OFDM CHANNEL ESTIMATION BASED ON COMBINED ESTIMATION IN TIME AND FREQUENCY DOMAIN

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ABSTRACT

In this paper we present a novel channel impulse response estimation technique for block-oriented OFDM transmission based on combining estimators: the estimates provided by a Kalman Filter operating in the time domain and a Wiener Filter in the frequency domain are optimally combined by taking into account their estimated error covariances. The resulting estimator turns out to be identical to the MAP estimator of correlated jointly Gaussian mean vectors. Different variants of the proposed scheme are experimentally investigated in an IEEE 802.11a-like system setup. They compare favourably with known approaches from the literature resulting in reduced mean square estimation error and bit error rate. Further, robustness and complexity issues are discussed.

Index Terms— Orthogonal frequency division multiplexing, MAP estimation, Kalman filtering, Wiener filtering

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has received a lot of interest in recent years both for wireline and wireless communications for its spectral efficiency and implementation simplicity [1]. By transmitting a high-rate data stream by many low-rate streams in parallel, a frequency-selective channel is turned into a set of parallel non-frequency selective narrowband transmission channels, for which a simple one-tap equalization can be carried out.

In this paper we consider wireless packet-oriented coherent OFDM transmission, as is e.g. used in wireless LANs (IEEE 802.11a/b/g, Hiperlan/2), assuming a single-input and single-output system. Coherent OFDM detection requires, among others, channel estimation and tracking. To this purpose, a burst of OFDM symbols typically consists of a preamble with known symbols, followed by the payload carrying the data. Some subcarriers of the payload also carry known data and serve the purpose of frequency fine tuning [2]. Channel impulse response estimation can then be carried out on the known symbols of the preamble. In order to increase the size of the payload or for use in a scenario with high terminal velocity, semi-blind channel estimation techniques have been proposed, where the channel estimates obtained on the preamble serve as initial values for a (re)estimation on the data of the payload. Many algorithms have been developed for pilot-assisted and semi-blind channel estimation [3].

In this paper we also carry out channel estimation both on the preamble and on the payload. However, we seek for an optimal combination of the two. First, a Kalman Filter is used for estimation on the preamble in the time domain. This has been shown to be computationally more efficient and to result in lower variance of the estimates compared to a frequency-domain Kalman Filter [4]. Next we conduct channel frequency response estimation on the symbols of the payload, which consists of two steps. The first step is a Maximum-Likelihood estimation, either solely at the subcarriers of the interspersed pilot data or also on the data subcarriers using the Expectation-Maximization (EM) algorithm. In a second step the estimates are improved by Wiener filtering. The final step is the optimal combination of the Kalman and Wiener Filter estimates. The resulting estimation formulae are surprisingly simple and we will give an alternative interpretation of them.

The paper is organized as follows. In the next section the OFDM transmission model is outlined. Section 3 describes the proposed channel estimation. In Section 4 we present simulation results and discuss robustness issues, before finishing with conclusions drawn in section 5.

2. SYSTEM MODEL

We consider a block-oriented OFDM transmission over a multipath fading channel. Let

$$\tilde{\mathbf{a}}(k) = (\tilde{a}_0(k), \ldots, \tilde{a}_{M-1}(k))^T \tag{1}$$

denote the $(M \times 1)$ symbol vector in the frequency domain[1] at time $k = iB + n$, where B is the number of symbols per block, i counts the blocks (packets) and n the symbols within a block. For simplicity of notation, let us assume that the known preamble consists of one symbol at $n = 0$ and the payload of the remaining $B - 1$ symbols ($n = 1, \ldots, B - 1$). The OFDM modulated symbol vector is obtained as

$$\mathbf{x}(k) = (\mathbf{W}_{M\times M})^H \tilde{\mathbf{a}}(k). \tag{2}$$

Here, $\mathbf{W}_{M\times M}$ denotes the DFT matrix of dimension $(M \times M)$ with the (i,l)-th entry $(\mathbf{W})_{i,l} = \frac{1}{\sqrt{M}} \exp(-j2\pi il/M)$, and $(\cdot)^H$ denotes Hermitian transpose.

