CHANNEL TRACKING USING PRUNING FOR MIMO-OFDM SYSTEMS OVER GAUSS-MARKOV CHANNELS

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ABSTRACT

In this paper we investigate the problem of channel tracking and detection for MIMO-OFDM systems over fast varying channels obeying a Gauss-Markov model. We consider time domain tracking of the channel matrix taps with Kalman filter, whereas symbols detection is carried out by a zero-forcing (ZF) soft detector. A key assumption of the theory of Kalman filter is that the state-space model is perfectly known, while communication systems make use of the detected symbols as an input to the Kalman filter in order to form a suitable state-space model. This gives rise to error propagation due to misdetected symbols (model mismatch) and is usually solved by using frequently inserted pilot symbols, resulting in a reduced spectral efficiency. To overcome this problem, we suggest a novel approach to mitigate the error propagation due to misdetections without using frequent pilot symbols. In particular, we consider the reliability of the detections based on the soft detector and use only those outputs that have robust reliability to track the channel matrix taps, minimizing the effect of Kalman filter mismodeling. This method can significantly reduce the error propagation effect, leading to an improved bit error probability.

Index Terms— MIMO systems, Kalman filtering

1. INTRODUCTION

Modern wireless communication systems require high data rate transmission over wireless channels. Information theory indicates that a multi-input/multi-output (MIMO) system can provide enormous capacity improvement, relative to a single-input/single-output system [1]. OFDM has gained much attention over the last few years due to its ability to transform the frequency selective channel into many low-rate parallel streams, thereby increasing the symbol duration, canceling the inter block interference (IBI) and leading to simple equalization. Recent studies have shown that for frequency-selective channels, combining OFDM with MIMO system can provide a high rate and reliable transmission. However, this requires an accurate estimate of the channel state information at the receiver. The conventional decision-directed time-domain channel estimation using Kalman filter is suitable for relative small values of Doppler offset, where the channel fluctuates very slowly. However, at high velocities, decision-directed method would be prone to many errors, due to the rapid nature of the channel variations. These errors would propagate into the Kalman filter and may diverge it. In this paper we suggest a new receiver to overcome this inherent problem. Our approach is based on the fact that the number of observations given at each OFDM symbol is much larger than the number of channel taps. Therefore, it is possible to prune observations whose detection is unreliable, reducing the risk of errors propagating into the Kalman filter. We show that the proposed scheme can significantly reduce the error propagation effect, leading to a reduced BER that approaches the performance of a system with no error propagation.

2. MIMO-OFDM SYSTEM MODEL

We consider a MIMO-OFDM system shown in Figure 1. In order to keep the presentation simple we consider a 2-transmit/ 2-receive antenna configuration and *N* OFDM subcarriers. The generalization to N_t-transmit/ N_r-receive antennas is straightforward. For the n^{th} block, *2N* baseband symbols are multiplexed into two parallel streams, $s_i(n)$, $i = 1, 2$, for two transmit antennas, each stream containing *N* symbols, to form an OFDM block. Each OFDM block of complex-valued symbols from an M-ary modulation alphabet set $A = \{a_1, .., a_{|A|}\}$ is transformed using an inverse fast Fourier transform (IFFT) and transmitted by an antenna. We assume that the use of a cyclic prefix (CP) both preserves the orthogonality of the tones and eliminates inter block interference (IBI) between consecutive OFDM blocks. The length of the CP is assumed to be larger than that of the channel's impulse response. The received $2N \times 1$ vector, after cyclic prefix removal and FFT (fast Fourier transform) operation can be expressed as

The work was supported by ARC Discovery Project.

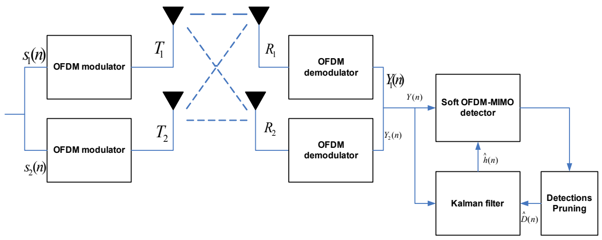

Fig. 1. MIMO-OFDM transceiver

$$\underbrace{\begin{bmatrix} \mathbf{Y_1(n)} \\ \mathbf{Y_2(n)} \end{bmatrix}}_{\mathbf{Y_n}} = \underbrace{\begin{bmatrix} \mathbf{D_1(n)} & \mathbf{D_2(n)} & \underline{\mathbf{0}} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{0}} & \mathbf{D_1(n)} & \mathbf{D_2(n)} \end{bmatrix}}_{\mathbf{D_n}} *$$

