PREAMBLE AND PILOT SYMBOL DESIGN FOR CHANNEL ESTIMATION IN OFDM

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ABSTRACT

The presence of null subcarriers in Orthogonal Frequency Division Multiplexing system complicates the design of both training preamble for channel estimation and pilot symbols for pilot-aided channel estimation that minimize the mean square error (MSE) of estimates of frequency-selective channels. In this paper, we numerically find optimal preambles, casting the MSE minimization problem into a semidefinite programming problem. Then, based on the optimal preamble, we also design pilot symbols for pilot-aided channel estimation. A design example under the same setting as IEEE802.11a is provided to verify the efficacy of our proposal.

Index Terms— Multipath channels, Parameter estimation

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a promising high-rate transmission technique, which mitigates inter-symbol interference (ISI) by inserting cyclic prefix (CP). If the channel delay spread is shorter than the duration of CP, ISI is completely removed. Moreover, if the channel remains constant within one OFDM symbol duration, OFDM renders a convolution channel into parallel flat channels, which enables simple one-tap frequency-domain equalization.

To obtain the channel state information (CSI), OFDM preambles or pilot symbols embedded in each OFDM symbol are utilized. OFDM preambles, or equivalently training OFDM symbols, are transmitted at the beginning of the transmitted record, while pilot symbols or pilot tones are embedded in each OFDM symbol, where they are separated from the information symbols in the frequency domain [1]. If the channel remains constant over several OFDM symbols, channel estimation by an OFDM preamble may be sufficient for symbol detection. But in the event of channel variation, OFDM preambles should be retransmitted to estimate the channel again to obtain reliable channel estimates for detection. On the other hand, pilot symbols inserted into every OFDM symbol enable channel estimation with each OFDM symbol. This is known as pilot-assisted channel estimation [2], which allows tracking of the channel variation.

When all subcarriers are available, OFDM preambles and pilot symbols have been well designed to enhance the channel estimation accuracy [3]. They can be optimally designed in terms of; i) minimizing the channel mean square estimation error [1]; ii) minimizing the bit-error rate when symbols are detected with channel estimates by pilot symbols [4]; iii) maximizing the lower bound on channel capacity with channel estimates [5]. It has been found that equi-distant and equi-powered pilot symbols are optimal with respect to several performance measure. Pilot symbols are also designed for OFDM systems with multiple antennas [6, 7, 8].

However, in practice, not all the subcarriers are available for transmission. It is often the case that null subcarriers are set on both edges of the allocated bandwidth to mitigate the interferences from/to adjacent bands [9]. For example, IEEE802.11a has 64 subcarriers among which 12 subcarriers at DC component and at the edges of the band are set to be null. Null subcarriers render equi-distant and equi-powered pilot symbols impossible to use. In [7], equi-powered pilot symbols are studied for channel estimation in multiple antenna OFDM system with null subcarriers. But they are not always optimal even for point-to-point OFDM system. Pilot sequences designed to reduce the channel mean square error (MSE) in multiple antenna OFDM system are also reported in [8] but they are not necessarily optimal. In this paper, we design optimal pilot sequences for channel estimation in OFDM with null subcarriers.

Our design criterion is the MSE in channel estimation. Although its expression can be readily derived, the optimal pilot sequence that minimizes MSE is unattainable in a closed form except for some special cases. To find the optimal sequence, we show that the MSE minimization problem can be cast into a semidefinite programming (SDP) problem [10]. With SDP, the optimal OFDM preamble which minimizes the channel MSE can be numerically found. Then, we select several significant subcarriers of the optimal OFDM preamble to design pilot symbols for pilot-aided channel estimation. We present a design example under the same setting as IEEE802.11a, which verifies that our optimal preamble has the least channel MSE. The designed pilot symbols when used for pilot-aided channel estimation exhibit comparable channel estimation accuracy with the long preamble of IEEE802.11a.

2. CHANNEL ESTIMATION IN OFDM

We consider point-to-point wireless OFDM transmissions over frequency-selective channels. Let the number of subcarriers be N. At the transmitter, a data sequence $\{s_0, s_1, \ldots, s_{N-1}\}$ is stacked into one OFDM symbol. Then, an N-point inverse discrete Fourier transform (DFT) follows to produce the N dimensional data, which is parallel-to-serial converted. A cyclic prefix (CP) of length N_{cp} is appended to mitigate the multipath effects.

