UPLINK CHANNEL ESTIMATION FOR OFDMA SYSTEM

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ABSTRACT

This paper proposes uplink channel estimation schemes for OFDMA system. If frequency responses of assigned subcarriers between corresponding users and base station (BS) are put together into one frequency selective fading channel, time domain expression of the channel, in other words, overall channel impulse response has a significant property, namely, only about the same number of elements as the length of CP have nonzero values. Based on the analysis, we propose a least square channel estimation scheme and a simple remedy for conventional channel estimation scheme. Computer simulation results show that the proposed schemes can significantly improve BER performance.

Index Terms— OFDMA, channel estimation, cyclic prefix

1. INTRODUCTION

Recently, much effort to apply block transmission schemes with cyclic prefix (CP), such as orthogonal frequency division multiplexing (OFDM)[1] and single carrier block transmission with CP (SC-CP)[2],[3], to mobile communications systems has been made as typified by WiMAX system, where OFDM access (OFDMA)[4] is adopted for the physical layer /medium access control layer protocol. One of the most serious difficulties in the OFDMA system is uplink channel estimation since the received signals are distorted by multiple frequency selective fading channels of number of active users, while, in the downlink, all received signal components are distorted by one frequency selective fading channel. Moreover, in the WiMAX system, no preamble symbol is employed in the uplink, which makes uplink channel estimation further difficult.

In this paper, we propose channel estimation schemes for OFDMA uplink. If frequency responses of assigned subcarriers between corresponding users and base station (BS) are put together into one frequency selective fading channel, time domain expression of the channel, in other words *overall channel impulse response* has a significant property. The derivation of the delay power spectrum of the overall impulse response reveals a fact that only the same number of (or a bit more) elements as the length of the CP have nonzero values, if consecutive subcarriers are allocated to each user. This means that we can remarkably reduce the number of parameters to be estimated. Based on the analysis, we propose a least square channel estimation scheme and a simple remedy for conventional channel estimation scheme. Computer simulation results show that the proposed schemes can significantly improve the bit error rate (BER) performance.

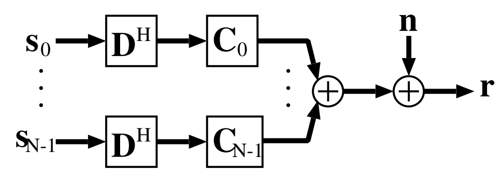

Fig. 1. System model.

2. SIGNAL MODELING

The following notations are used for describing the proposed schemes. M is the FFT size and L is the channel order, which is assumed to be the same as the guard interval. An $M \times M$ identity matrix will be denoted as $\mathbf{I}_M$, a zero matrix of size $A \times B$ will be denoted as $\mathbf{0}_{A\times B}$, all 1 matrix of size $A \times B$ as $\mathbf{1}_{A\times B}$, and a discrete Fourier transform (DFT) matrix of size $M \times M$, whose (i, j) element is $\frac{1}{\sqrt{M}}e^{-j\frac{2\pi(i-1)(j-1)}{M}}$, as $\mathbf{D}$. We will use $E[\cdot]$ to denote ensemble average, $(\cdot)^T$ for transpose, $(\cdot)^H$ for Hermitian transpose, diag$[\cdot]$ for diagonal matrix, and $(\cdot)^*$ for complex conjugate.

Fig. 1 shows a system model considered in this paper. N users transmit information bearing signal vectors $\mathbf{s}_n$ ($n = 0, \dots, N-1$) after performing inverse DFT (IDFT) and the insertion of the CP. The received signal vector at the BS after the CP removal is given by

$$\mathbf{r} = \begin{bmatrix} r_0 & \cdots & r_{M-1} \end{bmatrix}^T = \sum_{n=0}^{N-1} \mathbf{C}_n \mathbf{D}^H \mathbf{s}_n + \mathbf{n}, \qquad (1)$$

where $\mathbf{n} = [n_0, \dots, n_{M-1}]^T$ is an additive noise vector. $\mathbf{C}_n$ is a circulant channel matrix of the n-th user, whose first column is $[h_0^n \ \cdots \ h_L^n \ \mathbf{0}_{(M-L-1)\times 1}]^T$, where $\{h_0^n, \dots, h_L^n\}$ denotes a channel impulse response between the n-th user and the BS. In OFDMA system, each subcarrier is assigned to at most one user in order to obtain the orthogonality among the users. Although there are many subcarrier assignment protocols, in this paper, we assume that a consecutive set of subcarriers is assigned to a user and that the subcarriers are assigned in ascending order of the subcarrier index to the users in ascending order of the user index. This assumption is especially valid when adaptive modulation and coding (AMC) protocol is employed rather than partial usage of subchannels (PUSC) protocol. Defining the number of subcarriers assigned to the n-th user to be N_n, $\mathbf{s}_n$ is given by

