Joint Channel Tracking and MAP Detection for OFDMA Systems in Fading Channels

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Abstract— **In this research, we present a joint channel tracking and maximal *a posteriori* (MAP) symbol detection method for orthogonal frequency division multiple access (OFDMA) systems in time-varying fading channels. In contrast to some existing joint estimation and detection schemes that perform channel tracking in time-domain, we propose an algorithm that do both channel tracking and symbol detection in frequency domain, thus preventing the performance degradation due to the multiple access interference present in the time domain channel estimation. In conjunction with appropriate frequency allocation for each user, it is shown that a low-complexity algorithm can be obtained and, in the mean time, provides good performance in channel tracking.**

Keywords— **OFDM, OFDMA, EM, MAP, Joint estimation and detection.**

I. Introduction

Orthogonal frequency division multiplexing/multiple access (OFDM/OFDMA) are considered promising technologies for next-generation broadband mobile wireless access (BMWA) due to their outstanding architecture for multiple access and making combating severe inter-symbol interference (ISI) easy in BMWA. Despite the advantage of OFDM in combating ISI, the complexity of tracking all frequency-domain channel parameters of an OFDM system is high, due to its large number of frequency tones. To reduce the complexity in channel tracking, for OFDM systems, a common practice is to perform channel tracking in time-domain and then transform the time-domain channel estimates back into frequency-domain for symbol detection, using the fast Fourier transform (FFT). There has a number of research results obtained in this regard, among which [1, 2], *e.g.*, perform channel tracking using hard symbol decision feedbacks while [3, 4] do it with soft symbol information feedbacks based on the expectation-maximization (EM) algorithm. It has been shown in [4, 6], among others, that joint channel tracking and symbol detection based on the EM algorithm can outperform algorithms using hard decision feedbacks.

Despite the rich research results in joint channel tracking and symbol detection for OFDM systems, the idea of performing channel tracking in time-domain for OFDMA systems is more complicated and may lead to inferior performance, due to the severe multiple access interference (MAI) present in the time-domain received signal. Channel tracking in frequency-domain not only shields off the MAI present in time-domain channel estimation, it also avoids the algorithm complexity resulting from FFT and the suppression of MAI and ISI occurring in time-domain signal processing. Based on our previous results in [4, 6], in this research, we present a frequency-domain joint channel estimation and symbol detection approach for OFDMA systems and show that if appropriate frequency allocation can be made for each user in the system, then a low-complexity algorithm can be obtained and, in the mean time, provides good performance in channel tracking and symbol detection.

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II. System Model

We consider an OFDMA system where there is a number of users sharing the available bandwidth of the system. The number of total subcarriers of the system is denoted by N and the number of users in the system is K. The modulated symbols of a user is first put on its designated subcarriers and transformed with the inverse fast Fourier transform (IFFT). The resultant signal is then cyclic-prefixed and transmitted over a time-varying multiple-path fading channel $\mathbf{h}_k(t) = \{h_{k:0}(t), \ldots, h_{k:L_k-1}(t)\}$. The number of channel taps, L_k, for each user is considered smaller than or equal to the length of the cyclic-prefix, L_{cp}, which is a system parameter designed to combat ISI.

At the receiver, the received time-domain signal collected for the m-th OFDM symbol interval is $\mathbf{y}_m = \{y_{(m-1)(N+L_{cp})+L_{cp},\cdots,m(N+L_{cp})-1)}\}$, after removing the cyclic prefix. In this research, we assume signal synchronization has been achieved before channel estimation and symbol detection. Thus, without considering the effects of frequency and timing offsets, the frequency-domain received signal vector Y_m of dimension $N \times 1$, after the FFT of $\mathbf{y}_m$, is given by

$$Y_m = H_m X_m + N_m \tag{1}$$

where X_m is the vector of the frequency-domain transmitted symbols and H_m is the corresponding channel response in frequency-domain. The noise vector N_m is considered zero-mean and additive-white complex Gaussian (AWGN) distributed, and is denoted by $\mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$. We note that in this paper a bold-faced lower-case character $\mathbf{x}$ represents a vector in time-domain, yet an upper-case X stands for a vector in frequency-domain. In addition, a bold-faced upper-case $\mathbf{X}$ denotes a matrix either in time- or frequency-domain.

