{\mathbf{w}}_k(\mathbf{h}_j) = [\Re(\mathbf{w}_k(\mathbf{h}_j))^T, \Im(\mathbf{w}_k(\mathbf{h}_j))^T]^T$ , where  $\mathbf{y}_j[n]$  denotes the received signal at the *j*-th antenna, we can write

$$\tilde{\mathbf{y}}_j[n] = \sum_{k=1}^{M'} \tilde{\mathbf{w}}_k(\mathbf{h}_j) s_k[n] + \tilde{\mathbf{n}}_j[n] = \tilde{\mathbf{W}}(\mathbf{h}_j) \mathbf{s}[n] + \tilde{\mathbf{n}}_j[n],$$

where  $\mathbf{\tilde{W}}(\mathbf{h}_j) = [\mathbf{\tilde{w}}_1(\mathbf{h}_j) \cdots \mathbf{\tilde{w}}_{M'}(\mathbf{h}_j)]$ , and  $\mathbf{\tilde{n}}_j[n]$  is the real noise of variance  $\sigma^2/2$ . Finally, stacking all the received signals into  $\mathbf{\tilde{y}}[n] = [\mathbf{\tilde{y}}_1^T[n], \dots, \mathbf{\tilde{y}}_{n_R}^T[n]]^T$ , we can write

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n],$$

where  $\tilde{\mathbf{W}}(\mathbf{H}) = \left[\tilde{\mathbf{W}}^T(\mathbf{h}_1) \cdots \tilde{\mathbf{W}}^T(\mathbf{h}_{n_R})\right]^T$ , and  $\tilde{\mathbf{n}}[n]$  is defined analogously to  $\tilde{\mathbf{y}}[n]$ .

In the case of orthogonal STBCs (OSTBCs), the matrix  $\tilde{\mathbf{W}}(\mathbf{H})$  satisfies [1]

$$\tilde{\mathbf{W}}^{T}(\mathbf{H})\tilde{\mathbf{W}}(\mathbf{H}) = \|\mathbf{H}\|^{2}\mathbf{I},$$
(1)

which reduces the complexity of the coherent maximum likelihood (ML) decoder to find the closest symbols to the estimated signal

$$\hat{\mathbf{s}}_{\mathrm{ML}}[n] = rac{ ilde{\mathbf{W}}^T(\mathbf{H}) ilde{\mathbf{y}}[n]}{\|\mathbf{H}\|^2}$$

i.e., the OSTBC-MIMO channel response vectors  $\tilde{\mathbf{w}}_k(\mathbf{h}_j)$  can be seen as the ML equalizers.

#### 3. BLIND IDENTIFIABILITY OF OSTBC-MIMO CHANNELS

Recently, several methods for blind identifiability of OSTBC channels based on SOS have been proposed [2–5]. However, the identifiability conditions in [2–5] are related to the specific estimation criteria proposed in these works, and not to the blind channel estimation problem, which sometimes yields contradictory results. Furthermore, the relationship between the identifiability conditions associated to each of the proposed methods and the underlying structure of the OSTBC remains unclear.

In this section, the identifiability condition for the blind estimation of OSTBC-MIMO channels is studied, pointing out that it coincide with that of the method proposed in [4].

Let us start by introducing the following definition:

**Definition 1 (Non-identifiable OSTBC-MIMO channels)** The set given by an OSTBC code and a MIMO channel **H** is said to be a non-identifiable OSTBC-MIMO channel if there exists a channel  $\hat{\mathbf{H}} \neq c\mathbf{H}$ , with c a real constant, such that, for all  $\mathbf{s}[n]$ , we can find a signal  $\hat{\mathbf{s}}[n] \neq c^{-1}\mathbf{s}[n]$  satisfying

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{W}}(\mathbf{H})\mathbf{s}[n] + \tilde{\mathbf{n}}[n] = \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n] + \tilde{\mathbf{n}}[n].$$
(2)

Otherwise the OSTBC-MIMO channel is said to be identifiable.