$$\underbrace{\begin{bmatrix} \mathbf{W_L} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \mathbf{W_L} & \underline{\mathbf{0}} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{0}} & \mathbf{W_L} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \mathbf{W_L} \end{bmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} \mathbf{h_{11}(n)} \\ \mathbf{h_{12}(n)} \\ \mathbf{h_{21}(n)} \\ \mathbf{h_{22}(n)} \end{bmatrix}}_{\mathbf{h_n}}$$

$$+ \underbrace{\begin{bmatrix} \mathbf{z_1(n)} \\ \mathbf{z_2(n)} \end{bmatrix}}_{\mathbf{Z_n}} \tag{1}$$

where $\mathbf{Y_j(n)}$, $j = 1, 2$ is the $N \times 1$ observation vector at the j^{th} antenna, $\mathbf{D_j(n)}$, $j = 1, 2$ is an $N \times N$ diagonal matrix, with its main diagonal containing N symbols of the j^{th} transmit antenna, $\underline{\mathbf{0}}$ refers to a zero matrix with the appropriate size, $\mathbf{h_{ij}(n)}$, $i, j = 1, 2$ is the channel response vector between the j^{th} transmit and i^{th} receive antenna, and $\mathbf{z_j(n)}$, $j = 1, 2$ is an $N \times 1$ vector of independent identically distributed (i.i.d.) complex zero-mean Gaussian noise with covariance matrix $\mathbf{R} = \mathbf{I}\sigma^{\mathbf{2}}$ and assumed to be uncorrelated with the channel matrix. $\mathbf{W_L}$, is the $N \times L$ partial DFT matrix defined as $W_L \equiv \left\{e^{-j2\pi nk/N}\right\}_{n=0,\ldots,N-1;k=0,\ldots,L-1}$. In a compact matrix form the system model can be written as

$$\mathbf{Y_n} = \mathbf{D_n W h_n} + \mathbf{Z_n} \tag{2}$$

We consider a fading multipath channel according to Jake's model [2], consisting of L impulses and can be written as $h(t, \tau) = \sum_{l=0}^{L-1} \gamma_l(t)\, \delta(\tau - \tau_l)$ where τ_l is the delay of the l^{th} path, $\gamma_l(t)$ is the corresponding complex amplitude. $\gamma_l(t)$'s are wide-sense stationary complex Gaussian processes, independent of each other. This channel can be approximated by an auto regressive (AR) process, and we chose to work with the AR model of the first order, which is a reasonable approximation of the Jake's model parameters [3],

$$\mathbf{h_n} = \mathbf{A h_{n-1}} + \mathbf{v_n} \tag{3}$$

where $\mathbf{v_n}$ is the innovation process with a covariance matrix $\mathbf{Q}$. The unknown parameters $\mathbf{A}$ and $\mathbf{Q}$ are $4L \times 4L$ matrices and can be calculated using Yule-Walker equations [4].

3. RECEIVER STRUCTURE

In this section we present the decision-directed channel tracking and detection for the MIMO-OFDM system. Since the complexity of a maximum-likelihood (ML) detector is exponential, we instead use a zero-forcing (ZF) receiver that has a low complexity. We suggest a receiver consisting of two submodules: Soft detector and Kalman filter channel tracker using detections pruning. The receiver is shown in Figure 1. It consists of the following components:

3.1. Decision-Directed Zero-Forcing Soft Detector

The received signal $\mathbf{Y_n}$ and the estimated channels impulse response of the previous block $\widehat{\mathbf{h}}_{\mathbf{n-1}}$ are fed to the soft detector which yields the log likelihood ratio (LLR) of each of the bits of the n^{th} block. For clarity purposes, we will drop the time index n. Since the noise is independent between different subcarriers, the soft detector is implemented in the same manner for all N subcarriers. The MIMO-OFDM equation for the k^{th} subcarrier can be written as

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{Y_k}} = \underbrace{\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}}_{\mathbf{H_k}} \underbrace{\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}}_{\mathbf{d_k}} + \underbrace{\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}}_{\mathbf{w_k}}, \tag{4}$$

where y_j is the observation of the j^{th} receive antenna, $j = 1, 2$, H_{ij} is the channel frequency response between the j^{th} transmit antenna and the i^{th} receive antenna of the previous OFDM symbol, $j = 1, 2$, given by $H_{ij} = W_L^k \left(\widehat{h}_{ij}\right)_{n-1}$, where W_L^k is the k^{th} row of the partial DFT matrix W_L. d_i, $i = 1, 2$ are the data symbols of the i^{th} transmit antenna and w_j, $j = 1, 2$ is the additive complex circular Gaussian noise. The ZF receiver filter is given by

