STAR RECEIVER IN MIMO-CDMA MULTIPATH CHANNELS

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***Abstract*— This paper is concerned with the investigation of a MIMO-CDMA communication system which is based on the concept of SPATIOTEMPORAL ARRAY (STAR) manifold for handling multipath fading channels. Firstly a MIMO frequency selective channel is modelled as a function of both transmitter's and receiver's antenna-array manifold vectors. Then, assuming knowledge of the channel parameters of the desired user, two linear receivers are proposed. The proposed receivers are 'near-far' resistant and expressed in terms of the STAR manifold vector of the desired user. The investigation is supported by computer simulation studies where the performance of the proposed receivers is compared to the performance of conventional MMSE and RAKE receivers which have been also extended in order to be expressed in terms of the STAR manifold vectors.**

***Index Terms*— MIMO, CDMA, Array Processing, Subspace, STAR Manifold.**

NOTATION

a, A	Scalar
$\underline{a}, \underline{A}$	Column Vector
$\mathbb{A}$	Matrix
$\mathbb{I}_N$	Identity matrix of $N \times N$ dimension
$\underline{0}_N$	Zero vector of N elements
$\text{diag}(\underline{A})$	Diagonal matrix formed from vector $\underline{A}$
$\lfloor \cdot \rfloor$	Round down to integer
$\otimes$	Kronecker product

I. INTRODUCTION

With multiple antennas at both receiver and transmitter, Multiple Input Multiple Output (MIMO) wireless systems have shown great potential to provide high bandwidth efficiency for the next generation broadband mobile communications [1]. Recently, the study of multiuser MIMO systems has received much attention where the signals of different users overlap in both time and frequency resulting in multiple-access interference (MAI), which eventually limits the promised performance of MIMO systems [2].

A vast majority of the reported research on MIMO systems adopts the assumption of multiple independent elements at both sides of the transmission links [3]. However, fading between different links are not independent in real propagation environment [4]. The fading correlation depends on the physical parameters of multiple antennas and the scatterer characteristics.

Array processing and communication techniques have evolved into a well-established technology, from old conventional direction nulling and phase-arrays to advanced super-resolution arrays [5][6]. By exploiting array geometry and spatial-temporal information of the channel, e.g. direction and delay of multipath, an arrayed-MIMO system operating in a multiple-access environment provides enormous opportunities beyond the just added diversity, or array gain, benefits.

The paper is organised as follows: In section II, a multiuser MIMO channel is modelled as a function of the antenna-array manifold vector of both transmitter and receiver. The study concentrates upon the uplink of an asynchronous DS-CDMA channel. Both base station and mobiles are equipped with an array of antennas. In Section III, two spatial-temporal receivers are developed by applying array processing to project the received signal onto the null-space of the unwanted-signal subspace. In Section IV, simulation studies compare the performance of the proposed receivers to that of the conventional MMSE and RAKE receivers which have been also extended in order to be expressed in terms of the STAR manifold vectors. The paper is concluded in Section V.

II. SYSTEM MODEL

Consider the uplink of an M-user asynchronous MIMO-CDMA wireless communication system operating in a multipath propagation environment. The receiver has an array of N antennas whereas each transmitter is equipped with an array of $\overline{N}$ antennas[1]. It is assumed that the channel is frequency-selective with a delay spread less than one data symbol period.

The i^{th} user's baseband transmitted signal vector $\underline{m}_i(t) \in \mathcal{C}^{\overline{N} \times 1}$ can be written as

$$\underline{m}_i(t) = \sum_{n=-\infty}^{\infty} \underline{b}_i[n]\, c_{PN,i}(t - nT_{cs}) \qquad (1)$$
$$nT_{cs} \leq t \leq (n+1)T_{cs}$$

where T_{cs} is the data symbol period, $\underline{b}_i[n]$ is transmitted data during the n^{th} symbol period, and $c_{PN,i}(t)$ denotes one period of the PN-code sequence waveform of the i^{th} user

$$c_{PN,i}(t) = \sum_{k=0}^{N_c - 1} \alpha_i[k]\, p(t - kT_c) \qquad (2)$$

with T_c representing the chip period, $\alpha_i[k] = \pm 1$ (i^{th} user's PN sequence of period $N_c = T_{cs}/T_c$) and $p(t)$ is the rectangular chip waveform.