The above definition states that, if the observation vectors  $\tilde{\mathbf{y}}[n]$  can be due to several combinations of signals  $\hat{\mathbf{s}}[n]$  and MIMO channels  $\hat{\mathbf{H}}$  (not related by a real scale factor), then the channel (or signal) can not be identified without exploiting other properties of the sources such as their belonging to a finite alphabet, or a source correlation matrix with different eigenvalues [2, 4, 9]. Furthermore, we must note that the real scalar (c) ambiguity will be always present in the blind decoding process, then, from now on we can assume that  $\|\hat{\mathbf{H}}\| = \|\mathbf{H}\| = 1$ . From (2) it is easy to obtain

$$\frac{\hat{\mathbf{s}}^{T}[n]}{\|\hat{\mathbf{s}}[n]\|} \tilde{\mathbf{W}}^{T}(\hat{\mathbf{H}}) \tilde{\mathbf{W}}(\mathbf{H}) \frac{\mathbf{s}[n]}{\|\mathbf{s}[n]\|} = 1,$$

and since the above equality must be satisfied for all s[n], we have the following ambiguity condition

$$\tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\mathbf{H}) = \mathbf{Q},\tag{3}$$

where  $\mathbf{Q}$  is an orthogonal (i.e., real and unitary) matrix. Here, taking into account  $\|\hat{\mathbf{H}}\| = \|\mathbf{H}\| = 1$  and (1), we can find that (3) is equivalent to

range 
$$\left( \tilde{\mathbf{W}}(\mathbf{H}) \right) = \text{range} \left( \tilde{\mathbf{W}}(\hat{\mathbf{H}}) \right)$$
,

which is the ambiguity condition associated to the technique proposed in [4] (see also [9]). Thus, if the OSTBC channel can be identified from SOS with the only ambiguity of a real scale factor, the estimate can be obtained by means of the technique proposed in [4], which is equivalent to the relaxed blind ML estimator

$$\underset{\hat{\mathbf{H}};\hat{\mathbf{s}}[n]}{\operatorname{argmin}}\sum_{n=0}^{N-1} \|\tilde{\mathbf{y}}[n] - \tilde{\mathbf{W}}(\hat{\mathbf{H}})\hat{\mathbf{s}}[n]\|^2,$$

where N is the number of available blocks at the receiver.

#### 4. NEW RESULTS ON BLIND IDENTIFIABILITY OF OSTBC-MIMO CHANNELS

In the previous section we have presented the general identifiability condition for SOS-based blind identification of OSTBC-MIMO channels. However, the relationship with the underlying OSTBC structure remains unclear. In this section we present some new blind identifiability results which are directly related with the main OSTBC properties. Let us start by introducing the following definitions:

**Definition 2 (Identifiable OSTBCs)** An OSTBC is said to be identifiable iff there exists at least one channel **H** such that the associated OSTBC-MIMO channel is identifiable.

**Definition 3 (Non-identifiable OSTBCs)** An OSTBC is said to be non-identifiable iff, for all **H**, the associated OSTBC-MIMO channel is non-identifiable.

Constellation	$n_T$	M	L	R = M/L	$R_{th}$	Identifiable	Design	Multiplicity $(n_R = 1)$	Multiplicity $(n_R > 1)$
real	2	2	2	1	2	No	Alamouti	2	2
real	3	4	4	1	1	Yes	gen. ort	2	1
real	4	4	4	1	1	No	gen. ort	4	4
real	5	8	8	1	2/3	Yes	gen. ort	2	1
real	6	8	8	1	2/3	Yes	gen. ort	2	1
real	7	8	8	1	1/2	Yes	gen. ort	2	1
real	8	8	8	1	1/2	Yes	gen. ort	2	1
complex	2	2	2	1	1	No	Alamouti	4	4
complex	3	4	8	1/2	1/2	Yes	gen. ort	2	1
complex	4	4	8	1/2	1/2	No	gen. ort	4	4
complex	5	8	16	1/2	1/3	Yes	gen. ort	2	1
complex	6	8	16	1/2	1/3	Yes	gen. ort	2	1
complex	7	8	16	1/2	1/4	Yes	gen. ort	2	1
complex	8	8	16	1/2	1/4	Yes	gen. ort	2	1
complex	3	3	4	3/4	1/2	Yes	amicable	2	1
complex	4	3	4	3/4	1/2	Yes	amicable	2	1
complex	5	4	8	1/2	1/3	Yes	amicable	1	1
complex	6	4	8	1/2	1/3	Yes	amicable	1	1
complex	7	4	8	1/2	1/4	Yes	amicable	1	1
complex	8	4	8	1/2	1/4	Yes	amicable	1	1

Table 1. Identifiability characteristics for the most common OSTBCs.

The main identifiability results are given as five Theorems. Due to the lack of space, we only provide here a sketch of the proofs, and we refer the interested reader to [10]. The first theorem, which has also been proven in [8], ensures blind channel identifiability for any number of receive antennas:

**Theorem 1 (See also [8])** If an OSTBC code transmits an odd number of real symbols (M' odd), then the OSTBC-MIMO channel is identifiable regardless of the number of receive antennas.

