

ON RESIDUAL CARRIER FREQUENCY OFFSET MITIGATION FOR 802.11N

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ABSTRACT

Due to the preamble design for mixed-mode transmission of 802.11n, carrier-frequency offset (CFO) estimation errors (or residual CFO) may cause significant errors in estimating the high-throughput (HT) MIMO channel because of the time-varying phase error as a result of residual CFO. In this paper, we propose a joint residual CFO and channel estimation technique using HT preambles. We show that the proposed technique is robust against residual CFO. We also present a simple technique for performing residual CFO estimation utilizing the 802.11n cyclic structure of pilot patterns. We demonstrate that with the proposed techniques, the performance loss due to residual CFO can be essentially recovered.

Index Terms— Carrier frequency offset (CFO), channel estimation, 802.11n, MIMO, OFDM

1. INTRODUCTION

The emerging 802.11n standard employs multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) transmission techniques to significantly increase the throughput up to 288.9 Mbps in a 20 MHz bandwidth [1]. The standard mandates a so-called mixed-mode high-throughput (HT) transmission mode where the preambles for HT data with the MIMO transmission technique are inserted following the legacy (L) short training field (STF), long training field (LTF) and signal field (SIG) preambles, as illustrated in Fig. 1 [1].

To ensure reliable communication with OFDM, carrier frequency offset (CFO) must be estimated and compensated for. Techniques for CFO estimation using L-STF and L-LTF preambles have been well studied (e.g., [2] and [3]). For most practical systems, autocorrelation-based techniques [3] are employed to estimate CFO, due to its low complexity and ease of implementation. However, in the low-to-medium SNR region, the CFO estimation error (or *residual* CFO) is not negligible.

For mixed-mode HT transmission, as shown in Fig. 1, the time lag between L-LTF and the last HT-LTF is up to 32 μ s with four HT-LTFs. HT-LTF preambles are used to estimate the HT MIMO channel that is used in demodulating the HT data. The CFO estimation error made while processing L-LTF

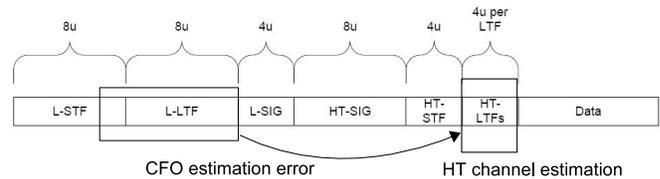


Fig. 1. Preamble structure for mixed-mode transmissions [1].

may cause significant errors in HT channel estimation using HT-LTFs, thus leading to substantial performance loss.

Since the packet may contain up to near 100 OFDM symbols [1], it is also necessary to estimate and compensate for residual CFO in the data demodulation process. Unlike the fixed pilot pattern design in 802.11a, in 802.11n, the pilot patterns for HT data are designed to shift cyclically from one OFDM symbol to another. Thus existing residual CFO estimation techniques for 802.11a are not applicable.

In this paper, we first propose a joint residual CFO and channel estimation technique using HT preambles. We show that the proposed technique is robust against residual CFO. We then present a simple residual CFO estimation technique that can fully utilize the new 802.11n pilot pattern design. We demonstrate that with the proposed techniques, there is little loss in performance. To our knowledge, there are no existing publications on these two subjects for 802.11n.

In Section 2, we briefly discuss signal model. In Section 3, we present the proposed joint residual CFO and channel estimation technique. In Section 4 we present residual CFO estimation using cyclically varying pilots. In Section 5 we discuss results.

2. SYSTEM MODEL

Due to space limitations, we will not elaborate on the 802.11n physical layer except for those of concern to this paper. Details on 802.11n can be found in [1]. During MIMO transmission, the coded user data is split into $N_{ss} \leq 4$ independent spatial data streams. When space-time coding is employed, these spatial streams are coded into $N_{sts} \leq 4$ space-time streams ($N_{sts} \geq N_{ss}$, with equality if no space-time coding

is employed). After spatial mapping (or beamforming), the space-time streams are transmitted with N_t transmit antennas. We assume that there is a single oscillator in the receiver so that the CFO is identical for all receive antennas. Further, the packet preambles received at each receive antenna are processed together so that the residual CFO is identical for all receive antennas.

