OPTIMAL PRECODER FOR AMPLIFY-AND-FORWARD HALF-DUPLEX COOPERATIVE SYSTEM

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ABSTRACT

In this paper, we design an optimal precoder for the amplify-and-forward (AF) half-duplex cooperative system. We first present asymptotic pairwise error probability (PEP) expression and identify the corresponding terms for diversity and coding gains. By employing the properties of the Farey sequence in number theory, we obtain a closed form optimal precoder to achieve both full diversity as well as maximum coding gain for square quadrature amplitude modulation (QAM) signals. Simulation results indicate that the proposed design significantly improves the BER performance of the relay system.

Index Terms— Cooperative system, half-duplex, amplify-and-forward (AF), pairwise error probability, diversity gain function, precoder.

1. INTRODUCTION

Diversity techniques, and in particular, those of spatial diversity, have been employed in practical wireless communication systems to overcome channel fading. To achieve a higher spatial diversity gain, multiple transmitter and receiver antennas are usually desirable. However, this is often impractical for the mobile units due to size and complexity limitation. Recently, another form of spatial diversity called *cooperative diversity* has been proposed for mobile wireless communications [1–6]. Protocols to implement the cooperative diversity have been proposed [2–4]. These protocols can generally be classified into two types, Decode-and-Forward (DF) and Amplify-and-Forward (AF) protocols. In a DF protocol the relay nodes decode the message received from the source node first, re-encode it and then transmit it to the destination. In an AF protocol, the relay nodes retransmit a scaled version of the received signal to the destination. An AF protocol usually has lower complexity than a DF protocol. In [6–8] distributed space time block codes are designed for both the DF and AF protocols. In this paper, we consider the problem of optimal precoder design for the AF protocol. We focus our attention on the AF protocol proposed in [2, 4]. It has been proved that the protocol can achieve the optimal diversity-multiplexing tradeoff [4, 9]. For such a protocol, we first present an asymptotic pairwise error probability (PEP) expression. Then, by employing the properties of the Farey sequence in number theory, we obtain a closed form optimal precoder so that the full diversity as well as maximum coding gain can be achieved if square QAM signals are transmitted.

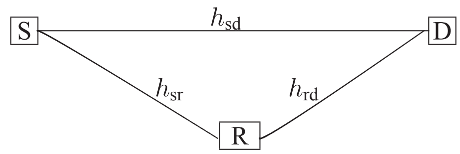

Fig. 1. A single relay system

2. SYSTEM MODEL

A diagram of a single relay system is shown in Fig 1. The relay node R assists the transmission from the source node S to the destination node D. The relay operates in a half-duplex mode by either transmitting or receiving signals, but not doing both at the same time. All nodes in the system are equipped with one antenna. The channel gain from the source node to the destination is denoted by h_{sd} whereas those from the source node to the relay node and from the relay node to the destination are denoted by h_{sr} and h_{rd} respectively. We consider a symmetric relay network in which all channel gains are assumed to be independent and identically distributed (IID) circularly Gaussian with zero mean and unit variance, and they do not change within the period of observation. The destination has the full channel knowledge and the source knows the second order statistics. The information bearing symbols are equally probable from a square M-ray-QAM constellation set $\mathcal{S}$ and are processed by a precoder before being transmitted. The transmission is carried out by blocks. We confine ourselves to blocks of length 2 since the optimal precoder design which are based on the criterion of maximum coding gain is very complicated for longer data block length. In the 1st time-slot, the source node transmits the first data symbol to both the destination and the relay node, and in the 2nd time-slot, it sends the second data symbol only to the destination while the relay amplifies and forwards what it received in the 1st time-slot to the destination. We denote the original information symbol vector by $\mathbf{s} = [s(1) \quad s(2)]^T$ with $s(t),\ t = 1, 2$ being the information symbol at the t-th time slot. We also assume that the symbols satisfy $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_2$ where $\mathbf{I}_2$ is an identity matrix of size 2. The data block is then processed by a precoder $\mathbf{F}$ such that

$$\mathbf{x} = [x(1) \quad x(2)]^T = \mathbf{F}\, [s(1) \quad s(2)]^T \tag{1}$$

The received symbols at the destination can be written as,

$$\mathbf{r} = \sqrt{E_\mathrm{p}}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

where E_p is the average power for transmitting a symbol at each node, and $\mathbf{H}$ is the channel matrix

$$\mathbf{H} = \begin{bmatrix} h_\mathrm{sd} & 0 \\ b h_\mathrm{sr} h_\mathrm{rd} & h_\mathrm{sd} \end{bmatrix} \tag{3}$$

