MIMO-IDMA: UPLINK MULTIUSER MIMO COMMUNICATIONS USING INTERLEAVE-DIVISION MULTIPLE ACCESS AND LOW-COMPLEXITY ITERATIVE RECEIVERS

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ABSTRACT

Interleave-division multiple access (IDMA) has recently been proposed as an alternative to CDMA. IDMA employs user-specific interleavers combined with low-rate channel coding for user separation. It can outperform coded CDMA when iterative receivers are used, and it allows the design of multiuser detectors with moderate complexity. In this paper, we extend IDMA to MIMO multiuser systems employing spatial multiplexing. We also develop an iterative receiver for MIMO-IDMA that incorporates an efficient soft multiuser detector whose complexity is linear in the number of users. Both flat-fading and frequency-selective MIMO channels are considered. The performance of the proposed MIMO-IDMA system is assessed through simulation results.

Index Terms—MIMO-IDMA, spatial multiplexing, multiuser detection, iterative receivers, multiple access.

1. INTRODUCTION

In multiuser communications, CDMA is widely used for multiple access because of its attractive properties. With CDMA, user separation is obtained through signature (spreading) sequences, and fading effects are combated by an interleaver placed between the forward error correction (FEC) coding and the spreading. The performance of CDMA systems is limited by multiple-access interference (MAI) and intersymbol interference (ISI) [1]. Because of the high complexity of optimal joint detection and decoding, suboptimal receiver techniques are commonly used. A turbo-type receiver where the multiuser detector and the channel decoders exchange extrinsic information [2] has been presented in [3]. This receiver uses soft interference cancellation by means of an MMSE filter.

In [4], *interleave-division multiple access* (IDMA) has been proposed and demonstrated to outperform coded CDMA when iterative receivers are used. IDMA employs user-specific interleavers combined with low-rate channel coding for user separation. It is a wideband scheme that allows the use of multiuser detectors with moderate complexity. IDMA shares with CDMA several desirable features, especially diversity against fading and mitigation of user interference from other cells [4].

In this paper, we generalize IDMA to multiple-antenna (MIMO) multiuser systems in which the users employ spatial multiplexing [5] to achieve high data rates. The resulting *MIMO-IDMA* scheme extends all advantages of IDMA to the MIMO case. Furthermore,

generalizing the receiver proposed in [4], we develop an iterative receiver for MIMO-IDMA that employs an efficient soft multiuser detector whose complexity is linear in the number of users. Both flat-fading and frequency-selective MIMO channels are considered.

The paper is organized as follows. In Section 2, we describe the structure of the new MIMO-IDMA system, including the iterative receiver. The MIMO-IDMA soft multiuser detector is developed in Section 3. Finally, simulation results demonstrating the performance of MIMO-IDMA are presented in Section 4.

2. THE PROPOSED MIMO-IDMA SYSTEM

2.1. Basic System Model

We consider an uplink multiple-access scenario where each user (transmitter) has $M_{\rm T}$ antennas and employs spatial multiplexing [5], and the base station (receiver) has $M_{\rm R}$ antennas. The MIMO channels are allowed to be time-varying and frequency-selective. At a given time instant n, the received vector $\mathbf{r}[n] = (r_1[n] \cdots r_{M_{\rm R}}[n])^T$ can thus be expressed as

$$\mathbf{r}[n] = \sum_{m=1}^{M} \sum_{l=0}^{L-1} \mathbf{H}^{(m)}[n,l] \mathbf{x}^{(m)}[n-l] + \mathbf{w}[n], \qquad (1)$$

where $\mathbf{x}^{(m)}[n] = (x_1^{(m)}[n] \cdots x_{M_T}^{(m)}[n])^T$ is the data vector transmitted by the *m*th user, $\mathbf{H}^{(m)}[n, l]$ is the $M_{\mathsf{R}} \times M_{\mathsf{T}}$ MIMO channel matrix from the *m*th user to the base station (*l* denotes the discrete delay), $\mathbf{w}[n] = (w_1[n] \cdots w_{M_{\mathsf{R}}}[n])^T$ is a noise vector, *L* is the channel length, and *M* is the number of users. We assume that the data symbols $x_k^{(m)}[n]$ are from a BPSK alphabet $\mathcal{A} = \{-1, 1\}$ and the elements of the noise vector $\mathbf{w}[n]$ are i.i.d. Gaussian with variance σ^2 .

