A SIMPLE ALGORITHM FOR NEIGHBOR DISCOVERY IN WIRELESS NETWORKS

Daniele Angelosante
DAEIMI
Università di Cassino, Italy
email: d.angelosante@unicas.it

Ezio Biglieri
Departament de Tecnologia
Universitat Pompeu Fabra, Barcelona, Spain
email: ezio.biglieri@upf.edu

Marco Lops
DAEIMI
Università di Cassino, Italy
email: lops@unicas.it

ABSTRACT

We consider the neighbor-discovery problem in a fixed wireless network where each node is identified by its signature, and the signatures are chosen so as to limit collisions. Limited-complexity constraints lead to a simple algorithm based on incoherent detection of nodes. The performance of this algorithm is evaluated by computing the probability of a false alarm and of a missed detection of a single node.

Index Terms— Wireless networks, neighbor discovery.

1. INTRODUCTION

In a wireless network, neighbor discovery (ND), i.e., the detection of all neighbors with which a given reference node may communicate directly, is a crucial task, especially when the nodes are mobile, and hence the network configuration is time-varying. As noted in [1], ND may be the first algorithm run in a network, and the basis of medium access, clustering, and routing algorithms. In this paper we study a simple ND algorithm, intended to identify neighbor identities under a number of simplifying assumptions on the network behavior. Many of these assumptions can be removed, as we plan to do in further studies: for example, we may consider a more general setting in which a dynamic model is known for the variation of the node parameters (e.g., their power), and their estimate is also of interest. If the searching node is equipped with a sectorized (or steerable) receive antenna, and each node transmits omnidirectionally, neighbor discovery can be performed for each sector, thus obtaining a map of the neighbor directions.

Following in the footsteps of [1], the basic assumptions we make in our study are as follows:

1. A node is called a *neighbor* of the reference node if its power, received by the latter, exceeds a preassigned threshold.[1]
2. Nodes cannot transmit and receive simultaneously on the same channel.
3. The maximum number of active nodes is fixed and finite.
4. Each node is identified by its own unique signature, and every node keeps a list of all the signatures of the network.
5. The discovery algorithm runs in a finite period, called a *discovery session*, whose duration is denoted T_{D}. During T_{D}, every node also transmits, independently and randomly with probability ε, a number of signals containing one or more copies of its signature. Each signal has the same duration ("slot") $T = T_{\mathrm{D}}/N$.
6. During each session, the parameters of each node do not change appreciably.

Among the alternatives that should be considered in setting up a ND algorithm, we may list the following:

Deterministic vs. random networks. A deterministic network has a constant structure, and reorganizations are not necessary. A typical example [2] is a group of laptop-computer nodes in a conference room wishing to organize themselves into a wireless network without resorting to a centralized structure. A random network has a varying topography/topology, due to to entrance/exit of nodes or to their movement.

Synchronous vs. asynchronous detection. Synchronous detection can take place when all nodes transmit under a common reference frame, which may be allowed by the presence of a local clock that keeps synchronous time [2, 3]. This assumption may be realistic in the context of a small network, whose nodes are so close to each other that the time misalignments due to different arrival times can be disregarded. In asynchronous detection, the nodes maintain no cooperation among them, and hence their transmission slots are randomly misaligned, although they keep one and the same duration.

Coherent vs. noncoherent detection. Coherent detection may occur if the receiver is allowed to estimate, within a reasonable accuracy, the phase of the carrier of each signal. Otherwise, noncoherent detection should be selected.

[1] Note that this definition can be generalized: for example, one may define a neighbor as one whose power-to-interference plus noise ratio exceeds a given threshold [1]. In this paper we stick to a more restrictive definition, which allows simpler algorithms.

Collision vs. collision-free networks. Collisions may be fully resolved at *modulation level* [4] by transmitting orthogonal signatures. If this is not possible because of bandwidth restrictions, a moderate amount of collisions is kept under control by using signatures with small correlation.