The dispersive fading channel is characterized by the channel impulse response (CIR) vector

$$\mathbf{h}^{(0)}(k) = (h_0^{(0)}(k), \ldots, h_{L_h-1}^{(0)}(k))^T \tag{3}$$

of known length L_h. The $h_l^{(0)}(k)$, $l = 0, \ldots, L_h - 1$, are independent complex Gaussian random variables with a Jakes power density spectrum.

Prior to transmission, a cyclic prefix of length $L > L_h$ symbols is prepended and removed after transmission. The received signal can then be written as follows [1]:

$$\mathbf{r}(k) = \mathbf{X}(k)\mathbf{h}^{(0)}(k) + \mathbf{n}(k). \tag{4}$$

[1] We denote all frequency domain variables by a tilde $(\tilde{\cdot})$.

$\mathbf{X}(k)$ is a $(M \times L_h)$ circulant matrix formed from the vector $\mathbf{x}(k)$, and the additive white noise vector $\mathbf{n}(k)$ consists of M independent complex Gaussian random variables of variance σ_n^2 per dimension.

Throughout this paper we assume perfect timing and frequency synchronisation, and absence of phase noise.

3. CHANNEL ESTIMATION

The proposed channel estimator consists of a time domain Kalman Filter operating on the preamble, a frequency domain Wiener Filter working on the payload, and the combination of the estimates.

3.1. Kalman Filter

For the design of the Kalman Filter we assume signal propagation along $\hat{L}_h$ distinct paths, where the channel response of each path is described by a first-order state equation [5]:

$$h_l((i+1)B) = f \cdot h_l(iB) + g \cdot w_l(iB); \quad l = 0, \ldots, \hat{L}_h - 1, \quad (5)$$

where $w_l(iB)$ is complex white Gaussian noise of zero mean and unit variance, and

$$f = J_0(2\pi \hat{f}_d T_B) \quad (6)$$

$$g = \sqrt{(1-f^2)/\hat{L}_h}. \quad (7)$$

J_0 denotes the modified Bessel function of first kind and 0-th order. $\hat{f}_d$ is the assumed Doppler frequency, and T_B is the duration of a data block. In matrix notation, we have the following state equation

$$\mathbf{h}(i+1) = \mathbf{F}\mathbf{h}(i) + \mathbf{G}\mathbf{w}(i) \quad (8)$$

where $\mathbf{F} = f \cdot \mathbf{I}_{\hat{L}_h \times \hat{L}_h}$, $\mathbf{G} = g \cdot \mathbf{I}_{\hat{L}_h \times \hat{L}_h}$, and

$$E[\mathbf{h}(i)\mathbf{h}^H(i)] = (1/\hat{L}_h)\mathbf{I}_{\hat{L}_h \times \hat{L}_h}. \quad (9)$$

$\mathbf{I}_{\hat{L}_h \times \hat{L}_h}$ denotes the identity matrix of dimension $(\hat{L}_h \times \hat{L}_h)$. Here we left out B in the time argument for notational convenience. Note, in this model we assume for simplicity that each propagation path has the same power, because in practice the power profile is unknown at the receiver. Since the data of the preamble are known, eq. (4) may serve as measurement equation.

The Kalman Filter computes the a posteriori density of the channel impulse response vector, given all past observations, as a Gaussian density with mean $\mathbf{h}^{(KF)}(i|i)$, the MMSE estimate of the CIR, and covariance $\mathbf{P}^{(KF)}(i|i)$, which equals the covariance matrix of the estimation error [6]:

$$p(\mathbf{h}^{(0)}(i)|\mathbf{r}(0), \ldots, \mathbf{r}(i)) = \mathcal{N}(\mathbf{h}^{(0)}(i); \mathbf{h}^{(KF)}(i|i), \mathbf{P}^{(KF)}(i|i)). \quad (10)$$

The initial value $\mathbf{P}^{(KF)}(0|-1)$ of the covariance matrix is given by eq. (9).

