JOINT FREQUENCY OFFSET AND CHANNEL ESTIMATION FOR UL-MIMO-OFDMA SYSTEMS USING THE PARALLEL SCHMIDT KALMAN FILTERS

Kyeong Jin Kim^{†§}, Man-On Pun[‡], Tony Reid[†], Ronald A. Iltis[‡]

[†] Nokia Inc., 6000 Connection Dr., Irving, TX 75039

 [‡] Dept. of Electrical Engineering, Princeton University, Princeton, NJ 08544
 [‡] Dept. of Electrical and Computer Engineering University of California, Santa Barbara, CA 93106

ABSTRACT

Joint estimation of the carrier frequency offset (CFO) and channel response of each active user in the uplink of an OFDMA system over time-varying channels is investigated in this work. To cope with the enormous computational complexity involved in tracking the time variations of CFOs and channels, we propose to use the Parallel Schmidt Kalman Filter (PSKF) to break down the complicated optimization problem into multiple parallel but smaller optimization problems. This results in an estimation scheme whose complexity only grows linearly with the number of users. Simulations indicate that the proposed scheme can achieve high estimation accuracy.

Index Terms — Schmidt Kalman Filter, uplink, MIMO-OFDMA, frequency offset, channel.

1. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) has been considered as one of the leading multiple-access technologies for broadband wireless networks. In uplink(UL) OFDMA systems, active mobile users communicate with the base station (BS) by modulating the subcarriers exclusively assigned to them. In addition to its robustness to multipath fading and high spectral efficiency, OFDMA is particularly attractive due to its flexibility in allocating subcarriers to different users based on their different quality of service (QoS) requirements and channel conditions in a dynamic fashion [1]. Meanwhile, the recent developments in the multiple-input multiple-output (MIMO) techniques have led to considerable research interests in MIMO-OFDMA. Under the assumption that the propagation path between each pair of transmit and receive antennas undergoes independent fading, MIMO systems divide the original high-rate data stream into several parallel data substreams, each of which is transmitted from a corresponding transmit antenna. By exploiting this spatial multiplexing, MIMO OFDMA can achieve even higher spectral efficiency. As

a result, MIMO-OFDMA has been envisioned as a strong candidate for 4G mobile cellular systems.

In this paper we consider an uplink MIMO-OFDMA system. In the uplink, the MIMO-OFDMA system requires that active users must be synchronized in time and frequency in order to maintain orthogonality among different subcarriers, and subsequently different users. Furthermore, accurate channel estimation is indispensable in order to provide reliable data transmissions when coherent data detection is employed. Several schemes have been proposed to perform joint synchronization and channel estimation for uplink OFDMA systems, e.g. [2, 3, 4]. In particular, [2] proposes a maximum likelihood (ML) scheme to jointly estimate the carrier frequency offsets (CFOs), timing errors and channel impulse responses of all active users by resorting to the alternating projection technique, whereas a subspace-based CFO estimation scheme is proposed in [3]. These existing schemes have good performance but assume the channel is time-invariant. Since high Doppler shifts are expected in the outdoor environment due to high mobility of users, these approaches may not be suitable for practical time-varying environment. In this paper, we propose a new joint frequency offset and channel estimation scheme for UL-MIMO-OFDMA systems over time-varying channels, assuming perfect time synchronization has already been achieved.

In our previous works, nonlinear filtering techniques such as the Kalman filtering technique have been successfully applied to joint CFO and channel estimation for singleuser MIMO-OFDM systems [5, 6]. Unfortunately, a straightforward extension of the methods in [5, 6] to multi-user MIMO-OFDMA incurs prohibitively expensive computational complexity. To cope with this problem, we propose to employ a parallel Schmidt Kalman filter (PSKF) approach [7, 8] in this paper. The PSKF can substantially reduce the computational complexity required by the conventional Kalman filter without sacrificing significant performance loss. It will be evident in the later sections that the computational complexity required by PSKF only grows linearly with the total number of active users, which facilitates the development of a pipelined filtering processor where each

[§] Corresponding author: kyeong.j.kim@nokia.com.

filtering subblock performs one process for each user.