The balance of this paper is organized as follows. We list our basic assumptions, and describe the ND algorithm, in Section 2. Section 3 provides performance analysis, while Section 4 shows some numerical results.

2. FORMULATION OF THE PROBLEM; AN ND ALGORITHM

Our approach owes to the algorithms advocated in [1]. We consider a static,[2] deterministic, and synchronous network with noncoherent detection. We denote by $K+1$ the number of nodes, so that K is the maximum number of active neighbors of any node. All nodes are identified by their (nonorthogonal) signatures. Our ND algorithm is based on the transmission scheme illustrated in Fig. 1 (see also [1]). In every time interval ("slot"), each node transmits, independently of the other nodes, with the same probability ε. We also assume that the reference node receives with probability $1-\varepsilon$. Discovery

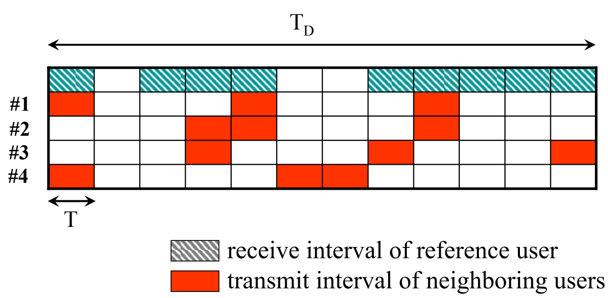

Fig. 1. *A scheme for synchronous neighbor detection.*

of neighbor k, $k=1,\dots,K$, is performed by the reference node by computing the correlation between the received signal and the signature of node k, and accumulating these correlations over the duration of the discovery session. Consider for example reference node 0, and discovery of node 1. The chip-wise signal collected from all potential neighbors during receiving slot t is

$$\mathbf{y}_t = \sum_{k=1}^{K} \xi_{k,t}\alpha_k \mathbf{s}_k + \mathbf{n}_t$$

[2]Consideration of a dynamic network, in which nodes may log in and out of the network, and change their parameters from session to session according to a known model, can be done with the aid of random-set theory (see [5] and references therein), and will be described elsewhere, along with an extension of the algorithm presented here intended to estimate the parameters (e.g., the transmitted power) of the neighboring nodes.

where α_k denote the received complex amplitudes,[3] $\mathbf{s}_k$ is the kth node signature, $\xi_{k,t}$ is a random variable taking value 1 if node k is transmitting at time t, and value 0 otherwise (so that $\mathbb{P}(\xi_{k,t}=1)=\varepsilon$), and $\mathbf{n}_t$ is additive white Gaussian noise. Correlation of $\mathbf{y}_t$ with $\mathbf{s}_1$ at receiving time t yields

$$r_{1,t} \triangleq (\mathbf{y}_t, \mathbf{s}_1) = \xi_{1,t}\alpha_1 + \sum_{k=2}^{K} \xi_{k,t}\alpha_k(\mathbf{s}_k, \mathbf{s}_1) + (\mathbf{n}_t, \mathbf{s}_1)$$

At the end of the discovery session, the resulting signal, obtained by summing up the contributions from each slot, is given by

$$r \triangleq \sum_{t\in\mathcal{R}_0} r_{1,t} = \nu_1\alpha_1 + \sum_{k=2}^{K} \nu_k\alpha_k\rho_{k,1} + n \qquad (1)$$

where $\mathcal{R}_0$ is the set of times at which the reference node is receiving, $\rho_{k,1} \triangleq (\mathbf{s}_k, \mathbf{s}_1)$, and

$$\nu_k \triangleq \sum_{t\in\mathcal{R}_0} \xi_{k,t} \qquad n \triangleq \sum_{t\in\mathcal{R}_0} (\mathbf{n}_t, \mathbf{s}_1)$$

(in words, ν_k is the number of slots in which node k is transmitting while node 0 is receiving). Node k is classified as a neighbor if its magnitude $|\alpha_k|$ exceeds the "activity threshold" τ_A. The receiver compares $|r|$ against the "discovery threshold" τ_D, and if this threshold is exceeded it includes node 1 in its neighbor list.[4] A "miss" occurs when a node exceeding the activity threshold is not detected, and a "false alarm" occurs when a node below the activity threshold is classified as a neighbor. The choice of the discovery threshold value is clearly a crucial design parameter, as its high value will cause a high probability P_M of a miss, while a low value will cause a high probability P_F of a false alarm. Plotting P_F versus P_M yields an indication of the performance of a discovery algorithm (this should be contrasted with the cost function of [1, Eq. (1)]).