In an OFDMA system, a single user is unlikely to occupy the entire bandwidth of the system. Therefore, without loss of generality, the set of frequency bins allocated to user K is defined as $\{f_k^1, \ldots, f_k^{N_k}\}$, where $1 \leq f_k^i \leq N$ and N_k is the number of bins used by user k. As a result, the corresponding frequency-domain received signal and channel response of user k are given, respectively, by

$Y_{k:m} \triangleq \{Y^1_{k:m}, \ldots, Y^{N_k}_{k:m}\}$ and $H_{k:m} \triangleq \{H^1_{k:m}, \ldots, H^{N_k}_{k:m}\}$. The frequency-domain model for the received signal of user k is thus expressed as

$$Y_{k:m} = H_{k:m} X_{k:m} + N_{k:m} \tag{2}$$

where $N_{k:m} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_k})$.

III. Model of the Frequency-Domain Channel Dynamics

To track the time-varying channel parameters, a model for the channel dynamics is usually required. A widely used model for fading channel is the auto-regressive (AR) model. Define an extended channel vector $\underline{\mathbf{h}}_{k:m-1} \triangleq [\mathbf{h}^T_{k:m-1}, \cdots, \mathbf{h}^T_{k:m-L_h}]^T$, using the AR model, the channel dynamics is characterized as

$$\begin{aligned}\mathbf{h}_{k:m} &= \mathbf{F}_k \underline{\mathbf{h}}_{k:m-1} + \mathbf{B}_k \mathbf{v}_{k:m} \\ &\triangleq [\mathbf{F}_{k:1}, \mathbf{F}_{k:2}, \ldots, \mathbf{F}_{k:L_h}]\, \underline{\mathbf{h}}_{k:m-1} + \mathbf{B}_k \mathbf{v}_{k:m}\end{aligned} \tag{3}$$

where L_h is the order of the AR model, and $\mathbf{F}_{k:i}$ and $\mathbf{B}_{k:i}$ are diagonal matrices of $L_k \times L_k$ for wide-sense stationary uncorrelated scattering (WSSUS) fading channels. The noise vector, $\mathbf{v}_{k:m}$, has the density function of $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{L_k})$.

The above dynamic model is essentially for channel parameters in time-domain, *i.e.* $\mathbf{h}_{k:m}$. Since we are interested in channel tracking in frequency-domain, a dynamic model for the fading channel parameters in frequency-domain is necessary. Denote the FFT matrix of dimension $N \times N$ by $\mathbf{W}_N$, it is obvious that the frequency-domain response of $\mathbf{h}_{k:m}$ is given by $\widetilde{\mathbf{W}}_{L_k} \mathbf{h}_{k:m}$, where $\widetilde{\mathbf{W}}_{L_k}$ is a sub-matrix of $\mathbf{W}_N$ formed from the column 1 to column L_k of $\mathbf{W}_N$. However, only the frequency-domain channel responses used by user k are interested, to reduced the complexity in channel tracking, we essentially require a dynamic model for $H_{k:m}$. Let e^i_k be a row vector of $1 \times N$ with the f^i_k-th index equal to one only and the others zero. If we define a sampling matrix for user k as $\mathbf{J}_k \triangleq [(e^1_k)^T, \cdots, (e^{N_k}_k)^T]^T$, then we have

$$H_{k:m} = \mathbf{J}_k \widetilde{\mathbf{W}}_{L_k} \mathbf{h}_{k:m} \triangleq \mathbf{W}_k \mathbf{h}_{k:m} \tag{4}$$

where we have $\mathbf{W}_k \triangleq \mathbf{J}_k \widetilde{\mathbf{W}}_{L_k}$, which is of dimension $N_k \times L_k$. We next apply this equation to derive an AR model for $H_{k:m}$.