2. SIGNAL AND CHANNEL MODELS FOR UPLINK OFDMA SYSTEMS

We consider an uplink OFDMA system with N subcarriers and K active users. The BS and each active user are equipped with N_r and N_t antennas, respectively. Each user is assigned N_k exclusive subcarriers, where $\sum_{k=1}^{\tilde{K}} N_k \leq N$. We denote the index set of carriers assigned to the k-th user as $\mathcal{I}_k \stackrel{\triangle}{=} \{i_1, i_2, \dots, i_{N_k}\}$ where $1 \leq i_l \leq N$ for l = $1, 2, \dots, N_k$. In this paper, we do not assume any specific carrier assignment scheme (CAS) in the derivation of the proposed scheme. As a result, the proposed scheme is applicable to any generalized CSA. Denote the N_k data symbols belonging to the *n*-th OFDMA block transmitted by the k-th user from the p-th antenna as $\boldsymbol{b}_{k}^{p}(n)$. For presentational convenience, we assume the data symbols are taken from the same complex-valued finite alphabet. $\boldsymbol{b}_{k}^{p}(n)$ is first mapped to form an vector $\tilde{d}_k^p(n)$ of length N. The *i*-th entry of $\tilde{d}_k^p(n)$, say $\tilde{d}_{k,i}^p(n)$, is non-zero if and only if $i \in \mathcal{I}_k$. Next, $\tilde{d}_{k}^{p}(n)$ is converted to the corresponding time-domain vector by an N-point inverse discrete fourier transform (IDFT). To prevent inter-symbol interference (ISI), a cyclic prefix (CP) of N_g symbols is appended in front of each IDFT output block. The resulting vector of length $N_d^g = N_d + N_g$ is digital-to-analog converted by a pulse-shaping filter, $p_D(t)$, with a finite support on $[0, T_d)$ where $T_d = NT_s$ with T_s being the data symbol interval. Finally, the analog signal from the pulse-shaping filter, $\tilde{s}_k^p(t)$, is transmitted over the channel from the *p*-th antenna.

The channel between the *p*-th transmit antenna of the *k*th user and the *q*-th receive antenna of the BS during the *n*th block, $\{h_{k,l}^{p,q}(n)\}$, is modeled as a tapped delay line (TDL) with delays $\tau_{k,l}^{p,q}$, where $0 \le l \le L_k^{p,q} - 1$ with $L_k^{p,q}$ being the channel order. Since $L_k^{p,q}$ is generally unknown, in practice we replace $L_k^{p,q}$ by L_f for all users and antenna pairs. We assume that the CP is sufficient to comprise the maximum path delay, i.e., $\max_{k,l,p,q}(\tau_{k,l}^{p,q}) \le N_g T_s$. Furthermore, we assume that $\{h_{k,l}^{p,q}(n)\}$ is constant over one OFDMA block but varies from block to block. Under such assumptions, in the *absence* of carrier frequency offsets, the received signal at the BS from the receiver antenna *q* corresponding to the *n*-th symbol is given as

$$r^{q}(t) = \sum_{k=1}^{K} \sum_{p=1}^{N_{t}} \sum_{l=0}^{L_{f}-1} h_{k,l}^{p,q}(n) \tilde{s}_{k}^{p}(t - lT_{s} - \tau_{k,l}^{p,q}) + v^{q}(t), \quad (1)$$

where $v^{q}(t)$ is a circularly symmetric white Gaussian noise.