Denote the exact CFO in Hz by ν . The time-varying phase error is $2\pi\nu t$. With a sampling rate of JB samples per second, the phase error is $2\pi\nu k/JB$, where B is the bandwidth and J is the oversampling factor. Since $B = N\Delta B$, where N is the number of OFDM subcarriers and ΔB is the inter-carrier spacing ($\Delta B = 312.5$ kHz in [1]), the phase error is $2\pi(\nu/\Delta B)k/(JN)$. Define $\epsilon \equiv \nu/\Delta B$, which can be considered as the CFO in number of samples. Let $\hat{\epsilon}$ be the estimated CFO using L-LTF or L-STF. Define *residual CFO* (the estimation error) $\Delta\epsilon \equiv \epsilon - \hat{\epsilon}$. The phase error due to residual CFO at time-sample k is $2\pi\Delta\epsilon k/(JN)$, which varies in time.

Let K_n be the time-sample index of the first element in the n -th OFDM symbol, skipping the cyclic prefix (CP). In the presence of residual frequency-offset error, the received signal for the m -th ($0 \leq m < N$) carrier and n -th OFDM symbol at n_r -th ($1 \leq n_r \leq N_r$) receive chain may be written as

$$R_{m,n,n_r} \cong e^{j2\pi\Delta\epsilon K_n/(JN)} \sum_{i_{sts}=1}^{N_{sts}} d_{m,n,i_{sts}} H_{m,i_{sts},n_r} + W_{m,n,n_r} \quad (1)$$

where W_{m,n,n_r} is complex white Gaussian noise with variance N_0 , and the equivalent channel H_{m,i_{sts},n_r} is a function of the transmitter spatial mapping (or beamforming) matrix, the transmitter cyclic-delay diversity value and the channel impulse response. (1) is a general expression that is applicable to any OFDM subcarrier in the HT portion of the packet.

In this paper we employ *per-tone* channel estimation, where the MIMO channel for each subcarrier is estimated independent of other subcarriers. The exact expression for H_{m,i_{sts},n_r} is not of concern to this paper. The channel used in this study is the IEEE indoor frequency-selective slow fading channel model [4]. We assume that the channel undergoes little change within the duration of one packet, which holds true for most indoor scenarios [4].

3. JOINT RESIDUAL CFO AND CHANNEL ESTIMATION

Let n_0 denote the index of the first OFDM symbol of the HT-LTF preamble ($n_0 = 9$ for mixed-mode transmission). To apply (1), we replace K_n by K_{n_0} , and replace $d_{m,n_0+k,i_{sts}}$ ($0 \leq k < N_{sts}$) by $L_m P_{i_{sts},k}$, where L_m is the known value for the m -th subcarrier and $\mathbf{P} = \mathbf{P}_{HTLTF}(1 : N_{sts}, 1 : N_{sts})$, with $\mathbf{P}_{HTLTF} = [1, -1, 1, 1; 1, 1, -1, 1; 1, 1, 1, -1; -1, 1, 1, 1]$ being the HT-LTF mapping matrix defined in [1]. From (1), combining all N_{sts} OFDM symbols corresponding to the HT-

LTF portion of the preamble, we have

$$\underline{R} = L_m \mathbf{E} \mathbf{P}^T \underline{H} \quad (2)$$

where the i -th element ($0 \leq i \leq N_{sts} - 1$) of \underline{R} is R_{m,n_0+i,n_r} , the i -th element ($1 \leq i \leq N_{sts}$) of \underline{H} is H_{m,i,n_r} , and the i -th diagonal entry ($0 \leq i \leq N_{sts} - 1$) of the diagonal matrix \mathbf{E} is $E_{i,i} = e^{j2\pi\Delta\epsilon K_{n_0+i}/(JN)}$. Notice that $K_{n_0+i} = K_{n_0+i-1} + J(N+N_g)$, where N_g is the length of the guard interval without oversampling. Obviously, (2) contains $N_{sts} + 1$ unknowns and N_{sts} equations.

Among possible ways to find $\Delta\epsilon$ and \underline{H} , such as MUSIC-like [5] algorithms, we seek to utilize more information from the received packet so that the number of equations can be increased to solve (2), which yields optimal estimates. Here, we use the HT-STF preambles to estimate $\Delta\epsilon$.

The HT-STF preamble occupies the $(n_0 - 1)$ -th OFDM symbol in the received packet. As stated in the standard [1], HT-STF is designed for automatic gain control for the HT portion of the packet. But its non-null subcarriers can certainly be utilized for our purpose.