3. PERFORMANCE ANALYSIS

Define Ψ to be a random variable describing the configuration of a discovery session, i.e., the pattern of receive/transmit activity of each node during each slot. In our analysis, we assume that the "interference" term in (1) is conditionally Gaussian given Ψ; specifically, the "fading gains" α_k are complex, circularly Gaussian with mean zero and common variance σ_k^2 for their real and imaginary parts. In addition, the correlations ρ_k are deterministic. The noise is also circularly Gaussian, with variance N_0 of its real and imaginary part.

[3]Our assumption of complex amplitudes implies that the signatures are not detected coherently. In addition, we assume $(\mathbf{s}_k, \mathbf{s}_k) = 1$ for all nodes.

[4]This receiver is suboptimum, since the interference term (the second one in the right-hand side of (1)) is not necessarily Gaussian, nor are the signatures orthogonal.

Formally, we have, for the discovery of node 1,

$$\begin{aligned} P_{\mathrm{M},1} &= \mathbb{E}_\Psi\left[\mathbb{P}(|r| < \tau_\mathrm{D} \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi)\right] \\ P_{\mathrm{F},1} &= \mathbb{E}_\Psi\left[\mathbb{P}(|r| > \tau_\mathrm{D} \,\big|\, |\alpha_1| < \tau_\mathrm{A}, \Psi)\right] \end{aligned} \tag{2}$$

To evaluate these probabilities, we first need the conditional distribution of $|r|$ given $|\alpha_1|$ and Ψ. Given the session configuration Ψ, $\{\nu_k\}_{k=1}^K$ are known, and hence $|r|$ is conditionally Rician:

$$\begin{aligned} &f\left(|r| \,\big|\, |\alpha_1|, \Psi\right) \\ &= \frac{2|r|}{\Sigma^2} \exp\left\{-\frac{|r|^2 + \nu_1^2|\alpha_1|^2}{\Sigma^2}\right\} I_0\left(\frac{2|r|\nu_1|\alpha_1|}{\Sigma^2}\right) \end{aligned} \tag{3}$$

where

$$\frac{\Sigma^2}{2} \triangleq \sum_{k=2}^{K} \nu_k^2 \rho_k^2 \sigma_k^2 + \bar{\nu}_0 N_0$$

$\bar{\nu}_0 \triangleq |\mathcal{R}_0|$ is the number of instants in which the reference node senses the channel, and $I_0(\cdot)$ is the modified Bessel function of first kind and order 0. We compute $P_{M,1}$ as the expectation over Ψ of

$$\begin{aligned} &\mathbb{P}\left(|r| < \tau_\mathrm{D} \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi\right) \\ &= 1 - \int_{\tau_\mathrm{D}}^{\infty} f\left(|r| \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi\right) \, d|r| \end{aligned} \tag{4}$$

The integrand of the above can be evaluated as

$$f\left(|r| \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi\right) = \frac{\mathbb{P}\left(|\alpha_1| > \tau_\mathrm{A} \,\big|\, |r|, \Psi\right) f(|r| \big| \Psi)}{\mathbb{P}(|\alpha_1| > \tau_\mathrm{A})}$$

To compute

$$\mathbb{P}\left(|\alpha_1| > \tau_\mathrm{A} \,\big|\, |r|, \Psi\right) = \int_{\tau_\mathrm{A}}^{\infty} f\left(|\alpha_1| \,\big|\, |r|, \Psi\right) \, d|\alpha_1|$$

we need

$$f\left(|\alpha_1| \,\big|\, |r|, \Psi\right) = \frac{f\left(|r| \,\big|\, |\alpha_1|, \Psi\right) f(|\alpha_1|)}{f\left(|r| \,\big|\, \Psi\right)} \tag{5}$$

Now,

① $f\left(|r| \,\big|\, |\alpha_1|, \Psi\right)$ is Rice (see (3)).