After the guard interval is removed from $r^q(t)$, we serialto-parallel convert the resulting signal and obtain the *n*-th received OFDMA block signal given as

$$\mathbf{r}^{q}(n) = \sum_{k=1}^{K} \sum_{p=1}^{N_{t}} \mathbf{D}_{k}^{p}(n) \mathbf{h}_{k}^{p,q}(n) + \mathbf{v}^{q}(n), \qquad (2)$$

where

$$\mathbf{v}^q(n) \sim \mathcal{N}(\mathbf{v}^q(n); \mathbf{0}, 2N_0/T_s \mathbf{I}_N),$$

$$\mathbf{h}_{k}^{p,q}(n) \stackrel{\triangle}{=} \left[h_{k,0}^{p,q}(n), h_{k,1}^{p,q}(n), \dots, h_{k,L_{f}-1}^{p,q}(n) \right]^{T}, \\ \mathbf{D}_{k}^{p}(n) \stackrel{\triangle}{=} \left[\begin{array}{ccc} d_{k,0}^{p}(n) & d_{k,N-1}^{p}(n) & \dots & d_{k,N-L_{f}+1}^{p}(n) \\ d_{k,1}^{p}(n) & d_{k,0}^{p}(n) & \dots & d_{k,N-L_{f}+2}^{p}(n) \\ \vdots & \vdots & \dots & \dots \\ d_{k,N-1}^{p}(n) & d_{k,N-2}^{p}(n) & \dots & d_{k,N-L_{f}}^{p}(n) \end{array} \right], \\ \mathbf{d}_{k}^{p}(n) = 1/\sqrt{N} \mathbf{W}^{H} \tilde{\mathbf{d}}_{k}^{p}(n).$$
 (3)

with $\mathbf{W} \in \mathbb{C}^{N \times N}$ being the fast Fourier transform (FFT) matrix. Note that $\mathbf{d}_k^p(n)$ is the IFFT output corresponding to input $\tilde{\mathbf{d}}_k^p(n)$ and $\mathcal{N}(\mathbf{x}; \mathbf{m}_x, \Sigma_x)$ denotes a circular Gaussian density with mean vector \mathbf{m}_x and covariance matrix Σ_x .

Unfortunately, due to the Doppler effect and oscillator mismatch between transmitter and receiver pairs, the received signal is usually distorted by carrier frequency offsets. Let $\varepsilon_k^{p,q}$ be the normalized carrier frequency offset with respect to the carrier spacing between the *p*-th transmit antenna of the *k*-th user and the *q*-th receive antenna of the BS. In the *presence* of CFOs, (2) becomes

$$\mathbf{r}^{q}(n) = \sum_{k=1}^{K} \sum_{p=1}^{N_{t}} \tilde{\mathbf{\Delta}}(\varepsilon_{k}^{p,q}(n)) \mathbf{D}_{k}^{p}(n) \mathbf{h}_{k}^{p,q}(n) + \mathbf{v}^{q}(n),$$
$$= \sum_{k=1}^{K} \tilde{\mathbf{D}}_{\varepsilon,k}^{q}(n) \mathbf{h}_{k}^{q}(n) + \mathbf{v}^{q}(n), \qquad (4)$$

where

$$\begin{split} \tilde{\boldsymbol{\Delta}}(\varepsilon_{k}^{p,q}(n)) & \stackrel{\triangle}{=} e^{j2\pi\varepsilon_{k}^{p,q}(n)((n-1)N_{d}^{q}+N_{g})}\boldsymbol{\Delta}(\varepsilon_{k}^{p,q}(n)), \\ \boldsymbol{\Delta}(\varepsilon_{k}^{p,q}(n)) & \stackrel{\triangle}{=} \operatorname{diag}(1, e^{j\frac{2\pi\varepsilon_{k}^{p,q}(n)}{N}}, \cdots, e^{j\frac{2\pi(N-1)\varepsilon_{k}^{p,q}(n)}{N}}), \\ \mathbf{h}_{k}^{q}(n) & \stackrel{\triangle}{=} \left[\mathbf{h}_{k}^{1,q}(n)^{T}, \mathbf{h}_{k}^{2,q}(n)^{T}, v, \mathbf{h}_{k}^{N_{t},q}(n)^{T}\right]^{T}, \\ \tilde{\mathbf{D}}_{\varepsilon,k}^{q}(n) & \stackrel{\triangle}{=} \left[\tilde{\boldsymbol{\Delta}}(\varepsilon_{k}^{1,q}(n))\mathbf{D}_{k}^{1}(n), \cdots, \tilde{\boldsymbol{\Delta}}(\varepsilon_{k}^{N_{t},q}(n))\mathbf{D}_{k}^{N_{t}}(n)\right]. \end{split}$$