For later use in Wiener filtering and estimator combination, the variables are transformed to the frequency domain:

$$\tilde{\mathbf{h}}^{(KF)}(i|i) = \mathbf{W}_{M \times \hat{L}_h}\mathbf{h}^{(KF)}(i|i) \quad (11)$$

$$\tilde{\mathbf{P}}^{(KF)}(i|i) = \mathbf{W}_{M \times \hat{L}_h}\mathbf{P}^{(KF)}(i|i)(\mathbf{W}_{M \times \hat{L}_h})^H. \quad (12)$$

Note that we adopted a block fading model here ($\mathbf{h}$ assumed constant for all B symbols of a block). We could have obtained individual estimates for each symbol interval $k = iB + n$ within the i-th burst by prediction, which we, however, did not do to save computations. Thus we set $\tilde{\mathbf{h}}^{(KF)}(k) = \tilde{\mathbf{h}}^{(KF)}(i|i)$ and $\tilde{\mathbf{P}}^{(KF)}(k) = \tilde{\mathbf{P}}^{(KF)}(i|i)$ for $k = iB, \ldots, (i+1)B - 1$.

3.2. Wiener Filter

The estimation of the channel frequency response (CFR) is executed in two stages: first Maximum-Likelihood (ML) estimates are computed for individual or all subcarriers, second an estimate for the frequency response on all subcarriers is obtained by Wiener filtering.

The received signal at subcarrier m, $m = 0, \ldots, M-1$, and symbol interval k, is given by

$$\tilde{r}_m(k) = \tilde{h}_m^{(0)}(k)\tilde{a}_m(k) + \tilde{n}_m(k), \quad (13)$$

where $\tilde{r}$, $\tilde{h}^{(0)}$ and $\tilde{n}$ denote the discrete Fourier transforms of r, $h^{(0)}$ and n, respectively. A ML estimate of the CFR on the payload containing unknown symbols can now be obtained either by decision-directed methods or by using the Expectation-Maximization (EM) algorithm [7].

For the results presented in this paper, however, we only consider a ML estimation on the N_p subcarriers $p(j)$, $j = 1, \ldots N_p$, containing embedded pilots. Here, the symbols are known and the ML estimate is obtained as:

$$\tilde{z}'_{p(j)}(k) = \frac{\tilde{r}_{p(j)}(k)\tilde{a}^*_{p(j)}(k)}{|\tilde{a}_{p(j)}(k)|^2}; \quad j = 1, \ldots, N_p. \quad (14)$$

Regarding the ML estimates as "observations", the purpose of the subsequent Wiener Filter is to obtain estimates of the CFR on all subcarriers and symbols by exploiting knowledge about the correlation in time and frequency direction. Instead of applying a two-dimensional linear filter, we first carry out a Wiener filtering along the time axis and second along the frequency axis. This is known to achieve almost identical performance, at, however, greatly reduced computational cost, as will be seen below.

Since the Wiener Filter along the time axis can be designed quite easily using estimates of the Doppler frequency and noise variance, it is not considered here in detail. Let the resulting estimates be denoted by $\tilde{\mathbf{z}}(k)$, where this vector has dimension N_p for the case considered here. We obtain the following linear observation model:

$$\tilde{\mathbf{z}}(k) = \mathbf{A}\tilde{\mathbf{h}}^{(0)}(k) + \tilde{\mathbf{n}}(k). \quad (15)$$

The matrix $\mathbf{A}$ is of dimension $(N_p \times M)$ and consists of zeros except for ones at the positions $(i, N_p(i))$, $i = 1, \ldots, N_p$. Furthermore, $E[\tilde{\mathbf{n}}(k)\tilde{\mathbf{n}}^H(k)] = \mathbf{\Lambda}_{\tilde{\mathbf{n}}}(k)$ is the diagonal covariance matrix of the estimation error of the Wiener Filter in time direction, which is a function of the position k within a burst. However, to save computations we used the same value, the value in the middle of the burst $(k = iB + B/2)$, for all k, making $\mathbf{\Lambda}_{\tilde{\mathbf{n}}}$ independent of the symbol index k.

The Wiener Filter along the frequency direction gives

$$\tilde{\mathbf{h}}^{(WF)}(k) = \mathbf{B}(k)\tilde{\mathbf{z}}(k), \quad (16)$$

where $\mathbf{B}$ is a solution of the Wiener-Hopf equation

$$\mathbf{B}(k)\mathbf{R}_{\tilde{\mathbf{z}}}(k) = \mathbf{R}_{\tilde{\mathbf{h}}\tilde{\mathbf{z}}}(k). \quad (17)$$