2.2. MIMO-IDMA Transmitter

The proposed MIMO-IDMA transmitter extends the IDMA transmitter of [4] to the multiuser MIMO spatial multiplexing case. As shown in Fig. 1(a), the data bit sequence of the *m*th user, $\mathbf{b}^{(m)} = (b^{(m)}[1] \cdots b^{(m)}[K])^T$, is encoded into a code bit sequence $\mathbf{c}^{(m)} = (c^{(m)}[1] \cdots c^{(m)}[N])^T$. Here, N = K/R where *R* is the code rate. The code is a serial concatenation of a terminated convolutional code for FEC and a low-rate repetition code.

Next, the code bit sequence $\mathbf{c}^{(m)}$ is interleaved by a user-specific interleaver $\pi^{(m)}(\cdot)$, resulting in the sequence $\mathbf{d}^{(m)}$ with $d^{(m)}[n] = c^{(m)}[\pi^{(m)}(n)]$. The *M* interleavers are assumed to be randomly

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Fig. 1. The proposed MIMO-IDMA system: (a) Transmitter, (b) receiver.

generated. The repeat code together with the user-specific interleaver performs a kind of spreading, somewhat similar to the spreading in a CDMA system. Finally, the interleaved bit sequence is BPSK-modulated using the mapping $0 \rightarrow -1, 1 \rightarrow 1$ and split into blocks of length $M_{\rm T}$ to yield the symbol vector $\mathbf{x}^{(m)}[n]$ transmitted by the $M_{\rm T}$ transmit antennas. The combined action of the BPSK mapping and serial-to-parallel conversion is expressed by

$$\mathbf{x}^{(m)}[n] = \begin{pmatrix} 2 d^{(m)}[nM_{\rm T}+1] - 1\\ 2 d^{(m)}[nM_{\rm T}+2] - 1\\ \vdots\\ 2 d^{(m)}[(n+1)M_{\rm T}] - 1 \end{pmatrix}, \quad n = 0, \dots, \frac{N}{M_{\rm T}} - 1.$$
(2)

2.3. Iterative MIMO-IDMA Receiver

The structure of the iterative (turbo-type [2–4]) MIMO-IDMA receiver is shown in Fig. 1(b). The receiver consists of two stages: a low-complexity soft MIMO multiuser detector—to be developed in Section 3—is followed by M parallel single-user soft-in/soft-out channel decoders (one corresponding to each user). These two stages are connected via deinterleavers and interleavers and exchange soft-information, as described in the following.

Soft MIMO Multiuser Detection. Assuming independent interleaved symbols $d^{(m)}[n]$ (i.e., ignoring the code dependencies), a symbolwise operation of the multiuser detector is optimum. At a given iteration, the detector then estimates the *a posteriori* log-likelihood ratios (LLRs) of the code bits $c^{(m)}[n]$,

$$\ell_{\rm mud}(c^{(m)}[n]) \triangleq \log \frac{P(c^{(m)}[n] = 1|\tilde{\mathbf{r}})}{P(c^{(m)}[n] = 0|\tilde{\mathbf{r}})},$$
(3)

for n = 1, ..., N and m = 1, ..., M. Here, $\tilde{\mathbf{r}}$ denotes the sequence of all received vectors $\mathbf{r}[n]$. In what follows, we will often suppress the dependence on n for simplicity of notation, e.g., we will write $c^{(m)}$ instead of $c^{(m)}[n]$. With Bayes' rule, (3) can be rewritten as

$$\ell_{\rm mud}(c^{(m)}) = \log \frac{P(\tilde{\mathbf{r}} | c^{(m)} = 1)}{P(\tilde{\mathbf{r}} | c^{(m)} = 0)} + \log \frac{P(c^{(m)} = 1)}{P(c^{(m)} = 0)}.$$
 (4)

The first term is the *extrinsic information* delivered by the MIMO multiuser detector and will be denoted as $\xi_{mud}(c^{(m)})$:

$$\xi_{\text{mud}}(c^{(m)}) \triangleq \log \frac{P(\tilde{\mathbf{r}} \mid c^{(m)} = 1)}{P(\tilde{\mathbf{r}} \mid c^{(m)} = 0)}.$$
(5)