② $f(|\alpha_1|)$ is Rayleigh: $f(|\alpha_1|) = \frac{|\alpha_1|}{\sigma_1^2} e^{-|\alpha_1|^2/2\sigma_1^2}$

③ $f\left(|r| \,\big|\, \Psi\right)$ is also Rayleigh: $f\left(|r| \,\big|\, \Psi\right) = \frac{|r|}{\sigma^2} e^{-|r|^2/2\sigma^2}$ where $\sigma^2 \triangleq \nu_1^2\sigma_1^2 + \Sigma^2/2$.

Combining ①, ②, and ③ we obtain

$$\begin{aligned} f(|\alpha_1| \big| |r|, \Psi) &= \frac{f(|r| \big| |\alpha_1|, \Psi) f(|\alpha_1|)}{f(|r| \big| \Psi)} \\ &= \frac{2|\alpha_1|}{\tilde{\Sigma}^2} \exp\left\{-\frac{|\alpha_1|^2 + |\tilde{r}|^2}{\tilde{\Sigma}^2}\right\} I_0\left(\frac{2|\alpha_1||\tilde{r}|}{\tilde{\Sigma}^2}\right) \end{aligned} \tag{6}$$

where $\tilde{\Sigma}^2 \triangleq \sigma_1^2\Sigma^2/\sigma^2$, and $|\tilde{r}| \triangleq \nu_1^2\sigma_1^2|r|/\sigma^2$. Since (6) is a Rice pdf, we have

$$\begin{aligned} P(|\alpha_1| > \tau_\mathrm{A} \big| |r|, \Psi) &= \int_{\tau_\mathrm{A}}^{+\infty} f(|\alpha_1| \big| |r|, \Psi) d|\alpha_1| \\ &= Q_1\left(|r|\sqrt{2}\frac{\nu_1}{\tilde{\Sigma}} \frac{\sigma_1^2}{\sigma^2}, \tau_\mathrm{A}\frac{\sqrt{2}}{\tilde{\Sigma}}\right) \end{aligned} \tag{7}$$

where $Q_1(\cdot,\cdot)$ is the generalized Marcum Q-function of order 1 [6, p. 47]. Finally,

$$\begin{aligned} &f(|r| \big| |\alpha_1| > \tau_\mathrm{A}, \Psi) \\ &= \frac{|r|}{\sigma^2} e^{-\frac{|r|^2}{2\sigma^2}} e^{\frac{\tau_\mathrm{A}^2}{2\sigma_1^2}} Q_1\left(|r|\sqrt{2}\frac{\nu_1}{\tilde{\Sigma}} \frac{\sigma_1^2}{\sigma^2}, \tau_\mathrm{A}\frac{\sqrt{2}}{\tilde{\Sigma}}\right) \end{aligned} \tag{8}$$

Now, from (4) we have

$$\begin{aligned} &\mathbb{P}(|r| < \tau_\mathrm{D} \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi) = 1 \\ &- \frac{\Sigma^2}{2\sigma^2} \sum_{n=0}^{\infty} \left(\frac{\nu_1^2\sigma_1^2}{\sigma^2}\right)^n Q_{n+1}\left(\frac{\sqrt{2}\nu_1}{\Sigma}\tau_\mathrm{A}, \frac{\sqrt{2}}{\Sigma}\tau_\mathrm{D}\right) \end{aligned} \tag{9}$$

where $Q_n(\cdot,\cdot)$ is the generalized Marcum Q-function of order n [6, p. 44].