We assume that our discrete-time baseband equivalent channel has maximum length L, and remain constant in at least one block, i.e., quasi-static. We also assume that N_{cp} is greater than the channel length L so that there is no inter-symbol interference (ISI) between OFDM symbols and denote the time-domain channel as $\{h_0, h_1, \ldots, h_{L-1}\}$.

At the receiver, we assume perfect timing synchronization. After removing CP, we take DFT of the received time-domain signals to obtain for $k \in [0, N-1]$ that

$$Y_k = H_k s_k + W_k, \tag{1}$$

where H_k is the channel frequency response at frequency $2\pi k/N$ given by $H_k = \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi kl}{N}}$ and W_k are i.i.d. circular Gaussian with zero mean and variance σ_w^2.

Let $\mathcal{K}$ be the set of active subcarriers and $|\mathcal{K}|$ be the number of elements in $\mathcal{K}$. For example, in IEEE802.11a, $\mathcal{K} = \{1, 2, \ldots, 26, 38, 39, \ldots, 63\}$ and $|\mathcal{K}| = 52$. If all $\{s_k\}$ for $k \in \mathcal{K}$ are pilot symbols, then the OFDM symbol is an OFDM preamble. For simplicity, the transmit power is normalized such that $\sum_{k\in\mathcal{K}} |s_k|^2 = 1$.

With the estimate of the channel frequency response H_k being $\hat{H}_k$, the mean square estimation error of the frequency-domain channel can be defined as

$$\eta_H := \sum_{k\in\mathcal{K}} E\{|\hat{H}_k - H_k|^2\}, \tag{2}$$

where $E\{\cdot\}$ stands for the expectation operator.

From (1), the channel frequency response H_k is easily estimated by Least Squares (LS) as Y_k/s_k, if all s_k for $k \in \mathcal{K}$ are non-zero. The mean square error (MSE) of each channel frequency response is then given by $E\{|\hat{H}_k - H_k|^2\} = \sigma_w^2/|s_k|^2$. If we distribute power equally to each pilot symbol, i.e., $|s_k|^2 = 1/|\mathcal{K}|$, then, the frequency-domain channel MSE is given by $\eta_H = |\mathcal{K}|^2\sigma_w^2$.

An alternative channel estimation scheme is to estimate the time-domain channel and then interpolate it to obtain the estimate of frequency-domain channel, as in pilot-assisted channel estimation [1]. For pilot-assisted channel estimation, we place $N_p (\le |\mathcal{K}|)$ pilot symbols $\{p_1, \ldots, p_{N_p}\}$ at subcarriers $\tilde{k}_1, \ldots, \tilde{k}_{N_P} \in \mathcal{K}$. We assume that $N_p \ge L$ so that the channel can be perfectly estimated if there is no noise.

Let $\text{diag}(\boldsymbol{a})$ be a diagonal matrix with the vector $\boldsymbol{a}$ on its main diagonal. Collecting the received signals having pilot symbols as $\tilde{\boldsymbol{Y}} = [Y_{\tilde{k}_1}, \ldots, Y_{\tilde{k}_{N_p}}]^T$, we obtain

$$\tilde{\boldsymbol{Y}} = \boldsymbol{D}_H \boldsymbol{p} + \tilde{\boldsymbol{W}}, \tag{3}$$

where $\boldsymbol{D}_H$ is a diagonal matrix with nth diagonal entry being $H_{\tilde{k}_n}$ such that $\boldsymbol{D}_H = \text{diag}[H_{\tilde{k}_1}, \ldots, H_{\tilde{k}_{N_p}}]$, and $\boldsymbol{p}$ is the pilot vector denoted as $\boldsymbol{p} = [p_1, \ldots, p_{N_p}]^T$.

We define an $N \times N$ DFT matrix as $\boldsymbol{F}$, whose $(m+1, n+1)$th entry is $e^{-j\frac{2\pi mn}{N}}$. Let us denote an $N \times L$ matrix $\boldsymbol{F}_L = [\boldsymbol{f}_0, \ldots, \boldsymbol{f}_{N-1}]^{\mathcal{H}}$ consisting of N rows and L columns of DFT matrix $\boldsymbol{F}$ and define an $N_p \times L$ matrix $\tilde{\boldsymbol{F}}_L$ having $\boldsymbol{f}_{\tilde{k}_n}^{\mathcal{H}}$ as its nth row, where $(\cdot)^{\mathcal{H}}$ denotes the complex conjugate transposition.