$$\mathbf{s}_n = \left[\mathbf{0}_{1\times\left(\sum_{k=0}^{n-1} N_k\right)}\ s_0^n \cdots s_{N_n-1}^n\ \mathbf{0}_{1\times\left(M-\sum_{k=0}^{n} N_k\right)}\right]^T, \tag{2}$$

where N_n satisfies $\sum_{n=0}^{N-1} N_n = M$.

After performing DFT at the BS, the discrete frequency domain received signal vector is given by

$$\boldsymbol{\rho} = \left[\rho_0 \ \cdots \ \rho_{M-1}\right]^T = \sum_{n=0}^{N-1} \mathbf{D}\mathbf{C}_n\mathbf{D}^H\mathbf{s}_n + \mathbf{D}\mathbf{n}, \tag{3}$$

$$= \sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{s}_n + \boldsymbol{\nu}, \tag{4}$$

where $\boldsymbol{\Lambda}_n = \mathbf{D}\mathbf{C}_n\mathbf{D}^H$ is a diagonal matrix, whose diagonal elements are the channel frequency response of the n-th user $\{\lambda_0^n, \ldots, \lambda_{M-1}^n\}$, and $\boldsymbol{\nu} = \mathbf{D}\mathbf{n}$. Defining a diagonal matrix as

$$\mathbf{P}_n = \text{diag}\left[\mathbf{0}_{1\times\left(\sum_{k=0}^{n-1} N_k\right)}\ \mathbf{1}_{1\times N_n}\ \mathbf{0}_{1\times\left(M-\sum_{k=0}^{n} N_k\right)}\right], \tag{5}$$

the frequency domain received signal vector can be further calculated as

$$\boldsymbol{\rho} = \sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{P}_n\mathbf{s}_n + \boldsymbol{\nu} \tag{6}$$

$$= \left(\sum_{n=0}^{N-1} \boldsymbol{\Lambda}_n\mathbf{P}_n\right)\left(\sum_{n=0}^{N-1} \mathbf{s}_n\right) + \boldsymbol{\nu} \tag{7}$$

$$= \boldsymbol{\Lambda}\mathbf{s} + \boldsymbol{\nu}, \tag{8}$$

where $\mathbf{s}$ is an overall information signal vector defined as

$$\mathbf{s} = \left[s_0 \quad \cdots \quad s_{M-1}\right]^T,$$

$$= \left[s_0^0 \cdots s_{N_0-1}^0 \quad \cdots \quad s_0^{N-1} \cdots s_{N_{N-1}-1}^{N-1}\right]^T, \tag{9}$$

and $\boldsymbol{\Lambda}$ is an overall frequency response matrix denoted as

$$\boldsymbol{\Lambda} = \text{diag}\left[\lambda_0 \quad \cdots \quad \lambda_{M-1}\right],$$

$$= \text{diag}\left[\lambda_0^0 \cdots \lambda_{N_0-1}^0 \quad \cdots \quad \lambda_0^{N-1} \cdots \lambda_{N_{N-1}-1}^{N-1}\right]. \tag{10}$$

From (8), we can see that, by defining $\mathbf{s}$ and $\boldsymbol{\Lambda}$ as (9) and (10), respectively, we obtain the same received signal model as that of conventional OFDM scheme.

3. OVERALL CHANNEL IMPULSE RESPONSE

Considerations on the time domain expression of the overall frequency response, namely, overall impulse response of the channel, reveal a significant property, although the impulse response itself has no physical meaning. In this section, we analyze the delay power spectrum of the overall channel.