Let $\underline{H}_{k:m-1} \triangleq [H^T_{k:m-1}, \cdots, H^T_{k:m-L_h}]^T$. We define an AR model for $H_{k:m}$ as

$$H_{k:m} = \mathcal{F}_k \underline{H}_{k:m-1} + \mathcal{B}_k V_{k:m} \tag{5}$$

where $\mathcal{F}_k$ is of dimension $N_k \times N_k L_h$ and $\mathcal{B}_k$ of $N_k \times N_k$. It is obvious that $E\{H_{k:m} \underline{H}^H_{k:m-1}\} = \mathcal{F}_k E\{\underline{H}_{k:m-1} \underline{H}^H_{k:m-1}\}$, where $E\{\cdot\}$ stands for expectation. Substituting (4) into this equation gives

$$\begin{aligned}&\mathbf{W}_k [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}](\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k) \\ &= \mathcal{F}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}_k) \underline{\mathbf{R}}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)\end{aligned} \tag{6}$$

where $\otimes$ stands for the Kronecker product. For WSSUS channels, $\mathbf{R}_{k:i} = E\{\mathbf{h}_{k:m-p} \mathbf{h}^H_{k:m-p\pm i}\}$ is a diagonal matrix, and $\underline{\mathbf{R}}_k = E\{\underline{\mathbf{h}}_{k:m-1} \underline{\mathbf{h}}^H_{k:m-1}\}$ is a block matrix with its sub-matrix at the i-th row and the j-th column equal to $[\underline{\mathbf{R}}_k]_{i,j} = \mathbf{R}_{k:|i-j|}$, $i, j = 1, \ldots, L_h$.

Recall that the dimension of $\mathbf{W}_k$ is $N_k \times L_k$. If $N_k > L_k$, then there exists the left inverse of $\mathbf{W}_k$, which is

$$\mathbf{W}^\dagger_k \triangleq (\mathbf{W}^H_k \mathbf{W}_k)^{-1} \mathbf{W}^H_k. \tag{7}$$

In this case, it is straightforward to show that

$$\begin{aligned}\mathcal{F}_k &= \mathbf{W}_k [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}] \underline{\mathbf{R}}^{-1}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^\dagger_k) &(8)\\ &= \mathbf{W}_k \mathbf{F}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^\dagger_k). &(9)\end{aligned}$$

The last equality results from the fact that $\mathbf{F}_k = [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}] \underline{\mathbf{R}}^{-1}_k$, which can be easily verified from (3). We note that the complexity of $\underline{\mathbf{R}}^{-1}_k$ is actually low due to the fact the sub-matrix $\mathbf{R}_{k:i}$ of $\underline{\mathbf{R}}_k$ is diagonal, and thus the sub-matrix of $\underline{\mathbf{R}}^{-1}_k$ is also a diagonal matrix. Moreover, let $\mathbf{R}_{k:i} = \mathrm{diag}\{r_{i,0}, r_{i,1}, \cdots, r_{i,L_k-1}\}$, $i = 0, \cdots, L_h - 1$, and define the sub-matrix of $\underline{\mathbf{R}}^{-1}_k$ to be $\mathbf{G}_{k:i} \triangleq \mathrm{diag}\{g_{i,0}, g_{i,1}, \cdots, g_{i,L_k-1}\}$, it is easy to verify that

$$\begin{bmatrix} g_{0,j} & g_{1,j} & \cdots & g_{L_h-1,j} \\ g_{1,j} & g_{0,j} & \cdots & \vdots \\ \vdots & \ddots & \ddots & g_{1,j} \\ g_{L_h-1,j} & \cdots & g_{1,j} & g_{0,j} \end{bmatrix} = \begin{bmatrix} r_{0,j} & r_{1,j} & \cdots & r_{L_h-1,j} \\ r_{1,j} & r_{0,j} & \cdots & \vdots \\ \vdots & \ddots & \ddots & r_{1,j} \\ r_{L_h-1,j} & \cdots & r_{1,j} & r_{0,j} \end{bmatrix}^{-1}, j = 0, \cdots, L_k - 1 \tag{10}$$

where for Rayleigh fading channels, $r_{i,j} = p_j J_0(2\pi i f_d T_s)$, p_j is the variance of path j, and $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first-kind. Since the channel order L_h is usually set to be 1 or 2, we can immediately obtain the coefficients $g_{i,j}, i = 0, \cdots L_h - 1$ and $j = 0, \cdots L_k - 1$, virtually without any matrix inversion.