Then, we can express (3) as

$$\tilde{\boldsymbol{Y}} = \boldsymbol{D}_p \tilde{\boldsymbol{F}}_L \boldsymbol{h} + \tilde{\boldsymbol{W}}, \tag{4}$$

where $\boldsymbol{D}_p = \text{diag}\left[p_1, \ldots, p_{N_p}\right]$, and $\boldsymbol{h} = [h_0, \ldots, h_{L-1}]^T$. The LS estimate of $\boldsymbol{h}$ is obtained by

$$\hat{\boldsymbol{h}} = \left(\boldsymbol{D}_p \tilde{\boldsymbol{F}}_L\right)^{\dagger} \tilde{\boldsymbol{Y}}, \tag{5}$$

where $(\cdot)^{\dagger}$ denotes the pseudo-inverse of a matrix. The time-domain channel MSE is then expressed as

$$\eta_h = E\{||\hat{\boldsymbol{h}} - \boldsymbol{h}||^2\} = \sigma_w^2 \,\text{tr}\left(\tilde{\boldsymbol{F}}_L^{\mathcal{H}} \boldsymbol{D}_p^{\mathcal{H}} \boldsymbol{D}_p \tilde{\boldsymbol{F}}_L\right)^{-1}, \tag{6}$$

where $||\cdot||$ is the Euclidean norm.

If all subcarriers are available and L is a divisor of N, then equi-distant and equi-powered pilot symbols are optimal in the sense that they minimize the channel MSE such that $\eta_h = L\sigma_w^2$ [1, 4]. However, in the presence of null subcarriers, equi-distant and equi-powered pilots are not always optimal. Equi-powered pilot symbols are investigated for frequency-domain channel estimation in multiple antenna OFDM system with null subcarriers [7], but they are not always optimal even for point-to-point OFDM systems. To reduce the time-domain channel MSE for multiple antenna OFDM system, $\boldsymbol{p}$ has been designed to satisfy $\tilde{\boldsymbol{F}}_L^{\mathcal{H}} \boldsymbol{D}_p^{\mathcal{H}} \boldsymbol{D}_p \tilde{\boldsymbol{F}}_L = \boldsymbol{I}_p$ in [8]. But such a pilot sequence does not always exists.

If the channel statistics are available, then the optimal interpolation filter to obtain the frequency-domain channel from the time-domain channel can be derived. Without channel statistics, from $H_k = \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi kl}{N}}$, it is reasonable to utilize $\hat{H}_k = \sum_{l=0}^{L-1} \hat{h}_l e^{-j\frac{2\pi kl}{N}}$ as the estimate of frequency-domain channel. Consequently, the frequency-domain channel MSE is given by

$$\eta_H = \sigma_w^2 \,\text{tr}\left[\left(\tilde{\boldsymbol{F}}_L^{\mathcal{H}} \boldsymbol{D}_p^{\mathcal{H}} \boldsymbol{D}_p \tilde{\boldsymbol{F}}_L\right)^{-1} \boldsymbol{R}\right], \tag{7}$$

where $\boldsymbol{R} = \sum_{k\in\mathcal{K}} \boldsymbol{f}_k \boldsymbol{f}_k^{\mathcal{H}}$.

Provided that $\boldsymbol{R} = c\boldsymbol{I}$ for a non-zero constant c, the minimization of the time-domain channel MSE is equivalent to the minimization of the frequency-domain channel MSE. However, when there exists some null subcarriers, then $\boldsymbol{R} \neq c\boldsymbol{I}$ except for some special cases and such equivalence does not hold true. In this paper, we numerically design optimal pilot sequences that minimize either frequency-domain channel MSE or time-domain channel MSE in the presence of null subcarriers.

3. PREAMBLE AND PILOT DESIGN WITH SDP

Let us consider the minimization of frequency-domain channel MSE. The optimal pilot symbol sequence can be obtained by minimizing η_H in (7) with respect to $\boldsymbol{p}$ under the constraints that $\boldsymbol{p}^{\mathcal{H}}\boldsymbol{p} = 1$. Except for some special cases, e.g., $\boldsymbol{R} = c\boldsymbol{I}$ for a non-zero constant c, there is no analytical solution. Fortunately, this minimization problem can be cast into a semidefinite programming (SDP) [10] problem, whose global solution can be efficiently and numerically found by the existing routines.