$$\mathbf{G_k} = \left(\mathbf{H_k^H H_k}\right)^{-\mathbf{1}} \mathbf{H_k^H} \tag{5}$$

and results in

$$\underbrace{\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}}_{\mathbf{R_k}} = \mathbf{G_k Y_k} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}}_{\mathbf{d_k}} + \underbrace{\left(\mathbf{H_k^H H_k}\right)^{-1} \mathbf{H_k^H w_k}}_{\overline{\mathbf{w}}_{\mathbf{k}}}, \tag{6}$$

where the transformed noise vector $\overline{\mathbf{w}}_{\mathbf{k}}$ has covariance matrix $\mathbf{R}_{\overline{\mathbf{w}}_{\mathbf{k}}} = \mathbf{E}\left(\overline{\mathbf{w}}_{\mathbf{k}} \overline{\mathbf{w}}_{\mathbf{k}}^{\mathbf{H}}\right) = \sigma^{\mathbf{2}} \left(\mathbf{H_k^H H_k}\right)^{-\mathbf{1}}$.
The LLRs of the soft detector are defined as

$$\Lambda_k^{(i,j)} = \ln \frac{f(b^{i,j} = 1 | \mathbf{R_k}, \overline{\mathbf{w}}_{\mathbf{k}})}{f(b^{i,j} = 0 | \mathbf{R_k}, \overline{\mathbf{w}}_{\mathbf{k}})} = \ln \frac{\sum_{b \in B_i^1} f(b | \mathbf{R_k}, \overline{\mathbf{w}}_{\mathbf{k}})}{\sum_{b \in B_i^0} f(b | \mathbf{R_k}, \overline{\mathbf{w}}_{\mathbf{k}})} \tag{7}$$

where $\Lambda_k^{(i,j)}$ is the LLR value of the i^{th} bit, $i = 1, ..., \log_2 |A|$ of the signal from the j^{th} layer (antenna), $j = 1, 2$, $b^{i,j}$ is the i^{th} bit of the j^{th} layer, $b \in B_i^v$ denotes the set of all symbols whose i^{th} bit is equal to v, where $v = 0, 1$. In order to compute the LLRs of the detector output, we use the approximation proposed in [5]

$$\Lambda_k^{(i,j)} \approx \frac{1}{\sigma_{\overline{w}_j}^2}\left[\min_{b\in B_i^0}|r_j-b|^2-\min_{b\in B_i^1}|r_j-b|^2\right] \quad (8)$$

where $\sigma_{\overline{w}_j}^2 \equiv (\mathbf{R}_{\overline{\mathbf{w}}_\mathbf{k}})_{j,j}$ denotes the noise variance of the transformed noise. The estimated symbols $\widehat{\mathbf{d}}_\mathbf{k} = \left[\widehat{\mathbf{d}}_\mathbf{1}\ \widehat{\mathbf{d}}_\mathbf{2}\right]^\mathbf{T}$ are produced by taking the hard decisions of the LLRs and mapping to the appropriate symbol point.

3.2. Channel Tracking

Consider the AR model for the channel dynamics (3), and the MIMO-OFDM equation (2). A state-space model of the system can be written down as

$$\mathbf{h_n} = \mathbf{A}\mathbf{h_{n-1}} + \mathbf{v_n} \quad (9)$$

$$\mathbf{Y_n} = \mathbf{D_n}\mathbf{W}\mathbf{h_n} + \mathbf{Z_n} \quad (10)$$

In order to track the channel we need to regenerate the symbols matrix $\mathbf{D_n}$, that was sent over the channel. However, $\mathbf{D_n}$ is not given at the receiver and therefor, the Kalman filter can not be used directly. Instead, as a suboptimal solution, we can use $\widehat{\mathbf{D}}_\mathbf{n}$, the estimated matrix of $\mathbf{D_n}$, for channel tracking. The estimated matrix $\widehat{\mathbf{D}}_\mathbf{n}$ is obtained by taking the hard decisions of the soft detector. Now, the Kalman filter can be applied to track the channel impulse response $\mathbf{h_n}$. The Kalman filter equations are [6]

$$\hat{\mathbf{h}}_\mathbf{n}^- = \mathbf{A}\hat{\mathbf{h}}_\mathbf{n-1} \quad (11)$$

$$\mathbf{P}_\mathbf{n}^- = \mathbf{A}\mathbf{P}_\mathbf{n-1}\mathbf{A}^\mathbf{H} + \mathbf{Q} \quad (12)$$