In addition to $\mathcal{F}_k$, it also requires $\mathcal{B}_k \mathcal{B}^H_k$ in channel tracking. Based on the result in (8) and the fact that $\mathbf{B}_k \mathbf{B}^H_k = \mathbf{R}_{k:0} - [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}] \underline{\mathbf{R}}^{-1}_k [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}]^H$, it is not difficult to show from (5) that

$$\mathcal{B}_k \mathcal{B}^H_k = \mathbf{W}_k \mathbf{B}_k \mathbf{B}^H_k \mathbf{W}^H_k. \tag{11}$$

On the other hand, for the case of $N_k \leq L_k$, $(\mathbf{I}_{L_h} \otimes \mathbf{W}_k) \underline{\mathbf{R}}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)$ in (6) becomes full-ranked, leading to

$$\begin{aligned}\mathcal{F}_k = \; &\mathbf{W}_k [\mathbf{R}_{k:1}, \cdots, \mathbf{R}_{k:L_h}] (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k) \times \\ &[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k) \underline{\mathbf{R}}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)]^{-1}.\end{aligned} \tag{12}$$

As a result, we have

$$\mathcal{B}_k \mathcal{B}^H_k = \mathbf{W}_k \mathbf{R}_{k:0} \mathbf{W}^H_k - \mathcal{F}_k [(\mathbf{I}_{L_h} \otimes \mathbf{W}_k) \underline{\mathbf{R}}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)] \mathcal{F}^H_k. \tag{13}$$

In (8) where $N_k > L_k$ or in (12) and (13) for the case of $N_k \leq L_k$, the computational complexity of $\mathbf{W}^\dagger_k$ or $[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k) \underline{\mathbf{R}}_k (\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)]^{-1}$ seems to be high. However the complexity can be greatly reduced if the frequency bins allocated to user k possess certain structure. We will give detailed discussions on the complexity reduction after we introduce the recursive expectation maximization (EM) algorithm for joint channel estimation and MAP detection.

IV. Joint Channel Estimation and Symbol Detection over Time-Varying Channels

In the absence of both the transmitted symbols and channel state information, the receiver must in some ways perform channel estimation and symbol detection jointly in order to recover the transmitted data. For time-varying channels, it has been shown in [5–7] that the recursive EM algorithm is more suitable for joint channel tracking and symbol detection. In this work, we employ the recursive EM algorithm to perform joint channel estimation and symbol detection both in frequency domain, aiming for a low-complexity algorithm for OFDMA systems.

We define the observation for user k up to time M as $\mathbf{Y}_{k:M} \triangleq \{Y_{k:M}, \cdots, Y_{k:1}\}$ and the unknown parameter set as $\Theta_{k:M} = [H_{k:M}, \cdots, H_{k:1}]$. The hidden state of the system is $\Psi_{k:M} \triangleq \{X_{k:M}, \cdots, X_{k:1}\}$. It is clear that the complete data for estimating the parameter set $\Theta_{k:M}$ is $\{\mathbf{Y}_{k:M}, \Psi_{k:M}\}$. Given that $\Psi_{k:M}$ is not in fact observed, the *incomplete* log likelihood of $\Theta_{k:M}$ is given by

$$\log P(\mathbf{Y}_{k:M}; \Theta_{k:M}) = \log E_{\Psi_{k:M}}\{P(\mathbf{Y}_{k:M}, \Psi_{k:M}; \Theta_{k:M})\}, \tag{14}$$

where $E_{\Psi_{k:M}}\{\cdot\}$ is the expectation w.r.t. $\Psi_{k:M}$. It is in general difficult to estimate $\Theta_{k:M}$ from this LLK. To reduce the complexity, we use the EM algorithm to approach it iteratively. To this end, we first define a Kullback-Liebler (K-L) measure of $\Theta_{k:M}$ at iteration ℓ as

$$\sum_{\{\Psi_{k:M}\}} \log\{P(\mathbf{Y}_{k:M}|\Psi_{k:M}; \Theta_{k:M})\} P(\Psi_{k:M}|\mathbf{Y}_{k:M}; \widehat{\Theta}^{\ell-1}_{k:M}), \tag{15}$$

where $\widehat{\Theta}^{\ell-1}_{k:M}$ is the estimate of $\Theta_{k:M}$ at iteration $\ell-1$. Given $P(X_{k:m}|\mathbf{Y}_{k:M}; \widehat{\Theta}^{\ell-1}_{k:M}),\ m = 1, \ldots, M$, the K-L measure can be rewritten as