[1]Note that a symbol with a 'bar' at the top will denote a transmitter's parameter, $(\bullet)^H$ represents the Hermitian, $(\bullet)^T$ the transpose, $(\bullet)^*$ the complex conjugate, $\mathcal{C}$ the Set of complex numbers and $\mathcal{R}$ the Set of real numbers.

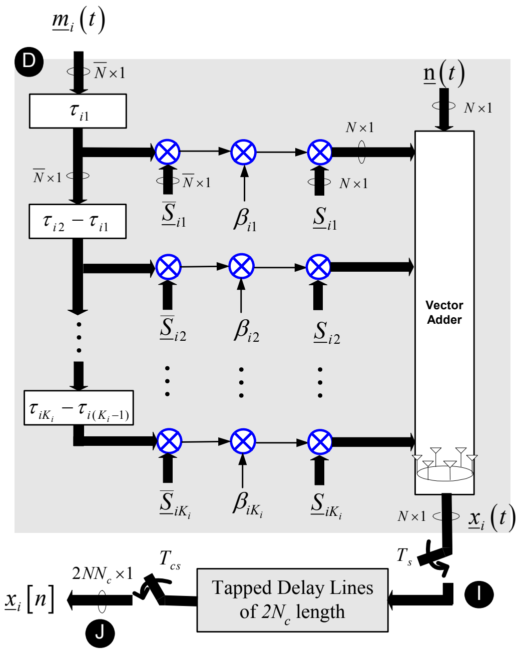

Fig. 1. Arrayed MIMO Channel of the i^{th} user.

Let us assume plane-wave propagation with transmitted signals of the i^{th} user arriving at the receiver via K_i multipaths. Furthermore, consider that the j^{th} path of the i^{th} user's direction of departure (DOD) (azimuth, elevation) $= \left(\overline{\theta}_{ij}, \overline{\varphi}_{ij}\right)$ and direction of arrival (DOA) $\left(\theta_{ij}, \varphi_{ij}\right)$. The array manifold vector of the receiver $\underline{S}_{ij}$ of the j^{th} path of the i^{th} user can be formulated as

$$\underline{S}_{ij} = \exp(-\mathrm{j} \cdot [\underline{r}_x, \underline{r}_y, \underline{r}_z] \cdot \underline{\mathrm{k}}_{ij}) \tag{3}$$

where $[\underline{r}_x, \underline{r}_y, \underline{r}_z] \in \mathcal{R}^{N\times 3}$ defined as the Cartesian coordinates of the receiver antenna array element and $\underline{\mathrm{k}}_{ij} = (2\pi F_c/c) \cdot [\cos\theta_{ij}\cos\varphi_{ij}, \sin\theta_{ij}\cos\varphi_{ij}, \sin\varphi_{ij}]^T$ is the wavenumber vector pointing towards the DOA of the j^{th} path of the i^{th} user. The transmitter array manifold vector as $\overline{\underline{S}}_{ij}$ for the j^{th} path of the i^{th} user can be defined in a similar fashion to Eqn. (3) but with the parameters associated with the transmitters. Without loss of generality, all the signals are assumed to propagate on x-y plane, i.e., $\overline{\varphi}_{ij} = \varphi_{ij} = 0$.

Based on the channel model depicted in Fig. 1, the received complex baseband signal vector at the input of the receive antenna array (point I) can be expressed as

$$\underline{x}(t) = \sum_{i=1}^{M}\sum_{j=1}^{K_i} \beta_{ij}\underline{S}_{ij}\overline{\underline{S}}_{ij}^H \underline{m}_i(t-\tau_{ij}) + \underline{\mathrm{n}}(t) \tag{4}$$

where β_{ij} and τ_{ij} represent the fading coefficient and the delay of the j^{th} path of the i^{th} user, and $\underline{\mathrm{n}}(t)$ denotes the complex white Gaussian noise vector with zero mean and covariance matrix $\sigma^2\mathbb{I}_N$. Note that transmit power is assumed identically distributed among all transmit antennas, and is absorbed by the fading complex coefficient β_{ij}.