$$\mathbf{K_n} = \mathbf{P}_\mathbf{n}^-\left(\hat{\mathbf{D}}_\mathbf{n}\mathbf{W_L}\right)^\mathbf{H}\left(\hat{\mathbf{D}}_\mathbf{n}\mathbf{W_L}\mathbf{P}_\mathbf{n}^-\mathbf{W}_\mathbf{L}^\mathbf{H}\hat{\mathbf{D}}_\mathbf{n}^\mathbf{H} + \mathbf{R}\right)^{\mathbf{-1}} \quad (13)$$

$$\hat{\mathbf{h}}_\mathbf{n} = \hat{\mathbf{h}}_\mathbf{n}^- + \mathbf{K_n}\left(\mathbf{Y_n} - \hat{\mathbf{D}}_\mathbf{n}\mathbf{W_L}\hat{\mathbf{h}}_\mathbf{n}^-\right) \quad (14)$$

$$\mathbf{P_n} = \mathbf{P}_\mathbf{n}^- - \mathbf{K_n}\hat{\mathbf{D}}_\mathbf{n}\mathbf{W_L}\mathbf{P}_\mathbf{n}^- \quad (15)$$

3.3. Channel Tracking using Detections Pruning

As long as $\mathbf{D_n} = \widehat{\mathbf{D}}_\mathbf{n}$ (meaning that there are no misdetections), the Kalman filter operates in its nominal values, that is to say, the state-space model is accurate and no mismodeling occurs. It achieves the minimum variance estimation error and is optimal in the MSE sense. However, if model inaccuracies due to misdetections occur ($\mathbf{D_n} \neq \widehat{\mathbf{D}}_\mathbf{n}$), there is a false impression that the filter is performing well, tracking the channel response, while in fact it is diverging from the true state. Moreover, one can not detect that the filter is diverging, tracking false trajectories. Incorrect detections may cause the Kalman filter to choose a wrong trajectory which in turn would cause more symbols to be misdetected and eventually fail the system completely. In order to overcome this problem we observe that the number of parameters to be estimated is *4L*, while the number of detections/observations is *2N*, and in a typical system *2N >> 4L*. This means that not all detections are needed in order to get an estimate of $\mathbf{h_n}$. Instead of using the whole block of data (*2N* observations), one can choose a subset, whose reliability in terms of detection confidence is high and use only those to track the channel impulse response. Conceptually, it would be like assigning weights to each subcarrier, and the weight factor can have a binary value $\{0, 1\}$, so that only subcarriers with a weight of 1 will be taken into account and the rest will be pruned. In this case the state-space would have the form of

$$\mathbf{h_n} = \mathbf{A}\mathbf{h_{n-1}} + \mathbf{v_n} \quad (16)$$

$$\mathbf{\Psi_n}\mathbf{Y_n} = \mathbf{\Psi_n}(\hat{\mathbf{D}}_\mathbf{n}\mathbf{W}\mathbf{h_n} + \mathbf{Z_n}) \quad (17)$$

where $\mathbf{\Psi_n}$ is a $2N \times 2N$ diagonal matrix and its diagonal entries contain one of two possible values $\{0, 1\}$. We define the following criterion for the reliability of the detection of the symbols of the k^{th} subcarrier:

$$\Lambda_k \equiv \begin{cases} 1, & \min\left\{\ \left|\Lambda_k^{(i,j)}\right|\right\} \geq\ T\ ;\left\{\begin{matrix}1\leq i\leq 2\log_2|A| \\ j=1,2\end{matrix}\right\} \\ 0, & else \end{cases} \quad (18)$$

where T is a predefined threshold value. This method assurers that if all $2\log_2|A|$ bits of the k^{th} subcarrier of both layers are reliable, we will use them for the purpose of channel tracking. The matrix $\mathbf{\Psi_n}$ can now be formed by assigning the values of Λ^k to the its main diagonal.

$$\mathbf{\Psi_n}(\mathbf{k},\mathbf{k}) \equiv \begin{cases} \Lambda_k, & 1 \leq k \leq N \\ \Lambda_k, & N+1 \leq k \leq 2N \end{cases} \quad (19)$$

In every block of data, a different set and number of detections may be used to track the channel $\mathbf{h_n}$. It is important to point out that while only a subset of the whole measurements are used for channel tracking, all $2N\log_2|A|$ bits are detected, since the output of the system at each block is $2N\log_2|A|$ detected bits. Finding the optimal threshold is a very complex task, which involves the analysis of the error propagation of misdetections of the Kalman filter. In general, there is no closed form for finding the optimal threshold and we need to resort to simulations.