Defining matrices $\mathbf{D}_n$ and $\mathbf{D}_{n,c}$ for $n = 0, \ldots, N-1$ as

$$\mathbf{D}_n = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} 1 & e^{-j\frac{2\pi}{M}\sum_{k=0}^{n-1} N_k} & \cdots & e^{-j\frac{2\pi L}{M}\sum_{k=0}^{n-1} N_k} \\ \vdots & \vdots & & \vdots \\ 1 & e^{-j\frac{2\pi}{M}\{(\sum_{k=0}^{n} N_k)-1\}} & \cdots & e^{-j\frac{2\pi L}{M}\{(\sum_{k=0}^{n} N_k)-1\}} \end{bmatrix}, \tag{11}$$

and

$$\mathbf{D}_{n,c} = \frac{1}{\sqrt{M}} \cdot \begin{bmatrix} e^{-j\frac{2\pi(L+1)}{M}\sum_{k=0}^{n-1} N_k} & \cdots & e^{-j\frac{2\pi(M-1)}{M}\sum_{k=0}^{n-1} N_k} \\ \vdots & & \vdots \\ e^{-j\frac{2\pi(L+1)}{M}\{(\sum_{k=0}^{n} N_k)-1\}} & \cdots & e^{-j\frac{2\pi(M-1)}{M}\{(\sum_{k=0}^{n} N_k)-1\}} \end{bmatrix}, \tag{12}$$

respectively, so that we have

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_0 & \mathbf{D}_{0,c} \\ \vdots & \vdots \\ \mathbf{D}_{N-1} & \mathbf{D}_{N-1,c} \end{bmatrix}, \tag{13}$$

the frequency response of the n-th user is given by

$$\begin{bmatrix} \lambda_0^n \\ \vdots \\ \lambda_{M-1}^n \end{bmatrix} = \mathbf{D} \begin{bmatrix} h_0^n \\ \vdots \\ h_L^n \\ \mathbf{0}_{(M-L-1)\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0\mathbf{h}_n \\ \vdots \\ \mathbf{D}_{N-1}\mathbf{h}_n \end{bmatrix}, \tag{14}$$

where $\mathbf{h}_n = [h_0^n \cdots h_L^n]^T$. Therefore, the overall frequency response can be written as

$$\begin{bmatrix} \lambda_0 \\ \vdots \\ \lambda_{M-1} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0\mathbf{h}_0 \\ \vdots \\ \mathbf{D}_{N-1}\mathbf{h}_{N-1} \end{bmatrix}. \tag{15}$$

By performing IDFT, we obtain the overall channel impulse response $[h_0 \cdots h_{M-1}]^T$ as

$$\begin{bmatrix} h_0 \\ \vdots \\ h_{M-1} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_0^H\mathbf{D}_0 & \cdots & \mathbf{D}_{N-1}^H\mathbf{D}_{N-1} \\ \mathbf{D}_{0,c}^H\mathbf{D}_0 & \cdots & \mathbf{D}_{N-1,c}^H\mathbf{D}_{N-1} \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 \\ \vdots \\ \mathbf{h}_{N-1} \end{bmatrix}, \tag{16}$$

$$= \begin{bmatrix} \sum_{n=0}^{N-1} \mathbf{D}_n^H\mathbf{D}_n\mathbf{h}_n \\ \sum_{n=0}^{N-1} \mathbf{D}_{n,c}^H\mathbf{D}_n\mathbf{h}_n \end{bmatrix}. \tag{17}$$

Moreover, since we have

$$\lim_{i \to l} \frac{1 - e^{j\frac{2\pi}{M}(i-l)N_n}}{1 - e^{j\frac{2\pi}{M}(i-l)}} = N_n, \tag{18}$$

the i-th element of the overall impulse response can be obtained as

$$h_i = \frac{1}{M} \sum_{n=0}^{N-1} \sum_{l=0}^{L} \frac{e^{j\frac{2\pi(i-l)}{M}\sum_{v=0}^{n-1} N_v}\left(1 - e^{j\frac{2\pi(i-l)}{M}N_n}\right)}{1 - e^{j\frac{2\pi}{M}(i-l)}} h_l^n. \tag{19}$$

Using (19) and assuming

$$E[h_l^n h_{l'}^{n'H}] = \begin{cases} \sigma_{l,n}^2, & l = l',\ n = n' \\ 0, & else \end{cases}, \tag{20}$$

the variance of the i-th element of the overall impulse response is written as

$$E[|h_i|^2] = \frac{1}{M^2} \sum_{n=0}^{N-1} \sum_{l=0}^{L} \frac{\sigma_{l,n}^2 \left(1 - \cos\frac{2\pi}{M}(i-l)N_n\right)}{1 - \cos\frac{2\pi}{M}(i-l)}. \tag{21}$$