Since according to (2) $x_k^{(m)}[n]$ corresponds to $d^{(m)}[nM_{\rm T} + k] = c^{(m)}[\pi^{(m)}(nM_{\rm T}+k)] =: c_{\pi,k}^{(m)}[n]$ via the BPSK mapping, the extrinsic information for $x_k^{(m)}[n]$ equals that for $c_{\pi,k}^{(m)}[n]$, i.e., $\xi_{\rm mud}(x_k^{(m)}) = \xi_{\rm mud}(c_{\pi,k}^{(m)})$. The second term in (4) is the *a priori* LLR of $c^{(m)}$. An estimate of this term is provided by the channel decoder of the *m*th user at the previous iteration. For the first iteration, equally likely bits are assumed, i.e., the *a priori* LLR values are set to zero. Finally, the sequence of extrinsic informations $\xi_{\rm mud}(c^{(m)}[n])$ is deinterleaved by the deinterleaver of the *m*th user and fed into the corresponding soft channel decoder as *a priori* information for the next iteration. The detailed operation of the soft MIMO multiuser detector will be described in the Section 3.

Soft Channel Decoding. The channel decoder for the *m*th user estimates the *a posteriori* LLR of the code bits

$$\ell_{\rm dec}(c^{(m)}) \triangleq \log \frac{P(c^{(m)} = 1|\tilde{\mathbf{r}})}{P(c^{(m)} = 0|\tilde{\mathbf{r}})},$$

based on the extrinsic information from the multiuser detector, $\xi_{mud}(c^{(m)})$, and the code structure. To estimate $\ell_{dec}(c^{(m)})$, the repeat code is soft-decoded by summing the *a priori* LLR values of the appropriate symbols (this presupposes that the corresponding received symbols are independent due to the combined action of the interleaver and the noisy fading channel) and the convolutional code is soft-decoded by the BCJR algorithm [6]. As in (4), $\ell_{dec}(c^{(m)})$ can be expressed as the sum of an extrinsic information $\xi_{dec}(c^{(m)})$ and an *a priori* LLR [3]. The sequence of extrinsic informations $\xi_{dec}(c^{(m)}[n])$ is interleaved and fed back to the multiuser detector as *a priori* information for the next iteration. Additionally, the channel decoder estimates the *a posteriori* LLRs of the information bits; at the final iteration, it performs a decision on the information bits by applying the sign function.

3. A LOW-COMPLEXITY SOFT MULTIUSER DETECTOR FOR MIMO-IDMA

We will now develop a low-complexity soft multiuser detector for MIMO-IDMA. As discussed above, the purpose of the soft multiuser detector is to calculate the extrinsic informations $\xi_{\text{mud}}(c_{\pi,k}^{(m)})$ based on the current received vector **r** and the extrinsic informations $\xi_{\text{dec}}(x_k^{(\nu)}), \nu = 1, \ldots, M$ produced by the channel decoders at the previous iteration. Our development extends the IDMA soft multiuser detector of [4] to the MIMO spatial-multiplexing case.

3.1. Flat-Fading Channel

For simplicity, we first consider a flat-fading channel; the frequencyselective case will be considered in the next subsection. In the flatfading case, (1) simplifies to

$$\mathbf{r}[n] = \sum_{m=1}^{M} \mathbf{H}^{(m)}[n] \mathbf{x}^{(m)}[n] + \mathbf{w}[n].$$
 (6)

This can be formulated as (again, we omit the time instant n)

$$\mathbf{r} = \mathbf{h}_k^{(m)} x_k^{(m)} + \boldsymbol{\eta}_k^{(m)}.$$
 (7)

Here, $\mathbf{h}_{k}^{(m)}$ is the *k*th column of $\mathbf{H}^{(m)}$ and $\boldsymbol{\eta}_{k}^{(m)}$ consists of the interference from the symbols transmitted from all other antennas of the same user, the interference from all other users, and the noise, i.e.,

$$\eta_k^{(m)} = \sum_{\substack{j=1\\ j \neq k}}^{M_{\rm T}} \mathbf{h}_j^{(m)} x_j^{(m)} + \sum_{\substack{\nu=1\\ \nu \neq m}}^{M} \mathbf{H}^{(\nu)} \mathbf{x}^{(\nu)} + \mathbf{w} \,.$$