The received m -th carrier in the HT-STF preamble can be written as $R_{m,n_0-1,n_r} = e^{j2\pi\Delta\epsilon[K_{n_0}-J(N+N_g)]/N} S_m \sum_{i_{sts}=1}^{N_{sts}} H_{m,i_{sts},n_r} + W_{m,n_0-1,n_r}$, where S_m is the known m -th element of the training sequence defined for the HT-STF preamble. In [1], there are 12 non-null S_m for 20MHz and 24 for 40MHz.

We now have the same number of equations as the number of unknowns. Notice that since the residual CFO value is identical for all carriers, we only need to choose N_n non-null carriers from HT-STF together with (2) to estimate $\Delta\epsilon$. Then the channel can be easily estimated. To account for channel noise, we seek to minimize $C(\Delta\epsilon) = \left| \text{Tr} \left[\text{diag} \left(\frac{\mathbf{P}^{-T} \mathbf{E}^{-1} \underline{R}}{L_m} \right) \right] - \frac{R_{m,n_0-1,n_r}}{S_m} e^{-j2\pi\Delta\epsilon \frac{K_{n_0}-JN-JN_g}{JN}} \right|^2$ and we propose a simple MMSE/LS estimator.

Let $\tilde{\mathbf{P}} = \mathbf{P}^{-T}$. For a non-null HT-STF carrier of index m , we can rewrite $C(\Delta\epsilon)$ as $L_m R_{m,n_0-1,n_r} / S_m = \tilde{W}_m + \sum_{k=1}^{N_{sts}} \sum_{l=1}^{N_{sts}} \tilde{P}_{lk} R_{m,n_0+k-1,n_r} e^{-j2\pi\Delta\epsilon k \frac{N+N_g}{N}}$, where the variance of \tilde{W} is $N_{sts} + 1$ times the variance of W in (1). If we select N_n non-null subcarriers from HT-STF, then

$$\tilde{\underline{R}} = \mathbf{G} \underline{u} + \tilde{\underline{W}} \quad (3)$$

where for $1 \leq i \leq N_n$, the i -th element of the length- N_n vector $\tilde{\underline{R}}$ is $L_m R_{m_i,n_0-1,n_r} / S_{m_i}$, the (i,k) -th element of the $N_n \times N_{sts}$ matrix $G_{i,k}$ is $\left(\sum_{l=1}^{N_{sts}} \tilde{P}_{l,i} \right) R_{m_i,n_0+k,n_r}$, and $u_k = e^{-j2\pi\Delta\epsilon k(N+N_g)/N}$ ($1 \leq k \leq N_{sts}$) is the vector to be estimated, containing the residual CFO $\Delta\epsilon$. Therefore, the general expression of an estimate of \underline{u} is

$$\hat{\underline{u}} = [\mathbf{G}^H \mathbf{G} + (N_{sts} + 1) N_0 \mathbf{I}]^{-1} \mathbf{G}^H \tilde{\underline{R}}. \quad (4)$$

Note that due to the definition of \mathbf{P} , when $N_{sts} = 2$, $\tilde{\mathbf{P}} = \mathbf{P}/2$, and $\sum_{l=1}^{N_{sts}} \tilde{P}_{l,2} = 0$. Thus (4) can be simplified to $\underline{g}^H \tilde{\underline{R}}$ where \underline{g} is the first column of \mathbf{G} .

The optimal estimate of $\Delta\epsilon$ is the one that minimizes $\|\underline{u}(\Delta\epsilon) - \hat{\underline{u}}\|^2$. After simplifying $dJ(\Delta\epsilon)/d\Delta\epsilon = 0$, we have

$$\sum_{k=1}^{N_{sts}} k (\Re[\hat{u}_k] \sin \theta_{\Delta\epsilon, k} + \Im[\hat{u}_k] \cos \theta_{\Delta\epsilon, k}) = 0 \quad (5)$$

where $\theta_{\Delta\epsilon, k} = 2\pi\Delta\epsilon(N + N_g)k/N$. (5) is a nonlinear equation, and there is no closed-form exact expression for its solutions. Obviously, one can again apply the least squares fit as an approximation. However, here we propose a polynomial approximation approach.