In the literature, a number of approaches have been proposed to model the time-varying channels and frequency offsets in mobile environments, e.g. [5, 6, 9]. We adopt a parametric model that uses the first order autoregressive (AR) model to model the channel and frequency offset as follows.

$$\varepsilon_{k}^{p,q}(n) = \theta_{k,\varepsilon}^{p,q} \varepsilon_{k}^{p,q}(n-1) + w_{k,\varepsilon}^{p,q}(n), \\
\mathbf{h}_{k}^{p,q}(n) = \mathbf{\Theta}_{k,h}^{p,q} \mathbf{h}_{k}^{p,q}(n-1) + \mathbf{w}_{k,h}^{p,q}(n).$$
(5)

Note that $\{\theta_{k,\varepsilon}^{p,q}, \theta_{k,h}^{p,q}\}$ are mainly determined by users' mobile speeds. In (5), $w_{k,\varepsilon}^{p,q}(n) \sim \mathcal{N}(w_{k,\varepsilon}^{p,q}(n); 0, q_{k,\varepsilon}^{p,q})$ and $\mathbf{w}_{k,h}^{p,q}(n) \sim \mathcal{N}(w_{k,h}^{p,q}(n); 0, \mathbf{Q}_{k,h}^{p,q})$, where $\mathbf{\Theta}_{k,h}^{p,d} = \operatorname{diag}(\theta_{k,h,0}^{p,q}, \cdots, \theta_{k,h,N_f-1}^{p,q})$ and $\mathbf{Q}_{k,h}^{p,q} \triangleq \operatorname{diag}(q_{k,h,0}^{p,q}, \cdots, q_{k,h,N_f-1}^{p,q})$.

3. SUBOPTIMAL SCHMIDT KALMAN FILTER FOR CHANNEL PARAMETERS ESTIMATION

According to (4), we have the following system and observation equations as

$$\mathbf{r}^{q}(n) = \mathbf{\tilde{D}}_{\varepsilon,k}^{q}(n)\mathbf{h}_{k}^{q}(n) + \mathbf{\tilde{D}}_{\varepsilon,\tilde{k}}^{q}(n)\mathbf{h}_{\tilde{k}}^{q}(n) + \mathbf{v}^{q}(n),$$

$$\mathbf{h}^{q}(n) = \mathbf{\Theta}_{h}^{q} \mathbf{h}^{q}(n-1) + \mathbf{w}_{h}^{q}(n),$$

$$\varepsilon^{q}(n) = \mathbf{\Theta}_{\varepsilon}^{q} \varepsilon^{q}(n-1) + \mathbf{w}_{\varepsilon}^{q}(n),$$
(6)

where we have defined the following quantities.