The baseband received signal vector $\underline{x}(t)$ is initially sampled with a period $T_s = T_c$, Thus, $\tau_{ij} \mod T_{cs} = \left(l_{ij} + \rho_{ij}\right)T_s$, where l_{ij} is the integer part, $l_{ij} = \left\lfloor \frac{\tau_{ij} \mod T_{cs}}{T_s} \right\rfloor$ and $\rho_{ij} \in [0,1)$ is the factional part. The phase shift $\exp\left(-j2\pi F_c\rho_{ij}T_s\right)$ is absorbed by the complex coefficient β_{ij}.

At point J, the contents of a bank of N tapped delay lines (TDL), each of length $2N_c$, are concatenated to obtain

$$\underline{x}[n] = \left[\ \underline{x}_1[n]^T,\ \cdots,\ \underline{x}_N[n]^T\ \right]^T \quad \in \mathcal{C}^{2NN_c\times 1} \tag{5}$$

where $\underline{x}_k[n] \in \mathcal{C}^{2N_c\times 1}$, $k = 1, \cdots N$, represents the contents of the TDL at k^{th} receiver antenna associated with the n^{th} data-symbol period. Note that because of the lack of symbol synchronization, $\underline{x}_k[n]$ contains contributions from not only the current (n^{th}) data-symbol, but also the $(n-1)^{\text{th}}$ (previous) and $(n+1)^{\text{th}}$ (next) data-symbol.

To model the above effects, the array manifold vector $\underline{S}_{ij}$ given in (3) is expanded to STAR manifold vector for the j^{th} path of i^{th} user,

$$\underline{S}_{ij} \otimes \left(\mathbb{J}^{l_{ij}}c_i\right) \quad \in \mathcal{C}^{2NN_c\times 1} \tag{6}$$

where $\mathbb{J} = \begin{bmatrix} \underline{0}_{2N_c-1}^T & 0 \\ \mathbb{I}_{2N_c-1} & \underline{0}_{2N_c-1} \end{bmatrix}$ and

$$c_i = \left[\alpha[0], \alpha[1], \cdots, \alpha[N_c-1], \underline{0}_{N_c}^T\right]^T \quad \in \mathcal{C}^{2N_c\times 1}$$

Using the STAR manifold vector defined by (6), the sampled baseband signal vector $\underline{x}[n]$ can be rewritten as

$$\underline{x}[n] = \sum_{i=1}^{M} \left\{ \begin{array}{l} \mathbb{H}_{i,\text{prev}}\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H \underline{b}_i[n-1] + \\ \mathbb{H}_i\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H \underline{b}_i[n] + \\ \mathbb{H}_{i,\text{next}}\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H \underline{b}_i[n+1] \end{array} \right\} + \underline{\mathrm{n}}[n]$$

where $\underline{\mathrm{n}}[n] \in \mathcal{C}^{2NN_c\times 1}$ denotes the sampled noise vector with identical independent Gaussian distribution, and

$$\mathbb{H}_i = \left[\ \underline{S}_{i1} \otimes \left(\mathbb{J}^{l_{i1}}c_i\right),\ \cdots,\ \underline{S}_{iK_i} \otimes \left(\mathbb{J}^{l_{iK_i}}c_i\right)\ \right] \quad \in \mathcal{C}^{2NN_c\times K_i}$$

$$\overline{\mathbb{S}}_i = \left[\ \overline{\underline{S}}_{i1},\ \overline{\underline{S}}_{i2},\ \cdots,\ \overline{\underline{S}}_{iK_i}\ \right] \quad \in \mathcal{C}^{\overline{N}\times K_i}$$

$$\underline{\beta}_i = \left[\ \beta_{i1},\ \beta_{i2},\ \cdots,\ \beta_{iK_i}\ \right]^T \quad \in \mathcal{C}^{K_i\times 1}$$

$$\mathbb{H}_{i,\text{prev}} = \left(\mathbb{I}_N \otimes \left(\mathbb{J}^T\right)^{N_c}\right)\mathbb{H}_i \quad \in \mathcal{C}^{2NN_c\times K_i}$$

$$\mathbb{H}_{i,\text{next}} = \left(\mathbb{I}_N \otimes \mathbb{J}^{N_c}\right)\mathbb{H}_i \quad \in \mathcal{C}^{2NN_c\times K_i}.$$

By defining the composite channel transfer matrix of the i^{th} user

$$\mathbb{G}_i = \mathbb{H}_i\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H \quad \in \mathcal{C}^{2NN_c\times\overline{N}}$$

the MIMO-CDMA system model can be simplified by

$$\underline{x}[n] = \sum_{i=1}^{M} \left[\mathbb{G}_{i,\text{prev}}, \mathbb{G}_i, \mathbb{G}_{i,\text{next}}\right] \overline{\underline{b}}_i[n] + \underline{\mathrm{n}}[n] \tag{7}$$