**Proof 1** The proof is based on the properties of the ambiguity matrix  $\mathbf{Q} = \tilde{\mathbf{W}}^T(\hat{\mathbf{H}})\tilde{\mathbf{W}}(\mathbf{H})$ , which can be rewritten as  $\mathbf{Q} = \alpha \mathbf{I} + \sqrt{1 - \alpha^2} \boldsymbol{\Delta}$ , where  $\alpha = \Re \left( \operatorname{Tr} \left( \hat{\mathbf{H}}^H \mathbf{H} \right) \right)$ , and  $\boldsymbol{\Delta} = -\boldsymbol{\Delta}^T$  is an orthogonal and skew symmetric matrix. Theorem 1 is a direct consequence of the non existence of orthogonal and skew-symmetric matrices of odd order [5, 8].

The following theorems state sufficient conditions for an OSTBC to be identifiable:

**Theorem 2** All the real OSTBCs with an odd number of transmit antennas  $n_T$  are identifiable.

**Proof 2** The proof proceeds by contradiction. If the code is non-identifiable, it is easy to prove that the code matrices must satisfy

$$\begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} = \mathbf{P}_{\mathbf{Q}} \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_{M'} \end{bmatrix} \mathbf{U}, \qquad (4)$$

where  $\mathbf{P}_{\mathbf{Q}}$  is an orthogonal and skew-symmetric block matrix constructed from the elements of  $\Delta$  and  $\mathbf{U}$  is a unitary and skew-Hermitian matrix of order  $n_T$ . For real OSTBCs the code matrices  $\mathbf{C}_k$  are real, and  $\mathbf{U}$  is real and skew-symmetric of order  $n_T$ , which implies that there do not exist non-identifiable real OSTBCs with an odd number of transmit antennas.

**Theorem 3** If an OSTBC code with  $n_T$  transmit antennas, and transmitting M' real symbols over L slots, is non-identifiable, then its code rate satisfies

$$\frac{M'}{L} \le \frac{2}{\left\lceil \frac{n_T}{2} \right\rceil}$$

**Proof 3** Taking into account that the eigenvalues of a unitary and skew-symmetric matrix only can take the values  $\pm j$ , and that any pair of orthogonal skew-symmetric matrices are orthogonally equivalent [11], (4) can be rewritten as

$$\mathbf{A}_{k}\begin{bmatrix} j\mathbf{I}_{p} & \mathbf{0} \\ \mathbf{0} & -j\mathbf{I}_{q} \end{bmatrix} = \mathbf{A}_{k+M'/2}, \qquad \qquad k = 1, \dots, M'/2,$$

where  $p + q = n_T$  and  $\mathbf{A}_k$  (k = 1, ..., M') are code matrices of an associated OSTBC obtained from linear operations over the code matrices  $\mathbf{C}_k$  (k = 1, ..., M'). Now, writing  $\mathbf{A}_k = \begin{bmatrix} \mathbf{F}_k & \mathbf{G}_k \end{bmatrix}$ , with  $\mathbf{F}_k \in \mathbb{C}^{L \times p}$  and  $\mathbf{G}_k \in \mathbb{C}^{L \times q}$ , and exploiting the properties of OSTBC code matrices [1], it is easy to prove that  $\mathbf{F}_k^H \mathbf{F}_l = \mathbf{0}$ ,  $\mathbf{G}_k^H \mathbf{G}_l = \mathbf{0}$ , for  $k, l = 1, ..., M'/2, k \neq l$ . This implies

$$M' \lceil n_T/2 \rceil \le M' \max(p,q) \le 2L,$$

and concludes the proof.

From Definition 2, we know that for any identifiable OSTBC there exists at least one channel **H** such that the OSTBC-MIMO channel is identifiable. The following theorem extends this result to all the full row rank channel matrices

**Theorem 4** Given an identifiable OSTBC and a full row rank channel matrix  $\mathbf{H}$  ( $n_R \ge n_T$ ), the associated OSTBC-MIMO channel is identifiable.

**Proof 4** The proof proceeds by contradiction. It is obvious that if **H** can not be identified, then any linear combination **HV** provides a non-identifiable OSTBC-MIMO channel. Since **H** is full row rank, all the possible channels can be written as **HV** for some **V**. Then, if **H** can not be identified, the OSTBC is non-identifiable.