Development of the Multiuser Detector. The proposed MIMO-IDMA multiuser detector is based on the assumption that $\eta_k^{(m)}$ has a Gaussian distribution; this will be approximately true for a large number of users. For given user channels $\mathbf{H}^{(m)}$, the conditional probability density function of \mathbf{r} given $x_k^{(m)}$ then is

$$p(\mathbf{r}|x_k^{(m)}) \sim \mathcal{N} \Big(\mathbf{h}_k^{(m)} x_k^{(m)} + \mathrm{E}(\boldsymbol{\eta}_k^{(m)}), \mathbf{C}_{\boldsymbol{\eta}_k^{(m)}} \Big) \,,$$

where $E(\eta_k^{(m)})$ and $C_{\eta_k^{(m)}}$ denote, respectively, the mean and covariance matrix of $\eta_k^{(m)}$ for given user channels $\mathbf{H}^{(m)}$. Using the Gaussian assumption, the extrinsic information in (5) becomes

$$\xi_{\text{mud}}(c_{\pi,k}^{(m)}) = \xi_{\text{mud}}(x_k^{(m)}) = 2\mathbf{h}_k^{(m)H} \mathbf{C}_{\eta_k^{(m)}}^{-1} \left[\mathbf{r} - \mathbf{E}(\boldsymbol{\eta}_k^{(m)})\right], \quad (8)$$

which is recognized as the MMSE estimate of $x_k^{(m)}$ from **r** using the linear model (7).

It remains to calculate $E(\boldsymbol{\eta}_k^{(m)})$ and $\mathbf{C}_{\boldsymbol{\eta}_k^{(m)}}$. Combining (6) and (7) yields $\boldsymbol{\eta}_k^{(m)} = \sum_{\nu=1}^M \mathbf{H}^{(\nu)} \mathbf{x}^{(\nu)} - \mathbf{h}_k^{(m)} x_k^{(m)} + \mathbf{w}$ and, thus,

$$E(\boldsymbol{\eta}_{k}^{(m)}) = \sum_{\nu=1}^{M} \mathbf{H}^{(\nu)} E(\mathbf{x}^{(\nu)}) - \mathbf{h}_{k}^{(m)} E(x_{k}^{(m)}).$$
(9)

Furthermore, from (7) we obtain

$$\mathbf{C}_{\eta_k^{(m)}} = \mathbf{C}_r - \mathbf{h}_k^{(m)} \operatorname{var}(x_k^{(m)}) \mathbf{h}_k^{(m)H}, \qquad (10)$$

where \mathbf{C}_r is the covariance matrix of the received vector \mathbf{r} . This expression is based on the assumption that, due to the interleaver, the symbols $x_k^{(m)}[n]$ are independent. We have

$$\mathbf{C}_{r} = \sum_{m=1}^{M} \mathbf{H}^{(m)} \mathbf{C}_{x^{(m)}} \mathbf{H}^{(m)H} + \sigma^{2} \mathbf{I}, \qquad (11)$$

with $\mathbf{C}_{x^{(m)}} = \operatorname{diag}\left\{\operatorname{var}(x_k^{(m)})\right\}_{k=1,\dots,M_{\mathrm{T}}}$. Woodbury's identity [7] gives the following expression of $\mathbf{C}_{n^{(m)}}^{-1}$:

$$\mathbf{C}_{\eta_{k}^{(m)}}^{-1} = \mathbf{C}_{r}^{-1} - \mathbf{C}_{r}^{-1} \mathbf{h}_{k}^{(m)} \left[\mathbf{h}_{k}^{(m)H} \mathbf{C}_{r}^{-1} \mathbf{h}_{k}^{(m)} - \frac{1}{\left[\operatorname{var}(x_{k}^{(m)}) \right]^{2}} \right] \mathbf{h}_{k}^{(m)H} \mathbf{C}_{r}^{-1}.$$
(12)

Thus, computation of (8) only requires a single matrix inversion.