$$\begin{split} \mathbf{h}^{q}(n) & \stackrel{\triangle}{=} & \left[\mathbf{h}_{1}^{q}(n)^{T}, \cdots, \mathbf{h}_{K}^{q}(n)^{T}\right]^{T} \in \mathbb{C}^{KN_{t}N_{f}}, \\ \varepsilon^{q}(n) & \stackrel{\triangle}{=} & \left[\varepsilon_{1}^{q}(n)^{T}, \cdots, \varepsilon_{K}^{q}(n)^{T}\right]^{T} \in \mathbb{R}^{KN_{t}}, \\ \tilde{\mathbf{D}}_{\bar{k}}^{q}(n) & \stackrel{\triangle}{=} & \left[\tilde{\mathbf{D}}_{l}(n), l = 1, \cdots, K, l \neq k\right] \in \mathbb{C}^{N \times (K-1)N_{t}N_{f}}, \\ \mathbf{\Theta}_{h}^{q} & = & \operatorname{diag}(\mathbf{\Theta}_{1,h}^{q}, \cdots, \mathbf{\Theta}_{K,h}^{q}), \\ \mathbf{\Theta}_{\varepsilon}^{q} & = & \operatorname{diag}(\mathbf{\Theta}_{1,\varepsilon}^{1,q}, \cdots, \mathbf{\Theta}_{K,\varepsilon}^{N_{t},q}), \\ \mathbf{\Theta}_{\varepsilon}^{q} & = & \operatorname{diag}(\mathbf{\Theta}_{1,\varepsilon}^{q}, \cdots, \mathbf{\Theta}_{K,\varepsilon}^{N_{t},q}), \\ \mathbf{\Theta}_{\varepsilon}^{q} & = & \operatorname{diag}(\mathbf{\Theta}_{1,\varepsilon}^{1,q}, \cdots, \mathbf{\Theta}_{K,\varepsilon}^{N_{t},q}), \\ \mathbf{\Theta}_{\varepsilon}^{q} & = & \operatorname{diag}(\mathbf{\Theta}_{1,\varepsilon}^{1,q}, \cdots, \mathbf{\Theta}_{K,\varepsilon}^{N_{t},q}), \\ \mathbf{\Theta}_{\varepsilon}^{q}(n) & \sim & \mathcal{N}(\mathbf{w}_{h}^{q}(n); \mathbf{0}, \mathbf{Q}_{h}^{q}), \\ \mathbf{w}_{\varepsilon}^{q}(n) & \sim & \mathcal{N}(\mathbf{w}_{\varepsilon}^{q}(n); \mathbf{0}, \mathbf{Q}_{\varepsilon}^{q}). \end{split}$$

Since the conventional Kalman filter requires prohibitively expensive computational complexity to obtain the Kalman gain, we proposed to use the suboptimal Schmidt Extended Kalman filter to break down the optimization problem above into K parallel and smaller optimization problems. This approach is promising since its computational complexity grows only linearly with the number of users.

3.1. Parallel Schmidt Kalman Filter (PSKF)

Following the notations in [7], we propose a parallel bank of Schmidt Kalman Filters (SKFs) whose size is K. In the k-th filter, we will divide the state vector into the essential state vector (ESV) related to the desired user k,

$$\mathbf{x}_{k}^{q}(n) \stackrel{\Delta}{=} \left[\varepsilon_{k}^{q}(n)^{T}, \mathbf{h}_{k}^{q}(n)^{T} \right]^{T}, \tag{7}$$

and the $nuisance\ {\rm state}\ {\rm vectors}\ ({\rm NSVs})$ related to the other users,

$$\mathbf{x}_{\tilde{k}}^{q}(n) \stackrel{\triangle}{=} [\varepsilon_{l}^{q}(n)^{T}, \mathbf{h}_{l}^{q}(n)^{T}, \forall l, l \neq k]^{T},$$
(8)

which is not coupled with ESV. Thus, we can obtain a new pair of linearized system and observation equations as

$$\begin{aligned} \tilde{\mathbf{r}}^{q}(n) &= \mathbf{J}_{k}^{q}(n)\mathbf{x}_{k}^{q}(n) + \mathbf{J}_{\tilde{k}}^{q}(n)\mathbf{x}_{\tilde{k}}^{q}(n) + \mathbf{v}^{q}(n), \\ \mathbf{x}_{k}^{q}(n) &= \operatorname{diag}(\mathbf{\Theta}_{k,\varepsilon}^{q}, \mathbf{\Theta}_{k,h}^{q})\mathbf{x}_{k}^{q}(n-1) + \begin{bmatrix} \mathbf{w}_{k,\varepsilon}^{q}(n-1) \\ \mathbf{w}_{k,h}^{q}(n-1) \end{bmatrix}, \end{aligned}$$