Since $\Delta\epsilon$ is normally much less than 0.1, $2\pi\Delta\epsilon k(N + N_g)/N$ with $k \leq 4$ is small enough so that we can apply Taylor series expansion of sine and cosine functions in (5) for closed-form approximation.

For the first order approximation, using $\sin(x) \approx x$ and $\cos(x) \approx 1$, we have

$$\Delta\epsilon \approx -\frac{1}{2\pi(1 + N_g/N)} \frac{\sum_{k=1}^{N_{sts}} k \Im[\hat{u}_k]}{\sum_{k=1}^{N_{sts}} k^2 \Re[\hat{u}_k]}. \quad (6)$$

For the second order approximation, using $\sin(x) \approx x$ and $\cos(x) \approx 1 - x^2/2$, we have $x = 2\pi(1 + N_g/N)\Delta\epsilon$ being a solution of $ax^2 - bx - c = 0$, where $a = \frac{1}{2} \sum_{k=1}^{N_{sts}} k^3 \Im[\hat{u}_k]$, $b = \sum_{k=1}^{N_{sts}} k^2 \Re[\hat{u}_k]$, and $c = \sum_{k=1}^{N_{sts}} k \Im[\hat{u}_k]$. It is easy to verify that the solution is $x = b/(2a) - \sqrt{b^2 + 4ac}/(2a)$.

After $\Delta\hat{\epsilon}$ is obtained, for per-carrier independent MMSE channel estimation, the HT MIMO channel for the m -th sub-carrier can be computed as

$$\hat{\underline{H}} = \left(\mathbf{P}\hat{\mathbf{E}}^H \hat{\mathbf{E}}\mathbf{P}^T + N_0\mathbf{I} \right)^{-1} \mathbf{P}\hat{\mathbf{E}}^H \underline{R}/L_m \quad (7)$$

where $(\cdot)^H$ denotes conjugate transpose.

4. RESIDUAL FREQUENCY-OFFSET ESTIMATION FOR HT DATA FIELD

Although a residual CFO estimate can be obtained from the HT channel estimation stage, simulations show that it is still necessary to perform residual CFO estimation and correction in the data demodulation process. This is because for long packets, the residual CFO estimated in the channel estimation stage may not be accurate enough to reduce to a negligible amount the time-varying phase shift due to the remaining residual CFO, as will be demonstrated later.

To further increase the estimation accuracy, we do not immediately correct $\hat{\epsilon}$ with the estimated residual CFO $\Delta\hat{\epsilon}$ in data demodulation so that the amount of residual CFO for each OFDM symbol is fixed. As a result, averaging may be performed to the estimated residual CFO before correction. For each data OFDM symbol, the final estimated residual CFO may be the (running) average of currently estimated value and previously estimated ones from past OFDM symbols.

Let n_d be the number of preamble OFDM symbols preceding the data symbols in the mixed mode, $P_{m,n,i_{sts}}$ be the cyclic pilot value defined in Table n71 and n72 in [1] (not repeated here due to space limitations), and p_{n+z} ($z = 3$ for mixed mode) be the polarity defined in [1]. Applying (1) with $d_{m,n,i_{sts}} = p_{n+z}P_{m,n,i_{sts}}$, for a pilot carrier in the HT data OFDM symbol, the received signal may be written as $R_{m,n,n_r} = e^{j2\pi\Delta\epsilon K_n/N} p_{n-n_d+z} \sum_{i_{sts}=1}^{N_{sts}} P_{m,n,i_{sts}} H_{m,n_r,i_{sts}} + W_{m,n,n_r}$.

Let L be the number of data OFDM symbols in the packet. Let N_p be the number of pilot carriers in the data OFDM symbols. To utilize the cyclic structure of the pilots, we group pilots in $N_c \leq N_p$ consecutive OFDM data symbols together starting from the first data OFDM symbol. Let the index of the first OFDM symbol in the l -th group be $n_l \geq 0$ ($1 \leq l \leq \lceil L/N_c \rceil$). At the n_r -th receive chain, for the l -th group, we have

$$\tilde{\underline{R}} = \underline{\Psi} \underline{v} + \tilde{\underline{W}} \quad (8)$$