Finally, we express $E(x_k^{(m)})$ and $var(x_k^{(m)})$ in terms of $\xi_{dec}(x_k^{(m)})$. In the iterative receiver scheme, the extrinsic information $\xi_{dec}(x_k^{(m)})$ produced by the channel decoders at the previous iteration is substituted for the *a priori* LLR of $c_{\pi,k}^{(m)}$ (recall that $x_k^{(m)}$ corresponds to $c_{\pi,k}^{(m)}$ via the BPSK mapping), i.e.,

$$\log \frac{P(c_{\pi,k}^{(m)} = 1)}{P(c_{\pi,k}^{(m)} = 0)} \stackrel{!}{=} \xi_{\text{dec}}(x_k^{(m)})$$

This is easily seen to imply the expressions

$$E(x_k^{(m)}) = \tanh \frac{\xi_{dec}(x_k^{(m)})}{2}, \quad \operatorname{var}(x_k^{(m)}) = 1 - [E(x_k^{(m)})]^2.$$

Complexity. The complexity of the proposed multiuser detector (using (12)) is linear in the number of users M. It is furthermore cubic in the number of receive antennas $M_{\rm R}$ because (12) requires the inversion of the $M_{\rm R} \times M_{\rm R}$ matrix C_r .

Scheduling Strategies. The updating of the *a priori* information at the input of the multiuser detector can be done in two different ways [8]. With *parallel scheduling*, the *a priori* information of all users at the input of the multiuser detector is updated by the channel decoders and used to calculate the extrinsic information of all users at the output of the multiuser detector concurrently. With *serial scheduling*, only the *a priori* information of all users at the output of the multiuser detector. In the next time step, the *a priori* information of another user is updated, and so forth.

3.2. Frequency-Selective Channel

Again generalizing [4], we now extend the MIMO-IDMA multiuser detector developed above to frequency-selective channels, i.e., channels introducing ISI. According to (1), the channel of the *m*th user is characterized by $\mathbf{H}^{(m)}[n, l]$, where $l \in \{0, \ldots, L-1\}$. To take ISI into account, we combine *L* successive received vectors into the super-vector $\mathbf{\bar{r}}[n] \triangleq (\mathbf{r}[n] \mathbf{r}[n+1] \cdots \mathbf{r}[n+L-1])^T$ of length $M_{\mathrm{R}}L$. Using (1), we can formulate $\mathbf{\bar{r}}[n]$ as

$$\bar{\mathbf{r}}[n] = \bar{\mathbf{h}}_{k}^{(m)}[n] x_{k}^{(m)}[n] + \bar{\boldsymbol{\eta}}_{k}^{(m)}[n].$$
(13)

Here, $\bar{\mathbf{h}}_{k}^{(m)}[n] \triangleq (\mathbf{h}_{k}^{(m)}[n,0] \mathbf{h}_{k}^{(m)}[n+1,1]\cdots\mathbf{h}_{k}^{(m)}[n+L-1,L-1])^{T}$, where $\mathbf{h}_{k}^{(m)}[n,l]$ is the *k*th column of $\mathbf{H}^{(m)}[n,l]$, and $\bar{\boldsymbol{\eta}}_{k}^{(m)}[n] \triangleq (\boldsymbol{\eta}_{k}^{(m)}[n] \boldsymbol{\eta}_{k}^{(m)}[n+1]\cdots\boldsymbol{\eta}_{k}^{(m)}[n+L-1])^{T}$. Because (13) has the form of the linear model (7), the extrinsic information $\xi_{\text{mud}}(c_{\pi,k}^{(m)})$ can be calculated as in (8) with obvious modifications:

$$\xi_{\rm mud}(c_{\pi,k}^{(m)}) = \xi_{\rm mud}(x_k^{(m)}) = 2\bar{\mathbf{h}}_k^{(m)H} \mathbf{C}_{\bar{\eta}_k^{(m)}}^{-1} \left[\bar{\mathbf{r}} - \mathbf{E}(\bar{\eta}_k^{(m)}) \right],$$

with $E(\bar{\eta}_k^{(m)})$, $C_{\bar{\eta}_k^{(m)}}$, and $C_{\bar{\eta}_k^{(m)}}^{-1}$ given by equations (9)–(12) suitably modified. The complexity still is linear in M and cubic in M_R ; it is furthermore cubic in the channel's impulse response length L.

4. SIMULATION RESULTS

We next present simulation results demonstrating the performance of the proposed MIMO-IDMA system.

Flat-fading channel. We first consider the case of a fast timevarying flat-fading channel. We simulated a MIMO-IDMA system with M = 16 users, $M_T = 4$ antennas per user, $M_R = 4$ base station antennas, and a terminated rate-1/2 convolutional code (code polynomial [23 35]₈) serially concatenated with a rate-1/16 repetition code (thus, the overall code rate was R = 1/32). The number of information bits per block was K = 128; this implies a total number of $K/(RM_T) = 1024$ transmit vectors per user. The channel matrices $\mathbf{H}^{(m)}[n]$ of size 4×4 were generated independently for each n, with elements that were i.i.d. Gaussian with zero mean and unit variance. The multiuser detector used parallel scheduling.