$$\mathbf{x}_{\tilde{k}}^{q}(n) = \operatorname{diag}(\boldsymbol{\Theta}_{\tilde{k},\varepsilon}^{q}, \boldsymbol{\Theta}_{\tilde{k},h}^{q}) \mathbf{x}_{\tilde{k}}^{q}(n-1) + \begin{bmatrix} \mathbf{w}_{\tilde{k},\varepsilon}^{q}(n-1) \\ \mathbf{w}_{\tilde{k},h}^{q}(n-1) \end{bmatrix},$$

where

$$\tilde{\mathbf{r}}^{q}(n) \stackrel{\triangle}{=} \delta \tilde{\mathbf{r}}^{q}(n) + \left[\mathbf{J}_{k}^{q}(n) \ \mathbf{J}_{\tilde{k}}^{q}(n) \right] \left[\begin{array}{c} \hat{\mathbf{x}}_{k}^{q}(n|n-1) \\ \hat{\mathbf{x}}_{\tilde{k}}^{q}(n|n-1) \end{array} \right],$$

$$\delta \tilde{\mathbf{r}}^{q}(n) \stackrel{\triangle}{=} \mathbf{r}^{q}(n) - \sum_{k=1}^{K} \tilde{\mathbf{D}}_{\hat{\varepsilon}(n|n-1),k}^{q}(n) \hat{\mathbf{h}}_{k}^{q}(n|n-1),$$

$$\mathbf{J}_k^q(n) \stackrel{\triangle}{=} \left[\mathbf{E} \tilde{\mathbf{D}}_{\hat{\varepsilon}(n|n-1),k}^q(n) \hat{\mathbf{h}}_k^q(n|n-1), \tilde{\mathbf{D}}_{\hat{\varepsilon}(n|n-1),k}^q(n) \right],$$

$$\mathbf{J}_{\tilde{k}}^{q}(n) \stackrel{\triangle}{=} \left[\mathbf{J}_{l}^{q}(n), \forall l, l \neq k\right], a \stackrel{\triangle}{=} (n-1)(N+N_{g}) + N_{g},$$

$$\mathbf{E} \stackrel{\triangle}{=} \operatorname{diag}\left(j2\pi a, \cdots, j2\pi(a+\frac{N-1}{N})\right). \tag{9}$$

Next, we decompose the covariance matrix $\mathbf{P}^{q}(n|n-1)$ and the Kalman gain matrix $\mathbf{K}^{q}(n)$ into the following form.

$$\mathbf{K}^{q}(n) = [\mathbf{K}_{k}^{q}(n)^{H} \mathbf{K}_{\bar{k}}^{q}(n)^{H}]^{H} = \begin{bmatrix} \mathbf{P}_{k,k}^{q}(n|n-1)\mathbf{J}_{k}(n)^{H} + \mathbf{P}_{k,\bar{k}}^{q}(n|n-1)\mathbf{J}_{\bar{k}}(n)^{H} \\ \mathbf{P}_{\bar{k},k}^{q}(n|n-1)\mathbf{J}_{k}(n)^{H} + \mathbf{P}_{\bar{k},\bar{k}}^{q}(n|n-1)\mathbf{J}_{\bar{k}}(n)^{H} \end{bmatrix} \mathcal{A},$$
$$\mathbf{P}^{q}(n|n-1) = \begin{bmatrix} \mathbf{P}_{k,k}^{q}(n|n-1) & \mathbf{P}_{k,\bar{k}}^{q}(n|n-1) \\ \mathbf{P}_{\bar{k},k}^{q}(n|n-1) & \mathbf{P}_{\bar{k},\bar{k}}^{q}(n|n-1) \\ \mathbf{P}_{\bar{k},\bar{k}}^{q}(n|n-1) & \mathbf{P}_{\bar{k},\bar{k}}^{q}(n|n-1) \end{bmatrix}, \quad (10)$$