$$Q_M(\Theta_{k:M}|\widehat{\Theta}^{\ell-1}_{k:M}) \triangleq -\log\{\pi^N \sigma_n^{2N_k}\}$$
$$-\sum_{m=1}^{M} E_{X_{k:m}}\left\{\frac{\|Y_{k:m} - X_{k:m}H_{k:m}\|^2}{\sigma_n^2}|\mathbf{Y}_{k:M}; \widehat{\Theta}^{\ell-1}_{k:M}\right\}, \tag{16}$$

where $E_{X_{k:m}}\{\cdot|\mathbf{Y}_{k:M}; \widehat{\Theta}^{\ell-1}_{k:M}\}$ is expectation w.r.t. $X_{k:m}$, using $P(X_{k:m}|\mathbf{Y}_{k:M}; \widehat{\Theta}^{\ell-1}_{k:M}) \propto P(\mathbf{Y}_{k:M}|X_{k:m}; \widehat{\Theta}^{\ell-1}_{k:M})$ in frequency domain. The EM algorithm can be stated as

E-step: Compute $Q_M(\Theta_{k:M}|\widehat{\Theta}^{\ell-1}_{k:M})$;

M-step: $\Theta^{\ell}_{k:M} = \arg\max_{\Theta_{k:M}} Q_M(\Theta_{k:M}|\widehat{\Theta}^{\ell-1}_{k:M})$.

Given the state $\widehat{\Theta}^{\ell-1}_{k:M}$, the new state $\widehat{\Theta}^{\ell}_{k:M}$ is obtained by maximizing the K-L measure w.r.t. $\Theta_{k:M}$. By iterating the above procedure, the EM algorithm is guaranteed to converge to a locally steady state $\widehat{\Theta}_{k:M}$ [8]. For dynamic channels, direct optimization w.r.t. $\Theta_{k:M}$ is extremely complicated. To alleviate the complexity of optimization, we use the recursive optimization method introduced in [9] to develop a recursive algorithm , which is written as

$$\underline{\widehat{H}}^{\ell}_{k:m} = \underline{\widehat{H}}_{k:m|m-1} - \left(\frac{\partial^2 Q_m(\Theta_{k:m}|\widehat{\Theta}^{\ell-1}_{k:m})}{\partial^2 \underline{H}_{k:m}}\bigg|_{\underline{\widehat{H}}_{k:m|m-1}}\right)^{-1}$$
$$\cdot\left(\frac{\partial Q_m(\Theta_{k:m}|\widehat{\Theta}^{\ell-1}_{k:m})}{\partial \underline{H}^*_{k:m}}\bigg|_{\underline{\widehat{H}}_{k:m|m-1}}\right),\ m = 0, \cdots, M, \tag{17}$$

where $\underline{H}_{k:m} = [H^T_{k:m}, \cdots, H^T_{k:m-L_h+1}]^T$, $\frac{\partial^2 Q_i(\cdot|\cdot)}{\partial^2 \underline{H}_{k:i}} \triangleq \frac{\partial^2 Q_i(\cdot|\cdot)}{\partial \underline{H}^*_{k:i}\partial \underline{H}^T_{k:i}}$, $\widehat{\Theta}^{\ell}_{k:m} = \{\hat{H}^{\ell}_{k:m}, \widehat{\Theta}^{\mathcal{L}}_{k:m-1}\}$, with $\mathcal{L}$ being the total number of iterations at each time step, and $\underline{\hat{H}}_{k:m|m-1} = f(\widehat{\Theta}^{\mathcal{L}}_{k:m-1})$ being the predicted value given by the dynamic evolution function of H_m. Based on the recursive EM algorithm, we will present two recursive estimators for blind dynamic channel tracking, one for the case of $N_k > L_k$, and the other for the case of $N_k \le L_k$.