$\mathbb{G}_{i,\text{prev}} = \mathbb{H}_{i,\text{prev}}\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H$, $\mathbb{G}_{i,\text{next}} = \mathbb{H}_{i,\text{next}}\,\mathrm{diag}\left(\underline{\beta}_i\right)\overline{\mathbb{S}}_i^H$ and

$$\overline{\underline{b}}_i[n] = \left[\underline{b}_i[n-1]^T,\ \underline{b}_i[n]^T,\ \underline{b}_i[n+1]^T\right]^T \quad \in \mathcal{C}^{3\overline{N}\times 1}$$

III. MIMO-CDMA Receivers

Assumed that the 1st user is the desired one, the decision variable vector during the n^{th} symbol period (after the receiver processor $\mathbb{W}_1$) can be decoupled into four terms representing the desired, ISI, MAI and noise terms:

$$\begin{aligned}
\underline{d}_1[n] &= \underline{d}_{1,\text{desired}}[n] + \underline{d}_{1,\text{isi}}[n] + \underline{d}_{1,\text{mai}}[n] + \underline{d}_{1,\text{noise}}[n] \\
&= \mathbb{W}_1^H \mathbb{G}_1 \underline{b}_1[n] + \mathbb{W}_1^H \left[\mathbb{G}_{1,\text{prev}}, \mathbb{G}_{1,\text{next}}\right] \begin{bmatrix} \underline{b}_1[n-1] \\ \underline{b}_1[n+1] \end{bmatrix} + \\
&\quad \mathbb{W}_1^H \sum_{i=2}^{M} \left[\mathbb{G}_{i,\text{prev}}, \mathbb{G}_i, \mathbb{G}_{i,\text{next}}\right] \overline{\underline{b}}_i[n] + \mathbb{W}_1^H \underline{\mathrm{n}}[n] \qquad (8)
\end{aligned}$$

Then the n^{th} data-symbol is detected by (for BPSK modulation) $\hat{\underline{b}}_1[n] = \operatorname{sign}\left(\operatorname{Re}\left\{\underline{d}_1[n]\right\}\right)$. Clearly, the optimum matrix $\mathbb{W}_1$ should be capable of coherently combining the multipath components of the desired signals as well removing the unwanted MAI and ISI. Note that the weight matrix is normalized to have unity norm.

A. Single-user Spatial-Temporal (ST) Receivers

Using the signal model proposed in (7), the two well-known single-user receivers (MMSE and Rake) can be extended to spatial-temporal receivers and redefined as

$$\begin{aligned}
&\text{Rake Receiver:} \quad && \mathbb{W}_1 = \mathbb{G}_1 \\
&\text{MMSE Receiver:} \quad && \mathbb{W}_1 = \mathbb{R}_{xx}^{-1} \mathbb{G}_1
\end{aligned}$$

where $\mathbb{R}_{xx} = \mathcal{E}\left\{\underline{x}[n]\,\underline{x}^H[n]\right\}$. These two receivers will be used for comparison with the two receivers proposed in the following subsection[2].

B. Subspace-based ST Receivers

With the aid of the array manifold, the directional parameters of the channel can be employed by the receiver to enhance the desired signals while alleviating the influence of unwanted signals. A subspace-based weight matrix is proposed as

$$\mathbb{W}_1 = \kappa_1 \mathbb{P}_n \mathbb{H}_1 \left(\mathbb{H}_1^H \mathbb{P}_n \mathbb{H}_1\right)^{-1} \overline{\mathbb{S}}_1^H \left(\overline{\mathbb{S}}_1 \operatorname{diag}\left(\underline{\beta}_1^*\right) \overline{\mathbb{S}}_1^H\right)^{-1} \qquad (9)$$

where κ_1 is the normalization factor and $\mathbb{P}_n$ represents the constructed projection-operator matrix onto the subspace spanned by the noise eigenvectors of the noise-plus-MAI/ISI space.