where

$$\mathcal{A}^{-1} = \mathbf{J}_{k}(n)\mathbf{P}_{k,k}^{q}(n+1|n)\mathbf{J}_{k}(n)^{H} + 2N_{0}/T_{s}\mathbf{I} + 2\operatorname{Re}\left(\mathbf{J}_{k}(n)\mathbf{P}_{k,\tilde{k}}^{q}(n+1|n)\mathbf{J}_{\tilde{k}}(n)^{H}\right)\mathbf{J}_{\tilde{k}}(n)\mathbf{P}_{\tilde{k},\tilde{k}}^{q}(n+1|n)\mathbf{J}_{\tilde{k}}(n)^{H}$$
(11)

The SKF forces $\mathbf{K}_{\tilde{k}}^{q}(n)$ to be zero, such that $\tilde{\mathbf{K}}^{q}(n) = [\mathbf{K}_{k,\text{skf}}^{q}(n)^{T}, \mathbf{0}^{T}]^{T}$. Note that $\mathbf{K}_{k,\text{skf}}^{q}(n)$ is optimal under the constaint that the NSVs are not estimated. But it is inferior to the original KF which uses $\mathbf{K}^{q}(n)$. According to the SKF, the estimated error covariance matrix $\mathbf{P}^{q}(n|n)$ is computed from the following equation:

$$\mathbf{P}^{q}(n|n) = \begin{bmatrix} \mathbf{P}^{q}_{k,k}(n|n) & \mathbf{P}^{q}_{k,\tilde{k}}(n|n) \\ \mathbf{P}^{q}_{\tilde{k},k}(n|n) & \mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n|n) \end{bmatrix}, \\ \mathbf{P}^{q}_{k,k}(n|n) = \\ \mathcal{B}\mathbf{P}^{q}_{k,k}(n|n-1)\mathcal{B}^{H} - 2\operatorname{Re}\{\mathcal{B}\mathbf{P}^{q}_{k,\tilde{k}}(n|n)\mathcal{C}^{H}\} + \\ \mathcal{C}\mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n|n-1)\mathcal{C}^{H} + 2N_{0}\mathbf{K}^{q}_{k,\mathrm{skf}}(n)\mathbf{K}^{q}_{k,\mathrm{skf}}(n)^{H}/T_{s}, \\ \mathbf{P}^{q}_{k,\tilde{k}}(n|n) = \mathcal{B}\mathbf{P}^{q}_{k,\tilde{k}}(n|n-1) - \mathcal{C}\mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n|n-1), \\ \mathbf{P}^{q}_{\tilde{k},k}(n|n) = \mathbf{P}^{q}_{k,\tilde{k}}(n|n)^{H}, \\ \mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n|n) = \mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n|n-1), \\ \mathbf{P}^{\Delta}_{\tilde{k},\tilde{k}}(n|n) = \mathbf{P}^{q}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{\Delta}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{\Delta}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{\Delta}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{A}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{A}_{\tilde{k},\tilde{k}}(n) \\ \mathbf{P}^{A}_{\tilde{k},\tilde{k}}(n|n) \\ \mathbf{P}^{A}_{\tilde{k},\tilde{k}}(n|n)$$

Note that the estimated error covariance matrix for NSVs is not updated. The estimation for the ESV is performed as follows.