Let $\widetilde{S}_{k:m} \triangleq E\{X_{k:m}|\mathbf{Y}_{k:m}; \widehat{\Theta}^{\ell-1}_{k:m}\}/\sigma_n^2$ and $\widetilde{C}_{k:m} \triangleq E\{X_{k:m}X^H_{k:m}|\mathbf{Y}_{k:m}; \widehat{\Theta}^{\ell-1}_{k:m}\}/\sigma_n^2$, which is a diagonal matrix. In addition, define $\mathbf{P}_{k:m} \triangleq \left(-\frac{\partial^2 Q_m(\Theta_{k:m}|\widehat{\Theta}^{\ell-1}_{k:m})}{\partial^2 \underline{H}_{k:m}}\Big|_{\underline{\widehat{H}}_{k:m|m-1}}\right)^{-1}$ which is essentially the variance of channel estimate. We give below the recursive minimum mean-squared error (MMSE) channel estimators for users with assigned tones $N_k > L_k$ and $N_k \le L_k$, respectively

A. Recursive MMSE channel Estimator for $N_k > L_k$

Define $\mathbf{R}^H_{k:m} \triangleq [\mathbf{I}_{L_k}| - \mathbf{F}^H_k](\mathbf{I}_{L_h+1} \otimes \mathbf{W}^{\dagger}_k)$ and

$$\mathbf{P}_{k:m|m-1} \triangleq \begin{bmatrix} \widetilde{C}^{-1}_{k:m} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{k:m-1} \end{bmatrix}. \tag{18}$$

The recursive MMSE channel estimator is given by

$$\underline{\widehat{H}}_{k:m} = \begin{bmatrix} \widetilde{C}^{-1}_{k:m}\widetilde{S}_{k:m}Y_{k:m} \\ \underline{\widehat{H}}_{k:m-1} \end{bmatrix} - \mathbf{P}_{k:m|m-1}\mathbf{R}_{k:m}\left[\mathbf{B}_k\mathbf{B}^H_k + \right.$$
$$\left.\mathbf{R}^H_{k:m}\mathbf{P}_{k:m|m-1}\mathbf{R}_{k:m}\right]^{-1}\mathbf{R}^H_{k:m}\begin{bmatrix} \widetilde{C}^{-1}_{k:m}\widetilde{S}_{k:m}Y_{k:m} \\ \underline{\widehat{H}}_{k:m-1} \end{bmatrix}. \tag{19}$$

The recursive update of $\mathbf{P}_{k:m}$ is given by

$$\mathbf{P}_{k:m} = \mathbf{P}_{k:m|m-1} - \mathbf{P}_{k:m|m-1}\mathbf{R}_{k:m}\left[\mathbf{B}_k\mathbf{B}^H_k + \right.$$
$$\left.\mathbf{R}^H_{k:m}\mathbf{P}_{k:m|m-1}\mathbf{R}_{k:m}\right]^{-1}\mathbf{R}^H_{k:m}\mathbf{P}_{k:m|m-1}. \tag{20}$$

We note that no exact information of $X_{k:m}$ is required in this estimator. The stochastic information of data is provided to the estimator via $\widetilde{C}_{k:m}$ and $\widetilde{S}_{k:m}$ through iterations.

B. Recursive MMSE channel Estimator for $N_k \le L_k$

For user k with $N_k \le L_k$, we use $\mathcal{F}_k$ and $\mathcal{B}_k\mathcal{B}^H_k$ defined in (12) and (13), respectively. Let

$$\mathbf{P}_{k:m|m-1} \triangleq \begin{bmatrix} \mathcal{B}_k\mathcal{B}^H_k + \mathcal{F}_k\mathbf{P}_{k:m-1}\mathcal{F}^H_k & \mathcal{F}_k\mathbf{P}_{k:m-1} \\ \mathbf{P}_{k:m-1}\mathcal{F}^H_k & \mathbf{P}_{k:m-1} \end{bmatrix}. \tag{21}$$

The recursive MMSE channel estimator is given by

$$\underline{\widehat{H}}_{k:m} = \begin{bmatrix} \mathcal{F}_k\underline{\widehat{H}}_{k:m-1} \\ \underline{\widehat{H}}_{k:m-1} \end{bmatrix} + \begin{bmatrix} \mathcal{B}_k\mathcal{B}^H_k + \mathcal{F}_k\mathbf{P}_{k:m-1}\mathcal{F}^H_k \\ \mathbf{P}_{k:m-1}\mathcal{F}^H_k \end{bmatrix}[\widetilde{C}^{-1}_{k:m} +$$
$$\mathcal{B}_k\mathcal{B}^H_k + \mathcal{F}_k\mathbf{P}_{k:m-1}\mathcal{F}^H_k]^{-1}(\widetilde{C}^{-1}_{k:m}\widetilde{S}_{k:m}Y_{k:m} - \mathcal{F}_k\underline{H}_{k:m-1}).$$