Next, we compute the conditional probability of a false alarm given Ψ. To this end, we compute $f(|r| \big| |\alpha_1| < \tau_\mathrm{A}, \Psi)$.

$$\begin{aligned} &f(|r| \,\big|\, |\alpha_1| < \tau_\mathrm{A}, \Psi) \\ &= \frac{\mathbb{P}(|\alpha_1| < \tau_\mathrm{A} \,\big|\, |r|, \Psi) f(|r| \,\big|\, \Psi)}{\mathbb{P}(|\alpha_1| < \tau_\mathrm{A})} \\ &= \frac{[1 - \mathbb{P}(|\alpha_1| > \tau_\mathrm{A} \,\big|\, |r|, \Psi)] f(|r| \,\big|\, \Psi)}{[1 - \mathbb{P}(|\alpha_1| > \tau_\mathrm{A})]} \\ &= \frac{\left[1 - Q_1\left(|r|\nu_1 \frac{\sigma_1^2}{\sigma^2} \frac{\sqrt{2}}{\tilde{\Sigma}}, \tau_\mathrm{A}\frac{\sqrt{2}}{\tilde{\Sigma}}\right)\right] f(|r| \big| \Psi)}{[1 - e^{-\frac{\tau_\mathrm{A}^2}{2\sigma_1^2}}]} \\ &= \frac{1}{2} e^{\frac{\tau_\mathrm{A}^2}{4\sigma_1^2}} \operatorname{csch}\left(\frac{\tau_A^2}{4\sigma_1^2}\right) \frac{|r|}{\sigma^2} e^{-\frac{|r|^2}{2\sigma^2}} \\ &\quad \times \left[1 - Q_1\left(|r|\nu_1 \frac{\sigma_1^2}{\sigma^2} \frac{\sqrt{2}}{\tilde{\Sigma}}, \tau_\mathrm{A}\frac{\sqrt{2}}{\tilde{\Sigma}}\right)\right] \end{aligned} \tag{10}$$

where $\operatorname{csch}(\cdot)$ is the hyperbolic cosecant. Finally, the conditional probability of false alarm is obtained as

$$\begin{aligned} &\mathbb{P}(|r| > \tau_\mathrm{D} \,\big|\, |\alpha_1| < \tau_\mathrm{A}, \Psi) \\ &= \int_{\tau_\mathrm{D}}^{+\infty} f(|r| \,\big|\, |\alpha_1| < \tau_\mathrm{A}, \Psi) \, d|r| \\ &= \frac{1}{2}\operatorname{csch}\left(\frac{\tau_\mathrm{A}^2}{4\sigma_1^2}\right) \\ &\times \left(e^{\frac{\tau_\mathrm{A}^2}{4\sigma_1^2}} e^{-\frac{\tau_D^2}{2\sigma^2}} - e^{-\frac{\tau_\mathrm{A}^2}{4\sigma_1^2}} \mathbb{P}\left(|r| > \tau_\mathrm{D} \,\big|\, |\alpha_1| > \tau_\mathrm{A}, \Psi\right)\right) \end{aligned} \tag{11}$$

Taking the expectation of (9) and (11) over Ψ yields $P_{\mathrm{M},1}$ and $P_{\mathrm{F},1}$, but these cannot be obtained in a closed form. For simple approximations, a Monte Carlo technique consists of generating M patterns of Ψ, denoted $\{\psi_{m=1}^{M}\}$, according to the actual node model, and using the estimates

$$\widehat{P}_{\mathrm{M},1} = \frac{1}{M}\sum_{\psi_m} \mathbb{P}(|r| < \tau_{\mathrm{D}} \,\big|\, |\alpha_1| > \tau_{\mathrm{A}}, \psi_m) \quad (12)$$

$$\widehat{P}_{\mathrm{F},1} = \frac{1}{M}\sum_{\psi_m} \mathbb{P}(|r| > \tau_{\mathrm{D}} \,\big|\, |\alpha_1| < \tau_{\mathrm{A}}, \psi_m) \quad (13)$$