where $v_k = e^{j2\pi\Delta\epsilon[K_{n_l+k}J(N+N_g)]/(JN)}$ for $0 \leq k \leq N_c - 1$, and for $0 \leq i \leq N_p - 1$, $\tilde{W}_i = \sum_{k=0}^{N_p-1} W_{\zeta(i),n_l+k,n_r}$ and $\tilde{R}_i = \sum_{k=0}^{N_c-1} R_{\zeta(i),n_l+k,n_r}$ with $\zeta(i)$ being the carrier index of the i -th pilot subcarrier. For example, for 20 MHz, the pilot carrier indices are $\zeta([0, 1, 2, 3]) = [-21, -7, 7, 21]$ [1]. The (i, k) -th entry of the $N_p \times N_c$ matrix $\underline{\Psi}$ is

$$(\underline{\Psi})_{i,k} = p_{n_l+k+z} \sum_{i_{sts}=1}^{N_{sts}} P_{\zeta(i),n_l+k,i_{sts}} H_{\zeta(i),i_{sts},n_r} \quad (9)$$

where $P_{\zeta(i),n_l+k,i_{sts}} = \Psi_{i_{sts},(n_l+k+i) \oplus k_p}^{(N_{sts})}$ varies cyclically with $\Psi_{i_{sts},(n_l+k+i) \oplus k_p}^{(N_{sts})}$ defined in Table n71 (20 MHz, $k_p = 4$) and n72 (40 MHz, $k_p = 6$) of [1]. For example, for $N_{sts} = 4$, $\Psi_{4 \times 4} = [1, 1, 1, -1; 1, 1, -1, 1; 1, -1, 1, 1; -1, 1, 1, 1]$ [1]. The MMSE estimate of \underline{v} for each receive chain is then

$$\hat{\underline{v}} = \left[\underline{\Psi}^H \underline{\Psi} + N_c N_0 \mathbf{I} \right]^{-1} \underline{\Psi}^H \tilde{\underline{R}}. \quad (10)$$

Similar to the previous section, one can again solve for $\Delta\epsilon$ by minimizing $\|\underline{v}(\Delta\epsilon) - \hat{\underline{v}}\|^2$. But since K_{n_l} may be very large, we can not directly apply polynomial approximations of sine and cosine functions. Rather than solving the nonlinear equation, for the l -th group of OFDM symbols, we can simply approximate $\Delta\epsilon$ as

$$\Delta\hat{\epsilon} = \sum_{k=1}^2 \frac{JN}{4\pi[K_{n_l} + (k-1)J(N+N_g)]} \angle \hat{v}_k. \quad (11)$$

When space-time coding is employed, two OFDM symbols are processed together. We can choose $N_c = 2$ so that the size of $\underline{\Psi}$ is reduced to $N_p \times 2$, which allows further simplification. As an example, for $N_{sts} = 4$ and 20 MHz transmission, with the polarity p_{n+z} removed in $\tilde{\underline{R}}$, $\underline{\Psi}_c = [\underline{\Lambda}_{c,1}, \underline{\Lambda}_{c,2}]^T$. At each receive antenna, for the first data OFDM symbol pair ($c = 0$), we have

$$\begin{aligned} \underline{\Lambda}_{c,1} &= [P_{-21,0}^T \underline{H}_{-21}, P_{-21,1}^T \underline{H}_{-21}; P_{-7,0}^T \underline{H}_{-7}, P_{-7,1}^T \underline{H}_{-7}] \\ \underline{\Lambda}_{c,2} &= [P_{7,0}^T \underline{H}_7, P_{7,1}^T \underline{H}_7; P_{21,0}^T \underline{H}_{21}, P_{21,1}^T \underline{H}_{21}], \end{aligned}$$

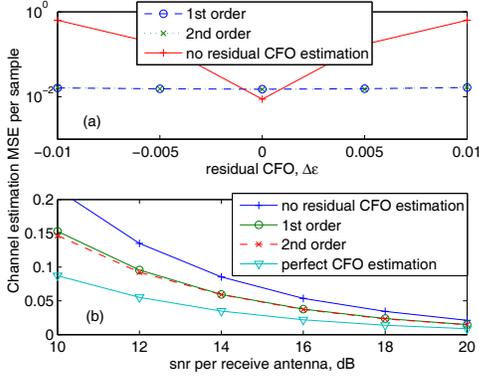


Fig. 2. (a) MSE per sample of estimated channel for first (6) and second order approximation of the sine/cosine functions at SNR per receive antenna of 20dB for 20MHz HT transmission. (b) Average channel estimation MSE at different SNR.