If the delay power spectra of all users' channel are assumed to be the same, namely, $\sigma_{l,n}^2 = \sigma_l^2$, $(n = 0, \ldots, N-1)$, and are modeled by an exponential decaying profile of

$$\sigma_l^2 = \frac{1}{C} a^l, \quad C = \sum_{l=0}^{L} a^l, \tag{22}$$

the variance of the i-th element of the overall impulse response can be further simplified as

$$E[|h_i|^2] = \frac{1}{CM^2} \sum_{n=0}^{N-1} \sum_{l=0}^{L} \frac{a^l(1 - \cos\frac{2\pi}{M}(i-l)N_n)}{1 - \cos\frac{2\pi}{M}(i-l)}. \tag{23}$$

Fig. 2 shows the delay power spectrum of the overall channel calculated by using (23), where the parameters are set to be $M = 1024$, $N_n = 64$ for all n, $N = 16$, $L = 128$ and $a = 1.0$. Also, Fig. 3 shows an example of $|h_i|^2$ obtained by computer simulation using the same parameters as Fig. 2. Here, a frequency selective Rayleigh fading channel of $L = 128$ is assumed. From the figures, we can see that the overall channel impulse response has nonzero values at only the first $L + 1$ (plus a bit more) and the last some elements. This means that only about $L + 1$ parameters are required to detect all users' signals, while the received signals are distorted by $N \times (L+1)$ independent channel coefficients. Note that we have assumed the consecutive subcarrier allocation and the equal delay power spectra among the users. Inconsecutive subcarrier allocation or unequal delay power spectra may result in looser support of the overall delay profile.

4. CHANNEL ESTIMATION SCHEMES

Based on the analysis in the previous section, we propose two channel estimation schemes.

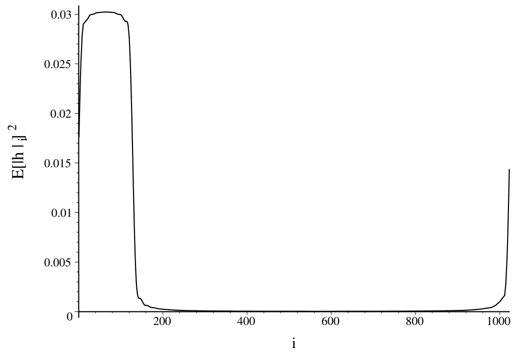

Fig. 2. Delay power spectrum: analysis.

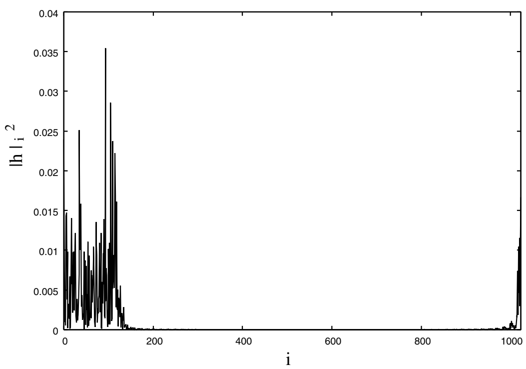

Fig. 3. Delay power spectrum: simulation.

4.1. Least Square Channel Estimation

Assuming the first $L + 1 + \alpha$ and the last α elements of $[h_0, \ldots, h_{M-1}]^T$ are nonzero, we define a vector $\mathbf{h}$ to be estimated as

$$\mathbf{h} = \left[h_0 \cdots h_{L+\alpha} \quad h_{M-\alpha-2} \cdots h_{M-1}\right]^T. \tag{24}$$

Using $\mathbf{h}$, the overall frequency response can be represented as

$$\left[\lambda_0 \ldots \lambda_{M-1}\right]^T = \mathbf{D}\left[h_0 \ldots h_{M-1}\right]^T = \mathbf{D}'\mathbf{h}, \tag{25}$$

where $\mathbf{D}'$ is an $M \times (L + 2\alpha + 1)$ matrix composed by the first $L + 1 + \alpha$ and the last α columns of $\mathbf{D}$.

The frequency response of a pilot subcarrier λ_k can be estimated as

$$\hat{\lambda}_k = \rho_k / s_k, \tag{26}$$

where k denotes an index of a pilot subcarrier. Extracting row elements from (25), which correspond to pilot subcarriers, we have

$$\boldsymbol{\lambda}_p = \mathbf{D}''\mathbf{h}, \tag{27}$$

where $\boldsymbol{\lambda}_p$ denote a frequency responses vector of the pilot subcarriers and the matrix $\mathbf{D}''$ is composed by the rows of

$\mathbf{D}'$, which also correspond to the pilot subcarriers. Therefore, if $\mathbf{D}''$ is tall, $\mathbf{h}$ can be estimated as

$$\hat{\mathbf{h}} = (\mathbf{D}''^H \mathbf{D}'')^{-1} \mathbf{D}'' \hat{\boldsymbol{\lambda}}_p, \quad (28)$$

where $\hat{\boldsymbol{\lambda}}_p$ is an estimated $\boldsymbol{\lambda}_p$ by using (26).