The corresponding variance of channel estimation is

$$\mathbf{P}_{k:m} = \mathbf{P}_{k:m|m-1} - \mathbf{P}_{k:m|m-1}\begin{bmatrix} \mathbf{I}_{N_k} \\ \mathbf{0} \end{bmatrix}[\widetilde{C}^{-1}_{k:m} +$$
$$\mathcal{B}_k\mathcal{B}^H_k + \mathcal{F}_k\mathbf{P}_{k:m-1}\mathcal{F}^H_k]^{-1}[\mathbf{I}_{N_k}\ \ \mathbf{0}]\mathbf{P}_{k:m|m-1}. \tag{22}$$

V. Low-Complexity Algorithm

As it has been shown in (19) and (20) that the complexity for implementing channel tracking for $N_k > L_k$ lies in the calculation of $\mathbf{W}^{\dagger}_k$ in $\mathbf{R}_k$ which, as shown in (7), requires matrix inversion of $L_k \times L_k$. On the other hand, for users with $N_k \le L_k$, the algorithm complexity is mainly dominated by the matrix inversion of $[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k)\underline{\mathbf{R}}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}^H_k)]$ which is required for the evaluation of $\mathcal{F}_k$ and $\mathcal{B}_k\mathcal{B}^H_k$ as shown in (12) and (13), respectively. If the matrices that require inversion can be reduced to diagonal matrices or matrices whose inversion can be implemented with fast algorithms, then the entire complexities of the schemes can be greatly reduced, even if matrices $\mathbf{W}_k^{\dagger}$, $\mathcal{F}_k$ and $\mathcal{B}_k\mathcal{B}_k^H$ can be essentially calculated off-line before real-time channel tracking.

For users with $N_k > L_k$, the complexity for $(\mathbf{W}_k^H\mathbf{W}_k)^{-1}$ could be quite large. However if the frequency bins associated with each user are partitioned into Q clusters, with Q being an even number, $L_k \leq Q \leq N_k/2$, and the clusters are evenly spaced among $0, \cdots, N-1$. In addition, if the frequency bins in every cluster are allocated to the same relative positions in each cluster, then it can be shown that $\mathbf{W}_k^{\dagger} = \frac{N}{L_k}\mathbf{W}_k^H$, *i.e.* $(\mathbf{W}_k^H\mathbf{W}_k)^{-1} = \frac{N}{L_k}\mathbf{I}_{L_k}$.

On the other hand, for users with $N_k \leq L_k$, if the frequency bins of a user are evenly spaced by N/N_k and $N_k = 2^p,\ N = 2^q,\ p, q = 2, 3, \cdots$ and $p < q$, then it can be shown that the sub-matrices $\mathbf{W}_k\mathbf{R}_{k:i}\mathbf{W}_k^H, i = 0, \cdots, L_h - 1$, of $[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k)\underline{\mathbf{R}}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}_k^H)]$ in $\mathcal{F}_k$ and $\mathcal{B}_k\mathcal{B}_k^H$ is circulant. This means that $\mathbf{W}_k\mathbf{R}_{k:i}\mathbf{W}_k^H$ can be factorized into $\mathbf{W}_{N_k}^H\Sigma_{k:i}\mathbf{W}_{N_k}$ where $\Sigma_{k:i}$ is a diagonal matrix of dimension $N_k \times N_k$ and $\mathbf{W}_{N_k}$ is a FFT matrix of dimension $N_k \times N_k$ too, with $N_k \leq L_k \leq L_{cp}$. Under this type of frequency allocation of user k, we have $[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k)\underline{\mathbf{R}}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}_k^H)] = [(\mathbf{I}_{L_h} \otimes \mathbf{W}_{N_k}^H)\underline{\Sigma}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}_{N_k})]$ and, hence, $[(\mathbf{I}_{L_h} \otimes \mathbf{W}_k)\underline{\mathbf{R}}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}_k^H)]^{-1} = [(\mathbf{I}_{L_h} \otimes \mathbf{W}_{N_k}^H)\underline{\Sigma}_k^{-1}(\mathbf{I}_{L_h} \otimes \mathbf{W}_{N_k})]$, with the submatrix of $\underline{\Sigma}_k$ at the i-th row and the j-th column equal to $\Sigma_{k:|i-j|}, i, j = 1, \cdots, L_h$. Now, using the same method that we applied to evaluate $\underline{\mathbf{R}}_k^{-1}$ in (10), it is very simple to obtain $\underline{\Sigma}_k^{-1}$ since L_h is usually one or two. Let $\underline{\Delta}_k \triangleq \underline{\Sigma}_k^{-1}$. Substituting the above results back into (12) and (13), it is straightforward to show that