The above theorem constitutes a sufficient condition for blind channel identifiability based on SOS. However, simulation results have shown that the full row rank condition on the channel matrix is not necessary for channel identification (see Section 5 and [4]). In order to extend the result of Theorem 4 we introduce the following conjecture, which has been validated by means of numerical results

**Conjecture 1** Let us consider an identifiable OSTBC and a multipleinput single-output (MISO) channel  $\mathbf{h}_1$  ( $n_R = 1$ ), then, the MISO channels  $\hat{\mathbf{h}}_1 = \left[ \Re(\hat{\mathbf{h}}_1^T), \Im(\hat{\mathbf{h}}_1^T) \right]^T$  satisfying (3) belongs to a subspace of dimension less or equal than 2 with probability one.

Finally, the following theorem ensures the blind identification for almost all the channels with  $n_R > 1$ .

**Theorem 5** Given an identifiable OSTBC and a MIMO channel H with  $n_R > 1$ , then, the associated OSTBC-MIMO channel is identifiable with probability 1.

**Proof 5** The proof is based on Conjecture 1. Considering an identifiable OSTBC, there does not exist any ambiguity matrix  $\mathbf{Q} \neq \mathbf{I}$  satisfying (3) for all **H**. Then, taking into account that **Q** is fixed by the first MISO channel, the OSTBC-MIMO channel is non-identifiable iff  $\mathbf{\tilde{h}}_2$  belongs to the subspace of channels with associated ambiguity matrix **Q**, which happens with probability zero.

## 5. FURTHER DISCUSSION AND EMPIRICAL ANALYSIS

The combination of Theorems 2, 3 and 5 allow us to explain previous results obtained by others authors. Table 1 shows the main results in [4], where we have added a column with the transmission rate R = M/L and the threshold derived from Theorem 3. As can be seen, any OSTBC transmitting at a rate  $R > R_{th}$  permits the channel identification with  $n_R > 1$  receive antennas, as predicted by Theorems 3 and 5. Furthermore, we must note that the condition on the transmission rate is very restrictive and there are only six OSTBC examples with  $R \leq R_{th}$ , which are the following:

- Alamouti codes: As pointed out in [2], it is impossible to achieve blind identification for the Alamouti code without assuming a correlation matrix **R**<sub>s</sub> with non-equal eigenvalues.
- **Real OSTBC**  $(n_T = M = L = 4)$ : Analogously to the Alamouti code, this is a non-identifiable code with practical application.
- **Real OSTBC**  $(n_T = 3, M = L = 4)$ : In this case, Theorem 2 implies that the code is identifiable, and Theorem 5 explains the blind identifiability of the channel with  $n_R > 1$ .
- Complex OSTBCs ( $n_T = 3, 4, M = 4, L = 8$ ): These codes have not practical interest due to the existence of OSTBCs with the same number of antennas ( $n_T$ ), a lower delay (L), and a higher transmission rate (R).

From the above results, and taking into account that the threshold in the transmission rate decreases rapidly with  $n_T$ , we can state that the only non-identifiable OSTBCs with practical interest are the Alamouti code and the real orthogonal design with  $n_T = M = L = 4$ . Furthermore, we must note that the obtained theoretical and empirical results ensure that certain OSTBCs, such as the real and complex codes with  $M = n_T = 8$ , are identifiable, which contradices the identifiability condition stated in [3]. Furthermore, in [5] the authors claim that the MIMO channel can not be identified when using the complex OSTBC with  $n_T = 3$ , M = 4, L = 8. The obtained theorems proves the identifiability of this code and validate the empirical results in [4] and Table 1.

Finally, we must point out that the study of SOS blind channel identifiability of OSTBC systems is still an open issue. Further lines include the derivation of necessary identifiability conditions, the proof of Conjecture 1 (which has been validated by means of numerical examples), and the derivation of tighter transmission rate thresholds for non-identifiable real OSTBCs.

## 6. CONCLUSIONS

In this paper we have presented identifiability conditions for blind multiple-input multiple-output (MIMO) channel identification based on second order statistics (SOS) of orthogonal space-time block coded (OSTBC) systems. The analysis, which does not exploit possible finite alphabet constraints on the information symbols, shows that, if the OSTBC is identifiable and the number of receive antennas is greater than one, the MIMO channel can be identified with probability one. The study reveals that the identifiability characteristics of OSTBCs are related to their underlying structure. Specifically, we have derived a threshold on the transmission rate, which is inversely proportional to the number of transmit antennas, and proved that any OSTBC with a higher transmission rate is identifiable. Finally, we have presented additional discussions and validated the obtained results by means of numerical examples.

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