We utilize the notation $\boldsymbol{A} \succeq 0$ for a symmetric matrix $\boldsymbol{A}$ to indicate that $\boldsymbol{A}$ is positive semidefinite and the notation $\boldsymbol{a} \succeq 0$ for a vector to signify that all entries of $\boldsymbol{a}$ are greater than or equal to 0.

Let us define $\lambda_n = |p_{\tilde{k}_n}|^2$ and

$$\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_{N_p}]^T. \tag{8}$$

By denoting the nth row of $\tilde{\boldsymbol{F}}_L$ as $\tilde{\boldsymbol{f}}_n^{\mathcal{H}}$, our MSE minimization problem can be stated as

$$\begin{aligned} &\min_{\boldsymbol{\lambda}} \operatorname{tr}\left[\left(\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1} \boldsymbol{R}\right] \qquad (9) \\ &\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0. \end{aligned}$$

Now let us introduce an auxiliary matrix variable $\boldsymbol{W}$ and consider the following problem:

$$\begin{aligned} &\min_{\boldsymbol{W},\boldsymbol{\lambda}} \operatorname{tr}(\boldsymbol{W}\boldsymbol{R}) \qquad (10) \\ &\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0 \\ &\boldsymbol{W} \succeq \left(\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1}. \qquad (11) \end{aligned}$$

Since $\boldsymbol{R} \succeq 0$, we have

$$\operatorname{tr}(\boldsymbol{W}\boldsymbol{R}) \geq \operatorname{tr}\left[\left(\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}}\right)^{-1} \boldsymbol{R}\right], \tag{12}$$

for $\boldsymbol{W} \succeq (\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}})^{-1}$. It follows that minimization of $\operatorname{tr}(\boldsymbol{W}\boldsymbol{R})$ is achieved if and only if $\boldsymbol{W} = (\sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}})^{-1}$, which proves that the minimization of $\operatorname{tr}(\boldsymbol{W}\boldsymbol{R})$ in (10) is equivalent to the original minimization in (9).

The constraint (11) can be rewritten by using Schur's complement as

$$\begin{bmatrix} \sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{W} \end{bmatrix} \succeq 0. \tag{13}$$

Using this, we finally reach the following minimization problem equivalent to the original problem.

$$\begin{aligned} &\min_{\boldsymbol{W},\boldsymbol{\lambda}} \operatorname{tr}(\boldsymbol{W}\boldsymbol{R}) \qquad (14) \\ &\text{subject to} \quad [1, \ldots, 1]\,\boldsymbol{\lambda} \leq 1, \quad \boldsymbol{\lambda} \succeq 0 \text{ and } (13) \end{aligned}$$

This is exactly an SDP problem where the cost function is linear and the constraints are convex, since they are in the form of linear matrix inequalities [10]. Thus, the globally optimal solution can be numerically found in polynomial time.

In the OFDM preamble, all subcarriers can be utilized for channel estimation, i.e., $N_p = |\mathcal{K}|$. Then, we can numerically obtain optimal preambles. In a pilot-assisted OFDM symbol, the number of pilot symbols is kept to as small as possible in order to reduce information transmission rate loss. In [5], the location, number, and power of the pilots are designed, using as criterion a lower bound on the average capacity when MMSE channel estimates by pilot symbols are adopted. However, it is not easy to design the location of pilot symbols when there exist null subcarriers, since analytical expressions for performance limits are in general unavailable.

As we have seen, for a given set of subcarriers, the optimal pilot symbols are obtained by resorting to numerical optimization. To determine the optimal set having N_p entries, i.e., the optimal location of N_p pilot symbols, we have to enumerate all possible sets, optimize pilot symbols for each set, and compare them. This design approach becomes infeasible as $|\mathcal{K}|$ gets larger. To design pilot symbols for a given N_p, we take a heuristic approach, which gives in general suboptimal solutions. First, we design an optimal preamble with SDP and denote its λ_i as $\lambda_{o,1}, \ldots, \lambda_{o,|\mathcal{K}|}$. After selecting a subcarrier index set with N_p entries corresponding to N_p largest $\lambda_{o,k}$, we optimize pilot power for the selected set, again with SDP.