Substituting (9) into (8), it can be derived that

$$\underline{d}_{1,\text{desired}}[n] = \mathbb{W}_1^H \mathbb{G}_1 \underline{b}_1[n] = \kappa_1 \underline{b}_1[n]$$

In the rest of this section, based on different channel knowledge, two approaches of constructing the projection operator $\mathbb{P}_n$ are proposed.

If the receiver is capable of obtaining the channel parameters (including the spreading codes) of all the users, the projector can be constructed by

$$\begin{aligned}
\mathbb{P}_n &= \text{projection operator onto subspace } \mathcal{L}\left[\mathbb{G}_1\right]^{\perp} \\
&= \mathbb{I}_{2NN_c} - \widehat{\mathbb{H}}_1 \left(\widehat{\mathbb{H}}_1^H \widehat{\mathbb{H}}_1\right)^{-1} \widehat{\mathbb{H}}_1^H
\end{aligned}$$

[2]Note that, all these linear receivers (including the following proposed receivers), only aim at handling the effect ISI/MAI. To cope with the effect of Inter-Channel-Interference, system designers will need to implement Successive Cancellation (SUC) algorithms [1].

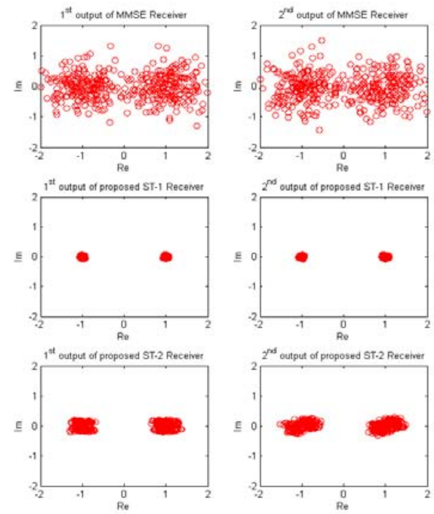

Fig. 2. Decision variables for 400 data symbols for a single run of the simulation. The subspace-based ST receivers show significant near-far resistant ability.

where $\widehat{\mathbb{H}}_1 = \left[\ \mathbb{H}_{1,\text{prev}}, \ \ \mathbb{H}_{1,\text{next}}, \ \ \mathbb{H}_{2,\text{total}} \ \ \cdots, \ \ \mathbb{H}_{M,\text{total}}\ \right]$ and $\mathbb{H}_{i,\text{total}} = \left[\ \mathbb{H}_{i,\text{prev}}, \ \ \mathbb{H}_i, \ \ \mathbb{H}_{i,\text{next}}\ \right]$. Eqn. (9) in connection with the above defined projection operator $\mathbb{P}_n$ will be called ST-1 receiver.

When the channel parameters of only the desired user are known, a interference–plus-noise-cancelling (INC) projection operator $\mathbb{P}_n$ can be obtained from the matrix $\mathbb{R}_{\text{unwanted}}$ associated with the last three terms of expression (8)

$$\mathbb{P}_n = \text{projection operator onto the noise subspace of } \mathbb{R}_{\text{unwanted}}$$

where $\mathbb{R}_{\text{unwanted}} = \mathbb{R}_{xx} - \mathbb{G}_1 \mathbb{G}_1^H$. This projection operator with (9) will be called ST-2 receiver.

With these two proposed approaches, the weight matrix $\mathbb{W}_1$ is made orthogonal to the estimated unwanted-signal subspace containing the contributions of ISI and MAI. The power of the residual interference is determined by the angle between the estimated unwanted-signal subspace and the actual MAI+ISI subspaces.

IV. Simulations

To evaluate the performance of the proposed receivers, firstly consider the uplink of a 3-user MIMO-CDMA system with the transmitters and receiver employing uniform linear array (ULA) of half-wavelength intersensor. The receivers operate in the presence of multipath effects from other users (MAI), as well as the ISI introduced in a frequency-selective fading channel. Using gold sequences of length 31, we arbitrarily assume 10 multipaths for 1st (desired) user, 8 multipaths for 2nd user and 6 multipaths for 3rd user. Furthermore, it is assumed that all required channel parameters have been successfully estimated, the input SNR associated with the desired user is 20 dB and the interfering signals are much stronger than the desired signal such that

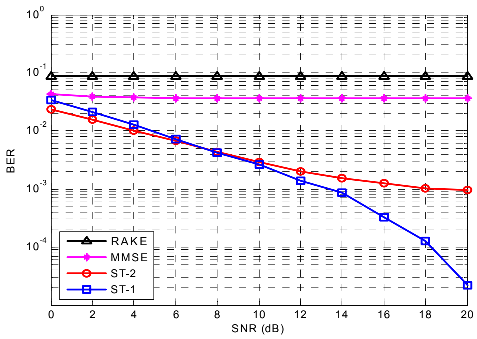

Fig. 3. BER performance of different receivers in a MIMO-CDMA system with 10 users, each with 3 multipaths. $\overline{N} = 2$ and $N = 2$, with half-wavelength spacing.