Using the linear model (15) we obtain

$$\mathbf{R}_{\tilde{\mathbf{z}}}(k) = E[\tilde{\mathbf{z}}(k)\tilde{\mathbf{z}}^H(k)] = \mathbf{A}\mathbf{R}_{\tilde{\mathbf{h}}}(k)\mathbf{A}^H + \mathbf{\Lambda}_{\tilde{\mathbf{n}}} \quad (18)$$

$$\mathbf{R}_{\tilde{\mathbf{h}}\tilde{\mathbf{z}}}(k) = E[\tilde{\mathbf{h}}^{(0)}(k)\tilde{\mathbf{z}}^H(k)] = \mathbf{R}_{\tilde{\mathbf{h}}}(k)\mathbf{A}^H, \quad (19)$$

where

$$\mathbf{R}_{\tilde{\mathbf{h}}}(k) = E[\tilde{\mathbf{h}}^{(0)}(k)(\tilde{\mathbf{h}}^{(0)})^H(k)] \quad (20)$$

is the correlation matrix of the unknown CFR, which has also been used as initial value of the error covariance matrix of the Kalman filter. We therefore set

$$\mathbf{R}_{\tilde{\mathbf{h}}}(k) = \tilde{\mathbf{P}}^{(KF)}(0|-1). \tag{21}$$

3.3. Estimator Combination

We now have two estimates for the CFR at symbol interval $k = iB + n$ within the i-th burst: $\tilde{\mathbf{h}}^{(KF)}(k)$ and $\tilde{\mathbf{h}}^{(WF)}(k)$. They can be optimally combined to an estimate $\tilde{\mathbf{h}}^{(c)}$, using their respective estimation error covariances [6]:

$$(\tilde{\mathbf{P}}^{(c)})^{-1}\tilde{\mathbf{h}}^{(c)} = (\tilde{\mathbf{P}}^{(WF)})^{-1}\tilde{\mathbf{h}}^{(WF)} + (\tilde{\mathbf{P}}^{(KF)})^{-1}\tilde{\mathbf{h}}^{(KF)}, \tag{22}$$

where

$$(\tilde{\mathbf{P}}^{(WF)})^{-1} = \mathbf{R}_{\tilde{\mathbf{h}}}^{-1} + \mathbf{A}^H\mathbf{\Lambda}_{\tilde{\mathbf{n}}}^{-1}\mathbf{A} \tag{23}$$

$$(\tilde{\mathbf{P}}^{(c)})^{-1} = (\tilde{\mathbf{P}}^{(WF)})^{-1} + (\tilde{\mathbf{P}}^{(KF)})^{-1} - \mathbf{R}_{\tilde{\mathbf{h}}}^{-1} \tag{24}$$

are the covariance matrices of the estimation error of the Wiener Filter and the combined estimator, respectively. Using (15) - (21) we obtain the surprisingly simple result

$$(\tilde{\mathbf{P}}^{(c)})^{-1}\tilde{\mathbf{h}}^{(c)} = \mathbf{A}^H\mathbf{\Lambda}_{\tilde{\mathbf{n}}}^{-1}\tilde{\mathbf{z}} + (\tilde{\mathbf{P}}^{(KF)})^{-1}\tilde{\mathbf{h}}^{(KF)}, \tag{25}$$

where

$$(\tilde{\mathbf{P}}^{(c)})^{-1} = \mathbf{A}^H\mathbf{\Lambda}_{\tilde{\mathbf{n}}}^{-1}\mathbf{A} + (\tilde{\mathbf{P}}^{(KF)})^{-1}. \tag{26}$$

A closer look at these equations reveals an alternative interpretation. The result is identical to the MAP estimation of the mean vector $\tilde{\mathbf{h}}^{(0)}$ of correlated jointly Gaussian random variables, given a Gaussian prior according to eqs (10) - (12) and the ML estimate $\tilde{\mathbf{z}}$ according to (15) on the data [8], [9].

4. SIMULATION RESULTS

4.1. Known Channel Model

In this section we present experimental results using a frame data structure which is similar to an IEEE 802.11a system. A burst consists of $B = 102$ symbols, of which the first two are the known preamble and the remaining form the payload. Of the total $M = 64$ subcarriers, channels no. 7, 21, 43 and 57 are reserved for known pilots. The available bandwidth in the 5 Ghz Band is chosen to 20 MHz, the data rate is 24 MBit/s with 96 data bits per OFDM symbol (coding rate $1/2$), and the modulation employed is 16-QAM.