$$\mathcal{F}_k = \mathbf{W}_{N_k}^H[\Sigma_{k:1}, \cdots, \Sigma_{k:L_h}]\underline{\Delta}_k(\mathbf{I}_{L_h} \otimes \mathbf{W}_{N_k}) \quad (23)$$

and

$$\begin{aligned}\mathcal{B}_k\mathcal{B}_k^H &= \mathbf{W}_{N_k}^H\Sigma_{k:0}\mathbf{W}_{N_k} - \\ &\mathbf{W}_{N_k}^H[\Sigma_{k:1}, \cdots, \Sigma_{k:L_h}]\underline{\Delta}_k[\Sigma_{k:1}, \cdots, \Sigma_{k:L_h}]^H\mathbf{W}_{N_k}. \quad (24)\end{aligned}$$

VI. Simulation Results

We now present some simulation results to demonstrate the performance of the proposed frequency-domain joint channel tracking and MAP detection scheme. The total number of tones in this OFDMA system is $N = 128$. There are five users in the system, with their corresponding numbers of tones being $\{24, 8, 16, 4, 2\}$, respectively. The length of cyclic prefix $L_{cp} = 8$, and the number of transmission paths L_k for each user is randomly generated with $L_k \leq L_{cp}$. The channel coefficients for each user are generated using Jake's model with the $f_dT_s = 0.01$ where T_s is the OFDM symbol time. To present the robustness of the proposed scheme, the receiver has no knowledge of L_k and simply assumes $L_k = 8$ for each user in doing channel tracking.

Fig. 1 shows the channel gain and phase for the first tone of user 3 and Fig. 2 presents the similar results for the first tone of user 5. It is clear that channel gains can be closely tracked, while there exist phase ambiguities in phase plots. This phase ambiguity problem can be circumvented by using differential encoding. In the simulations, we use DQPSK for each user.

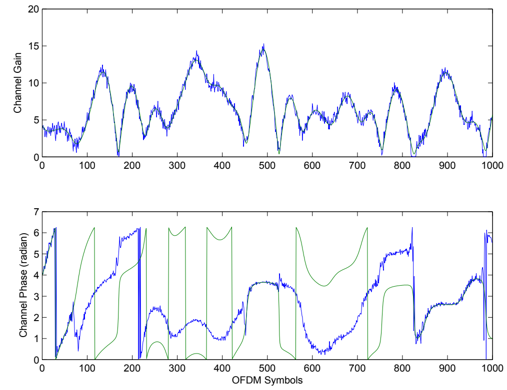

Fig. 1. Channel Tracking for the first tone of user 3 at $E_b/N_0 = 30$dB.

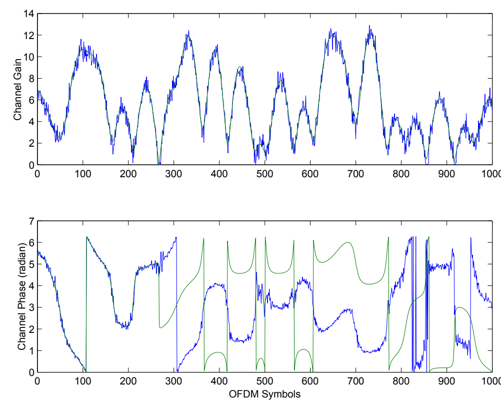

Fig. 2. Channel Tracking for the first tone of user 5 at $E_b/N_0 = 30$dB.

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