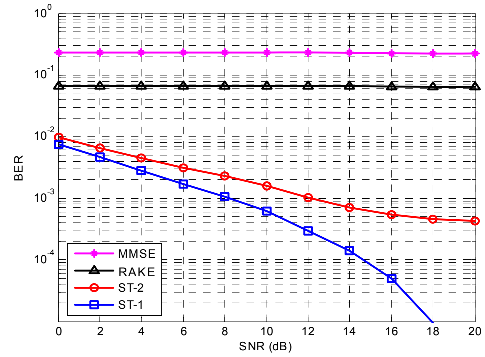

Fig. 4. BER performance of different receivers in a MIMO-CDMA system with 10 users, each with 3 multipaths. $\overline{N} = 2$ and $N = 4$. All arrays are ULA with half-wavelength spacing.

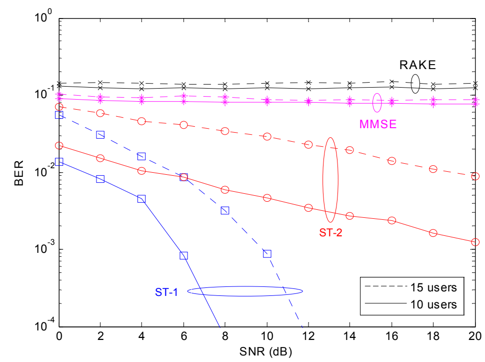

Fig. 5. BER performance of MIMO-CDMA system of 10 and 15 users, using 2-element ULA (half-wavelength spacing) at transmitters, 3-element ULA (half-wavelength spacing) at receivers, and assuming perfect channel estimation.

$$20 \log_{10} \left(\left\| \underline{\beta}_i \right\| / \left\| \underline{\beta}_1 \right\| \right) = 20 \text{ dB, for } i = 2, 3$$

The decision variables at the output of MMSE, ST-1 and ST-2 receivers are plotted in Fig. 2. It is shown that the proposed subspace-based ST receivers are near-far resistant.

In Fig. 3 and 4, the Bit-Error-Rate (BER) performance of various receivers is evaluated for $\overline{N} = 2$ and different value of N. There are 10 users, each with 3 multipaths. The simulation is executed for 1000 bursts of 400 symbols each. The results demonstrate that the two proposed ST receivers outperform RAKE and MMSE receivers. The BER performance of all ST receivers improves as the number of receive antennas increases from 2 to 4. However, the performance of the MMSE receiver decreases because with more receive antennas it needs more snapshots to properly estimated the receive correlation matrix. It is important to point out that because of near-far effect, the scenario considered here is MAI-dominant, which explains why the performance of MMSE or RAKE receivers do not improve by increasing the SNR.

When the transmitters and the receiver have roughly the same number of antennas, the ST-2 receiver performs better than ST-1 receiver in low SNR situations. This is because the ST-1 receiver only projects the received signal onto the null space of the interference, while the ST-2 receiver projects the received signal onto the null space of interference-noise space. When SNR is high, MAI becomes the dominant source of the error, so the ST-1 performs better.

The performance of the receivers is expected to decrease with the increase of the number of users in the CDMA system and simulation results presented in Fig. 5 show such effect. For a (2,3) MIMO channel with 2 multipaths per user. The performance improvement of the proposed ST receivers is clearly shown in Fig. 5.

V. Conclusions

In this paper, the uplink of a multiuser asynchronous MIMO-CDMA wireless communication system operating in frequency selective fading channel has been studied using the concept of STAR manifold vector. Two spatial-temporal receivers have been proposed based on projecting the received signals onto the null-space of the interference or interference–plus-noise subspace. Under the assumption that only the channel parameters of the desired user are available, simulation results show that, the subspace-based ST-2 receiver outperform RAKE and MMSE receivers.

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