We have discussed the design of pilot symbols minimizing the frequency-domain channel MSE. In terms of symbol detection, pilot symbols minimizing the frequency-domain channel MSE is in general more preferable than pilot symbols minimizing the time-domain channel MSE. If one wants the pilot symbols minimizing the time-domain channel MSE, one can construct them just by replacing $\boldsymbol{R}$ with $\boldsymbol{I}$ (cf. (6) and (7)) and applying the design procedure with the frequency-domain channel MSE described above.

4. DESIGN EXAMPLE

We design OFDM preambles and pilot symbols under the same setting as IEEE 802.11a. Out of $N = 64$ subcarriers, 12 subcarriers are null and $\mathcal{K} = \{\pm 1, \pm 2, \ldots, \pm 26\}$, where

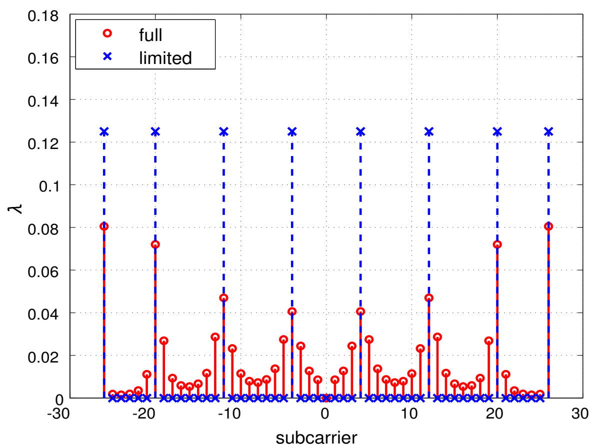

Fig. 1. Power of designed pilot symbols

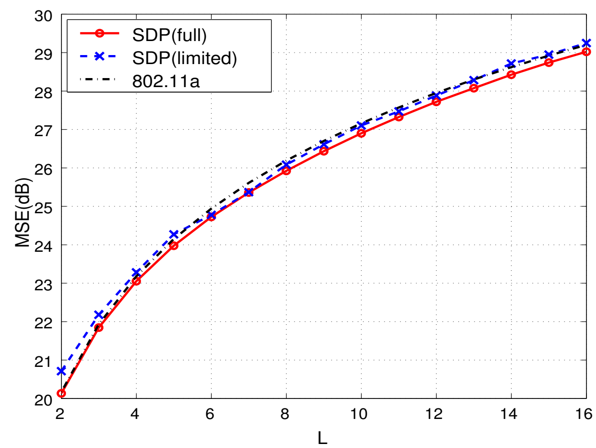

Fig. 2. Frequency-domain channel MSE

the subcarrier index $n > N/2$ is denoted as $n - N$ for convenience and $N_{cp} = 16$. Varying target channel length L from 2 to 16 and setting $\sigma_w^2 = 1$, we numerically minimize the frequency-domain channel MSE by SDP to obtain optimal OFDM preambles. For each L, to design pilot tones, we construct an index set corresponding to the subcarrier with significant pilot power in the optimal OFDM preamble.

Fig. 1 depicts the pilot symbol powers of the SDP optimized preamble for $L = 8$, marked by the notation $\circ$ labeled full. The power distribution is found to be symmetric around 0. This is due to the symmetry of our object function (7) in its arguments. The 8 significant pilot symbols are located at the subcarriers $\mathcal{P} := \{\pm 4, \pm 12, \pm 20, \pm 26\}$. Except for the middle portion and both edges, significant pilot symbols are uniformly distributed with spacing 8.

For the limited subcarriers $\mathcal{P}$, we optimize power allocation with SDP. The result is shown with x also in Fig. 1. Interestingly, numerically designed pilot symbols have the same power. This is reminiscence of the optimality of equi-distant and equi-powered pilots without null subcarriers. But, the same phenomena cannot always be seen for other L.

Fig. 2 presents the frequency-domain channel MSE η_H where the additive variance is set as 1. The curve (SDP(full)) with mark $\circ$ corresponds to the SDP optimized OFDM preamble where all the subcarriers in $\mathcal{K}$ are utilized, while the curve (SDP(limited)) with mark x corresponds to the pilot symbols at the selected subcarriers. For comparison, the result for IEEE 802.11a OFDM preamble with equi-powered pilot symbols at all subcarriers in $\mathcal{K}$ is also included. The SDP optimized OFDM preamble exhibits the least frequency-domain channel MSE but significant differences cannot be found between the three curves. The implication is that a similar channel estimation performance could be expected with a limited number of pilot symbols, which justifies the efficiency of the pilot-aided channel estimation.

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