4. SIMULATIONS

In this section we present the simulation results. The MIMO-OFDM setup is the following: The carrier frequency is $f_c = 4$ GHz, the number of subcarriers, *N*, is set to be 64. The available bandwidth is 1MHz and the modulation scheme is QPSK. The channel response $\mathbf{h_n}(\mathbf{l})$ is assumed to be a multipath with independent exponential decay power profiles, where

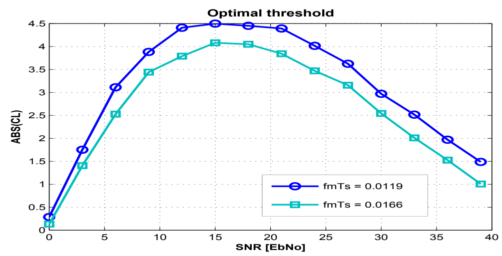

Fig. 2. Optimal thresholds

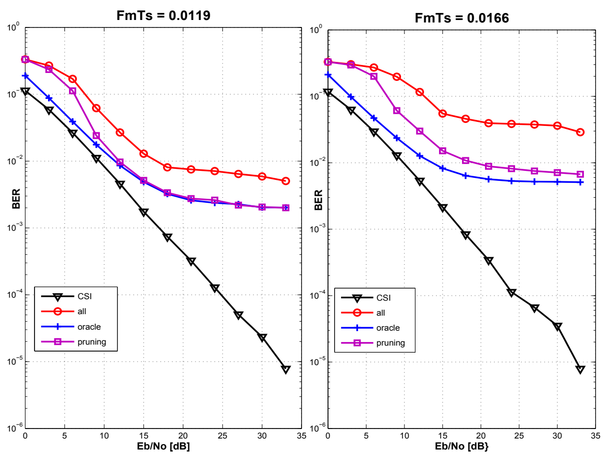

Fig. 3. Bit error rate for $F_m T_s = \{0.0119, 0.0166\}$

the length of the channel L is set to 4. We consider two normalized Doppler offsets: 0.0119 and 0.0166, corresponding to two terminal velocities of 50 km/hr and 70 km/hr, respectively. In order to find the optimal value of the threshold defined in (18), Monte Carlo simulations for different SNRs and threshold values were conducted. The chosen threshold for each SNR was the one that minimized the bit error rate (BER). The results for the optimal thresholds are depicted in Figure 2. The BER and the channel estimation mean square error (MSE) results are depicted in Figure 3 and Figure 4, respectively. Four methods are compared. The first one is the "all", in which all *2N* detections are taken into account. The second one is our proposed "pruning", using the optimal threshold. The third one is the "oracle" method, in which at each block, only correct detections are taken into account, thus, eliminating the error propagation effect. This is the lower bound for our algorithm. The forth method is the "CSI", where we assume that the channel impulse responses are ideally known at the receiver. "CSI" method serves as the lower bound for the BER of the system. It is clear that our algorithm is substantially better than the "all" method, in terms of both BER and MSE. Our algorithm almost achieves the "oracle" lower bound, while the "all" is unable to track the correct trajectories of the channel, as shown in the MSE in Figure 4.

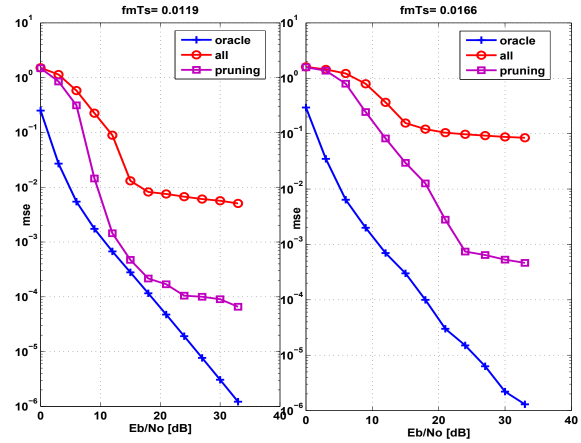

Fig. 4. Channel estimation error for $F_m T_s = \{0.0119, 0.0166\}$

5. CONCLUSIONS

In this paper we present a novel zero-forcing symbols detection and decision-directed channel tracking using Kalman filter for MIMO-OFDM systems. We address the problem of mismodeling of the state-space equations due to misdetected symbols, which causes the Kalman filter to diverge. We show that in order to perform channel tracking, not all observations are required, but instead, it is possible to take into account only those with high reliability. The simulation results show a considerable improvement over a system without pruning in terms of both channel estimation error and bit error rate.

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