$$\mathbf{K}_{k,\mathrm{skf}}^{q}(n) = \left(\mathbf{P}_{k,k}^{q}(n|n-1)\mathbf{J}_{k}(n)^{H} + \mathbf{P}_{k,\tilde{k}}^{q}(n|n-1)\mathbf{J}_{\tilde{k}}(n)^{H}\right)\mathcal{A}, \\
\hat{\mathbf{x}}_{k}^{q}(n|n) = \hat{\mathbf{x}}_{k}^{q}(n|n-1) + \mathbf{K}_{k,\mathrm{skf}}^{q}(n)\delta\mathbf{r}^{q}(n), \\
\delta\mathbf{r}^{q}(n) = (\mathbf{r}^{q}(n) - \hat{\mathbf{D}}_{\hat{\varepsilon}(n|n-1),k}^{q}(n)\hat{\mathbf{h}}_{k}^{q}(n|n-1)).$$
(13)

From these derivations, the proposed PSKF has K separated SKFs. Each SKF separately computes and updates $\mathbf{P}_{k,k}^q(n|n)$ and $\{\hat{\varepsilon}_k^q(n|n), \hat{\mathbf{h}}_k^q(n|n)\}$ as

$$\mathbf{P}_{k,k}^{q}(n+1|n) = \mathbf{\Theta}_{k,h}^{q} \mathbf{P}_{k,k}^{q}(n|n) (\mathbf{\Theta}_{k,h}^{q})^{T} + \mathbf{Q}_{k,h}^{q}, \\
\mathbf{x}_{k}^{q}(n+1|n) = \operatorname{diag}(\mathbf{\Theta}_{k,\varepsilon}^{q}, \mathbf{\Theta}_{k,h}^{q}) \mathbf{x}_{k}^{q}(n|n).$$
(14)

3.2. Complexity Analysis of the PSKF

To compute the Kalman gain defined (10), we require $O(N^3)$ multiplications and additions for \mathcal{A}^{-1} , $K \times O(N(K-1)(N_t(N_f+1))^2N)$ multiplications and additions for all $K_k^q(n)$, and $K \times O(N(K-1)^2(N_t(N_f+1))^2N)$ multiplications and additions for all $K_k^q(n)$. From these figures, we can easily find that, as the number of users increases, the proposed PSKF scheme has more advantages over the conventional Kalman filtering technique. Furthermore, it is worthwhile to note that the interference is not required to be modeled as simple AWGN in the proposed scheme.

4. SIMULATION RESULTS

In this section, we verify the proposed scheme through simulation. We employ the following system parameters:

- $N_t = N_r = 4$, K = 5, N = 64, and M = 10.
- $L_f = 2$, $||\mathbf{h}_k^{p,q}(n)||^2 = \{0.7740, 0.6332\}, \forall p, q, k.$
- AR coefficients: $\theta_{k,\varepsilon}^{p,q} = \theta_{k,h}^{p,q} = 0.999, \forall k, p, q.$

Figures 1-2 show the performance of the proposed joint channel and frequency offset scheme for a QPSK MIMO-OFDMA system over a moderate fading channel whose normalized Doppler frequency is $f_dT_d = 1e^{-2}$. A packet consists of ten OFDM symbols. In the simulations, we assume $\{\mathbf{D}_k^p(n)\}$ are perfectly known to the BS and zero initial conditions for $\hat{\varepsilon}_k^{p,q}(0)$ and $\hat{\mathbf{h}}_k^{p,q}(0)$.



Fig. 1. Performances of the joint estimation with equal user powers.

If the received signals from all users have an equal power, Figure 1 reveals that better performance can be achieved with more symbols (indicated by n) and higher SNR. Furthermore, Figure 2 shows the impact of the near-far effect on the performance of the proposed scheme.

In Figure 2, the received signal from the first user, k = 1, is $10 \log(K)$ [dB] stronger than that of other users. While the stronger user reaps better performance in terms of mean squared errors (MSE) of channel estimation, almost identical convergence behavior and performance in frequency offset estimation have been observed in Figure 2, irrespective of users' signal powers.

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