Fig. 2. Performance of a 4×4 MIMO-IDMA system with 16 users for a fast time-varying flat-fading channel: (a) Averaged BER versus E_b/N_0 and single-user bound after 10 iterations, (b) averaged BER versus the number of iterations, parametrized by E_b/N_0 .

In Fig. 2(a), the bit error rate (BER) averaged over all users after 10 iterations is shown versus the signal-to-noise ratio (SNR) E_b/N_0 (we note that $E_b/N_0 = M_T/(R\sigma^2)$). The single-user bound (BER for one user, i.e., M = 1) is shown as a performance reference. It is seen that the BER of the multiuser system approaches the single-user bound for high SNR. In Fig. 2(b), the convergence of the iterative receiver, i.e., BER as a function of the number of iterations, is depicted for various values of E_b/N_0 . It is seen that the system converges for E_b/N_0 larger than about 7 dB. For higher E_b/N_0 , the convergence is faster and the BER after convergence is lower. For high E_b/N_0 (about 9 dB), the receiver converges after about 5 iterations.

Frequency-selective channel. Next, we consider a time-varying frequency-selective channel with L = 3 taps. The channel was generated such that it stays constant for 50 transmit vectors and then a new realization is drawn independently. Because the matrices now have threefold size, a smaller system with MIMO dimension 2×2 and M = 8 users was implemented to reduce simulation times. All other system parameters were as before. Fig. 3 shows the BER as a function of E_b/N_0 after 20 iterations. The gap to the singleuser bound is smaller compared to the flat-fading case because of the smaller number of users and the delay diversity offered by the frequency-selective channel.



Fig. 3. BER of a 2×2 MIMO-IDMA system with 8 users versus E_b/N_0 for a time-varying frequency-selective channel.

5. CONCLUSIONS

We proposed a MIMO-IDMA system for uplink multiuser communications over frequency-selective channels. This system generalizes the IDMA system described in [4] to the MIMO spatial multiplexing case. It uses a turbo-type iterative MIMO-IDMA receiver incorporating an efficient soft MIMO multiuser detector whose complexity is linear in the number of antennas.

MIMO-IDMA is attractive due to its good performance and low complexity. The basic MIMO-IDMA system considered here can be extended in various ways. The simple convolutional code can be replaced by more sophisticated codes such as LDPC codes (an indepth study of the use of multiuser LDPC codes along with iterative receivers, including code optimization, is reported in [9]). Various strategies for selecting the order of the users in the serial scheduling scheme described in Section 3.1 can be used. A detailed assessment of the performance and complexity of these and other variants of MIMO-IDMA in comparison to a CDMA-based multiuser MIMO system is an interesting topic for future research.

6. REFERENCES

- [1] S. Verdú, *Multiuser Detection*. Cambridge (UK): Cambridge Univ. Press, 1998.
- [2] P. H. Siegel, D. Divsalar, E. Eleftheriou, J. Hagenauer, D. Rowitch, and W. H. Tranter, "The turbo principle: from theory to practice," *IEEE J. Sel. Areas Comm.*, vol. 19, pp. 793–799, May 2001.
- [3] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Comm.*, vol. 47, pp. 1046– 1061, July 1999.
- [4] L. Ping, L. Liu, and K. W. W. Leung, "Interleave-division multipleaccess," *IEEE Trans. Wireless Comm.*, vol. 5, pp. 938–947, Apr. 2006.
- [5] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Boston (MA): Cambridge University Press, 2005.
- [6] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, vol. 20, pp. 284–287, March 1974.
- [7] L. L. Scharf, *Statistical Signal Processing*. Reading (MA): Addison Wesley, 1991.
- [8] J. Boutros and G. Caire, "Iterative multiuser joint decoding: Unified framework and asymptotic analysis," *IEEE Trans. Inf. Theory*, vol. 48, pp. 1772–1793, July 2002.
- [9] A. Sanderovich, M. Peleg, and S. Shamai (Shitz), "LDPC coded MIMO multiple access with iterative joint decoding," *IEEE Trans. Inf. Theory*, vol. 51, pp. 1437–1450, Apr. 2005.