The channel is Rayleigh fading with four independent propagation paths with power loss and delay profile of $[0, -1, -3, -9]$ dB and $[0, 100, 200, 300]$ ns and Jakes Doppler spectrum, which corresponds to a typical urban type of scenario. The mobile terminal velocity is set to $v = 30.8$ km/h. First we assume the channel model parameters to be perfectly known.

Figs. 1 and 2 show the bit error rate (BER) of the decoder and the mean square estimation error (MSEE) of the CFR estimation, respectively. Here, the "observation" vector $\tilde{\mathbf{z}}(k)$ is the estimate of the CFR on the pilot subcarriers after Wiener filtering along the time axis. The combined estimator is compared with a Kalman Filter operating on the preamble and a Wiener Filter, which estimates the CFR on all subcarriers according to eqs. (15) - (20), but with $\mathbf{R}_{\tilde{\mathbf{h}}} = \tilde{\mathbf{P}}^{(KF)}(0|-1)$. This is the classical way of smoothing CFR estimates by exploiting the knowledge of the maximum delay $\hat{L}_h$ [3]. A lower bound for the BER is obtained by assuming the CFR to be perfectly known. As can be seen from these results, estimating

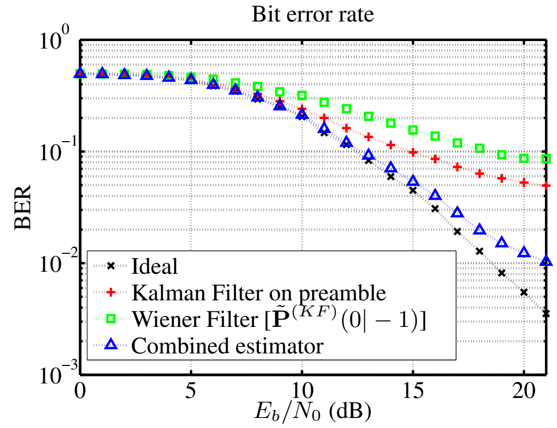

Fig. 1. BER of combined and individual estimators

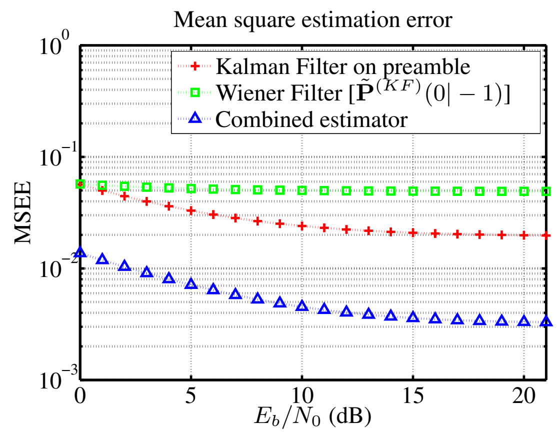

Fig. 2. MSEE of combined and individual estimators

the CFR on the pilot subcarriers alone performs very poorly. Also the Kalman filter, operating only on the preamble of a long burst, is not very effective. However, the combination of the two individually poor estimates is very powerful. This is probably due to the fact that the Frobenius norm of the covariance matrix $\tilde{\mathbf{P}}^{(KF)}(k)$ of the a posteriori density $p(\tilde{\mathbf{h}}^{(0)}(i)|\mathbf{r}(0),\ldots,\mathbf{r}(i))$, as provided by the Kalman Filter, is typically much smaller than the norm of the covariance matrix $\tilde{\mathbf{P}}^{(KF)}(0|-1)$ of the corresponding a priori density $p(\tilde{\mathbf{h}}^{(0)}(i))$. The more informative covariance matrix leads to more effective estimation.

4.2. Robustness

For the experiments reported so far we assumed that the Doppler frequency, the multipath profile (number and delay of paths) and the variance of the additive noise to be known. In this subsection we investigate the robustness of proposed channel estimator to incomplete knowledge of these parameters. Figs. 3 - 5 show the MSEE of the proposed combined and the individual estimators:

a) As a function of the E_b/N_0 assumed by the receiver, while the true channel-sided E_b/N_0 is 10 dB.
b) As a function of terminal velocity (and thus max. Doppler frequency) assumed by the terminal, while the true terminal velocity is 30.8 km/h at $E_b/N_0 = 14$ dB.