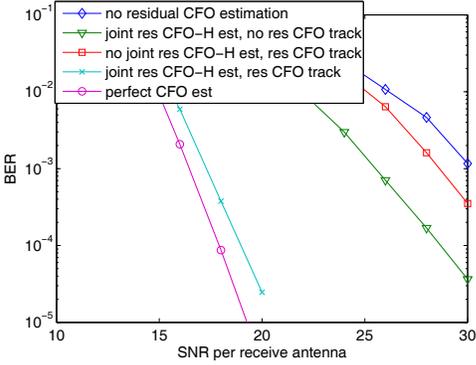


Fig. 3. BER performance of a space-time coded 4×2 system with and without proposed residual CFO tracking techniques.

where (9) is rewritten as $\underline{P}_{\zeta(i),n-n_d}^T \underline{H}_{\zeta(i)}$. For the next OFDM symbol pair, due to cyclic shift of pilot values, we have

$$\begin{aligned} \Lambda_{c+1,1} &= [\underline{P}_{7,0}^T \underline{H}_{-21}, \underline{P}_{7,1}^T \underline{H}_{-21}; \underline{P}_{21,0}^T \underline{H}_{-7}, \underline{P}_{21,1}^T \underline{H}_{-7}] \\ \Lambda_{c+1,2} &= [\underline{P}_{-21,0}^T \underline{H}_7, \underline{P}_{-21,1}^T \underline{H}_7; \underline{P}_{-7,0}^T \underline{H}_{21}, \underline{P}_{-7,1}^T \underline{H}_{21}]. \end{aligned}$$

As a result of further cyclic shifts, for the $(c+2)$ -th symbol pair, $\Lambda_{c+2,1} = \Lambda_{c,1}$ and $\Lambda_{c+2,2} = \Lambda_{c,2}$, and so on. Thus the receiver only needs to keep track of four unique 2×2 matrices for each packet per receive antenna, and the inverse operation in (10) can be performed very efficiently.

5. RESULTS

The proposed techniques are simulated for a 802.11n space-time coded system [1] over IEEE indoor frequency-selective fading channel model ‘E’ [4] with four transmit and two receive antennas. The system has two spatial streams, each

modulated with 16-QAM. Rate 1/2 convolutional coding is employed. OFDM is implemented with 800 ns guard interval ($N_g = N/4$). The CFO is set to 31.25 kHz and is estimated according to [3]. Per-carrier independent channel estimation is employed as given in (7). Each packet contains 1000 bytes of data without beamforming.

Fig. 2(a) illustrates the performance of the proposed joint residual CFO and channel estimation technique with the first order (6) and the second order polynomial approximations of the sine and cosine functions in (5) at SNR per receive antenna of 20 dB. Fig. 2(b) compares their performances at different SNR. Both the first and second order approximations yield about the same performance, with the second order slightly better as expected. Fig. 2(a) indicates that with the proposed joint estimation technique, the estimated channel is immune to initial CFO estimation errors. Fig. 2(b) demonstrates that the effect of residual CFO on channel estimation can be substantially reduced.

Fig. 3 illustrates the BER performance of mixed-mode transmission. “no residual CFO estimation” indicates that only initial CFO ϵ estimation [3] is employed and the residual CFO $\Delta\epsilon$ is not estimated afterwards, which leads a loss of over 13 dB at $\text{BER} = 10^{-3}$ compared with perfect ϵ estimation indicated by “perfect CFO est”. “joint res CFO-H est, no res CFO track” refers to applying the proposed joint $\Delta\epsilon$ and channel estimation technique, without estimating $\Delta\epsilon$ in data demodulation. The exact opposite case is given by “no joint res CFO-H est, res CFO track”. The former improves over the case without $\Delta\epsilon$ estimation by around 5 dB, and the latter improves around only 2 dB due to incorrect channel estimates. But either technique alone can not sufficiently remove the degradation due to $\Delta\epsilon$. When both techniques are employed, the performance is given by “joint res CFO-H est, res CFO track”, which effectively removes the defects due to $\Delta\epsilon$, with a gain of over 12 dB. The remaining loss due to CFO is suppressed to within 1 dB compared with perfect CFO estimation. This figure suggests that both techniques are necessary and very effective, and residual CFO must be estimated and corrected for 802.11n MIMO-OFDM systems.

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