Note that in order to achieve the channel estimation method total number of pilot subcarriers has to be greater than or equal to the nonzero elements of the overall channel impulse response, namely, $(L + 2\alpha + 1)$. Therefore, as far as sufficient number of pilot subcarriers are inserted and the assumptions of consecutive subcarriers allocation and the equal delay power spectrum are valid, it could be possible to estimate the overall impulse response or equivalently the overall frequency response, which is required to detect all the users' signals, even when some of the users do not transmit any pilot signal.

4.2. Remedy for Interpolation Approach

If the number of the pilot subcarriers are less than $(L+2\alpha+1)$, we cannot utilize the least square channel estimation method in 4.1. Also, the method utilizes a pseudoinverse matrix, which requires high computational complexity in general. Here, we propose a simple channel estimation scheme, which is applicable in an add-on manner to conventional channel estimation scheme using interpolation in frequency domain.

Let $[\hat{\lambda}_0 \cdots \hat{\lambda}_{M-1}]$ denote an estimated overall frequency response obtained by using (26) and interpolation. We firstly perform IDFT of the estimated frequency response to obtain a time domain expression $[\hat{h}_0 \cdots \hat{h}_{M-1}]$, and then force to be zero at the corresponding elements as $\hat{h}_i = 0$ for $i = L+\alpha+1, \ldots, M-\alpha-1$. Finally, we obtain improved overall channel frequency response by performing DFT of $[\hat{h}_0 \cdots \hat{h}_{M-1}]$.

4.3. Numerical Results

Figs. 4 and 5 show BER performances of uncoded OFDMA system with QPSK modulation using the proposed channel estimation schemes with the number of pilot subcarriers of 256 and 512, respectively. The system parameters are set to be $M = 1024$, $N_n = 64$ for all n, $N = 16$, $L = 128$ and $\alpha = 40$. Here, we have employed the 0th order interpolation in the frequency domain for the conventional scheme. From the figures, we can see that the proposed least square estimation scheme can significantly improve the BER performance and that the simplified remedy of the interpolation approach in 4.2 also can achieve considerable performance gain.

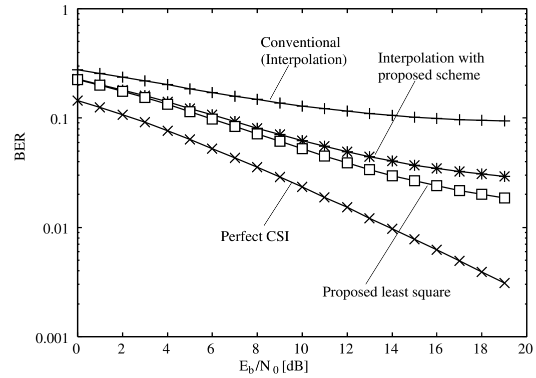

Fig. 4. BER performance (# of pilot subcarriers: 256).

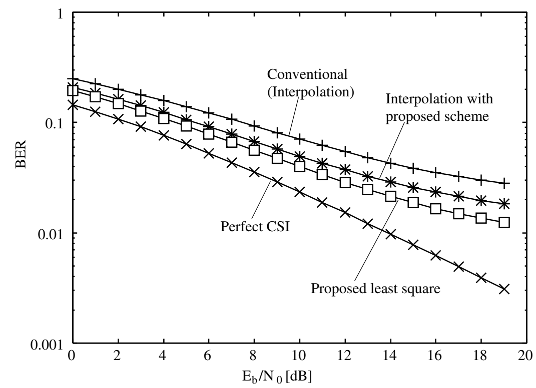

Fig. 5. BER performance (# of pilot subcarriers: 512).

5. CONCLUSION

In this paper, we have proposed two channel estimation schemes for OFDMA system based on the analysis of the delay power spectrum of overall channel impulse response. Although the received signals are distorted by $N \times (L+1)$ independent channel coefficients, only $L+1$ (plus a bit more) parameters are required for the detection of all users' signals. Computer simulation results show that the proposed channel estimation schemes can significantly improve the BER performance.

6. REFERENCES

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