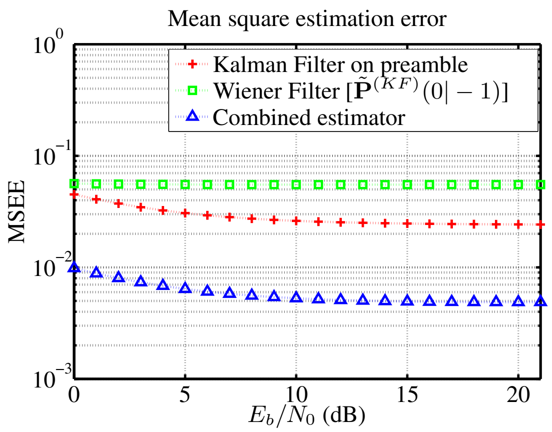

Fig. 3. MSEE at assumed E_b/N_0

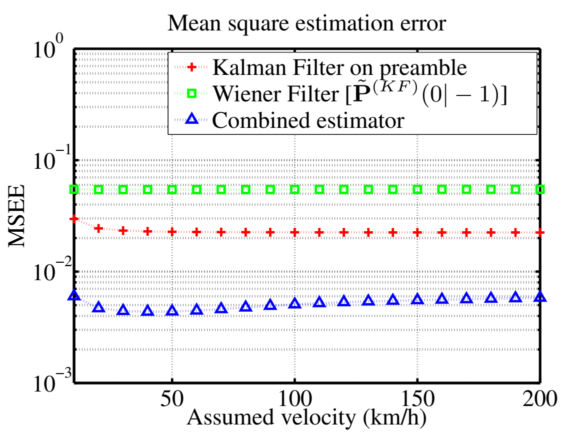

Fig. 4. MSEE at assumed velocities

c) For wrong assumptions concerning the multipath profile: The maximum delay (300 ns) is assumed to be known, but the profile is assumed to consist of 7 paths with relative delays of 50 ns.

For each setup, the system parameters not mentioned, are assumed to be known to the extent described in section 4.1. Figs. 3 - 5 show that the combined estimator consistently outperforms the individual estimators.

4.3. Computational Complexity

Compared to a Wiener Filter, the additional complexity of the proposed estimator resides in the Kalman Filter and the estimator combination. Note, however, that the Kalman Filter only operates on the preamble, i.e. only one iteration of the Kalman Filter equations is carried out per burst. The estimator combination (22) requires the inversion of $\tilde{\mathbf{P}}^{(WF)}$ and $\tilde{\mathbf{P}}^{(KF)}$. Since we have set $\tilde{\mathbf{P}}^{(KF)}$ constant within a burst and $\mathbf{\Lambda}_{\tilde{\mathbf{n}}}$ constant for the whole transmission, see remarks at the end of section 3.1 and in section 3.2, the estimator combination requires only one matrix inversion per burst, see eq. (26), resulting in only a small computational overhead due to estimator combination.

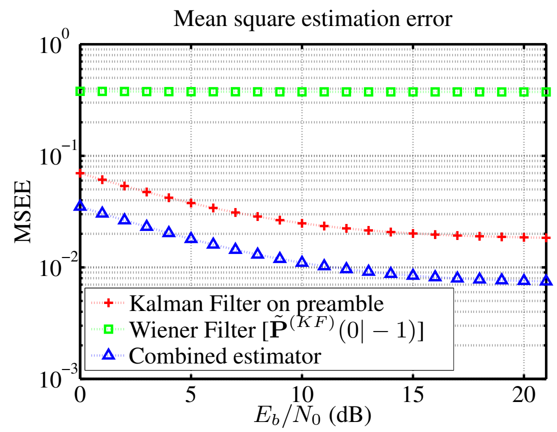

Fig. 5. MSEE at assumed multipath profile

5. CONCLUSIONS AND OUTLOOK

In this paper, a combined channel estimation algorithm based on Kalman filtering in the time and Wiener filtering in the frequency domain is proposed. While the individual estimators perform rather poorly, the combined estimation turned out to be very effective. Further, robustness issues und computational complexity are briefly discussed. In future research we try to further improve the performance by utilizing ML estimations of the CFR on the data subcarriers obtained by the EM algorithm. Further the effects of phase noise and non-perfect frequency synchronisation will be investigated.

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