4. NUMERICAL RESULTS

As a simple example of application, consider $K+1=4$ nodes whose signatures are length-seven m-sequences. The duration of the discovery session is set to be $T_D = 100T$, thus $N = 100$. Node amplitudes are zero-mean circularly-complex Gaussian with variances $2\sigma_1^2 = 1$ and $2\sigma_3^2 = 0.25$. The activity threshold is set to be $\tau_{\mathrm{A}} = 1$, and $\varepsilon = 0.5$. Figure 2 shows the probability of a false alarm vs. the probability of a miss in the discovery of node 1 when node 2 has a variance $2\sigma_2^2$ varying in the range $[0, 30]$ with $2N_0 = 1$. As expected, the algorithm we have described here is not near-far resistant. Figure 3 shows the probability of a false alarm vs. the probability of a miss in the discovery of node 1 obtained from simulation and from Monte Carlo approximation in (12)-(13) when node 1, 2, and 3 have variances $2\sigma_1^2 = 1$, $2\sigma_2^2 = 0.5$, and $2\sigma_3^2 = 0.25$, respectively. In addition, $\tau_{\mathrm{A}} = 1$, $\varepsilon = 0.5$, and $2N_0 = 1$.

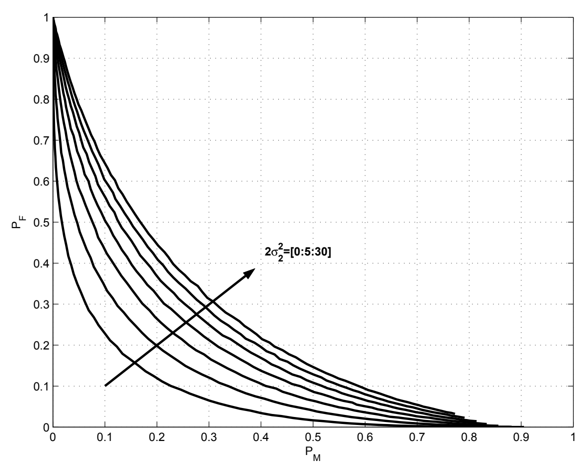

Fig. 2. *$P_{\mathrm{F},1}$ vs. $P_{\mathrm{M},1}$ where the variance of node 2 is increasing in the range $[0, 30]$ with step-size 5.*

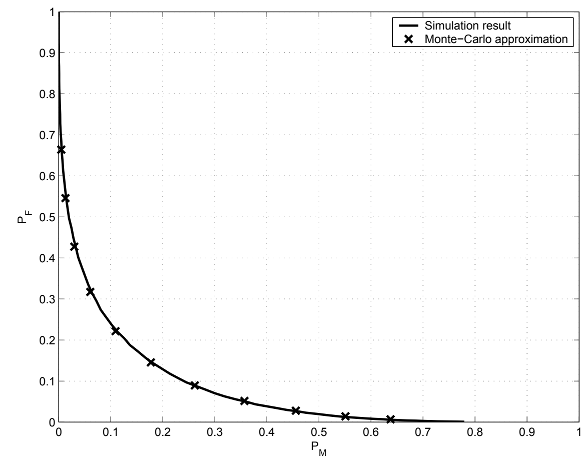

Fig. 3. *Comparison between simulation and Monte Carlo approximation of the probabilities of false alarm and missed detection of node 1.*

5. CONCLUSIONS

We have described a simple neighbor-discovery algorithm, based on [1] and including a transmission scheme based on nonorthogonal signatures. The algorithm performance is illustrated by using a combination of analysis and simulation. As expected, matched filter is not near-far resistant. Further work will relax some of the assumptions made here for simplicity, and introduce multiuser detection to better exploit the problem structure. More precisely, in [7], we introduce a number of ND algorithms which guarantee near-far resistance and derive closed formulas for the asymptotic values of the optimal thresholds.

6. ACKNOWLEDGMENTS

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