with b being the amplification coefficient at the relay node, and $\mathbf{n}$ is the zero-mean circularly Gaussian noise vector at the destination whose covariance matrix is $\sigma^2\mathbf{\Sigma}$ with σ^2 being the noise power and $\mathbf{\Sigma} = \begin{bmatrix} 1 & 0 \\ 0 & (1 + b^2|h_\mathrm{rd}|^2) \end{bmatrix}$. We assume that the relay node has knowledge of the second order statistics of channel gain h_sr between the source node to the relay node and the amplification coefficient satisfies $b = \sqrt{E_\mathrm{p}/(E_\mathrm{p} + \sigma^2)}$ [10]. For convenience in analysis, we rewrite $\mathbf{Hx}$ in Eq. (2) as $\mathbf{Hx} = \mathbf{Xh}$, where $\mathbf{X}$ is the signal matrix and $\mathbf{h}$ is the equivalent channel vector given below

$$\mathbf{X} = \begin{bmatrix} x(1) & 0 \\ x(2) & x(1) \end{bmatrix} \quad \text{and} \quad \mathbf{h} = \begin{bmatrix} h_\mathrm{sd} \\ b h_\mathrm{sr} h_\mathrm{rd} \end{bmatrix} \tag{4}$$

3. PAIR-WISE ERROR PROBABILITY

Suppose a maximum likelihood (ML) detector is applied to the relay system described in Eq. (2). For a given channel realization and a symbol block $\mathbf{s} \in \mathcal{S}^2$, the pairwise error probability is defined as the probability of deciding in favor of $\mathbf{s}' \neq \mathbf{s},\ \ \mathbf{s}' \in \mathcal{S}^2$, and is given by

$$P_e(\mathbf{s} \to \mathbf{s}'|\mathbf{h}) = Q\Big(\frac{d(\mathbf{s}, \mathbf{s}')}{2}\Big) \tag{5}$$

where function $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\big(-\frac{x^2}{2\sin^2\theta}\big)d\theta$ [11], and $d(\mathbf{s}, \mathbf{s}')$ is the Euclidean distance between $\mathbf{s}$ and $\mathbf{s}'$ at the ML detector

$$\begin{aligned} d^2(\mathbf{s}, \mathbf{s}') &= \frac{E_\mathrm{p}}{\sigma^2}\, (\mathbf{s} - \mathbf{s}')^H\mathbf{F}^H\mathbf{H}^H\mathbf{\Sigma}^{-1}\mathbf{H}\mathbf{F}(\mathbf{s} - \mathbf{s}') \\ &= \rho\, \mathbf{e}^H\mathbf{F}^H\mathbf{H}^H\mathbf{\Sigma}^{-1}\mathbf{H}\mathbf{F}\, \mathbf{e} \end{aligned} \tag{6}$$

with $\rho = E_\mathrm{p}/\sigma^2$ being the signal-to-noise ratio (SNR) and $\mathbf{e} \triangleq (\mathbf{s} - \mathbf{s}')$ the error vector. Writing $\mathbf{u} = [u(1) \ \ u(2)]^T \triangleq \mathbf{F}\,\mathbf{e} = \mathbf{F}(\mathbf{s} - \mathbf{s}') = \mathbf{x} - \mathbf{x}'$, then, we can re-write Eq. (6) as

$$d^2(\mathbf{s}, \mathbf{s}') = \rho\, \mathbf{h}^H\mathbf{U}^H\mathbf{\Sigma}^{-1}\mathbf{U}\,\mathbf{h} \tag{7}$$

where $\mathbf{U}$ is the error matrix after precoding such that

$$\mathbf{U} = \begin{bmatrix} u(1) & 0 \\ u(2) & u(1) \end{bmatrix} = \mathbf{X} - \mathbf{X}' \tag{8}$$

with $u(i) = x(i) - x'(i),\ i = 1, 2$, and $\mathbf{X}$ and $\mathbf{X}'$ as defined in Eq. (4). From Eqs. (5) to (7), the average PEP for the system in Eq. (2) can be expressed as

$$\begin{aligned} P_\mathrm{e}(\mathbf{s} \to \mathbf{s}') &= \mathbb{E}_\mathbf{h}\Big[Q\Big(\frac{d(\mathbf{s}, \mathbf{s}')}{2}\Big)\Big] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathbb{E}_\mathbf{h}\Big[\exp\Big(-\rho\frac{\mathbf{h}^H\mathbf{U}^H\mathbf{\Sigma}^{-1}\mathbf{U}\mathbf{h}}{8\sin^2\theta}\Big)\Big]d\theta \end{aligned} \tag{9}$$

For the relay system with a data block length of $2P,\ P \geq 1$, an asymptotic expression for the PEP is given by Theorem 1 in [12]. For the particular case of data block length of 2, i.e., $P = 1$, a more accurate expression can be obtained by extracting one more dominant term. Here, we state the result as a corollary:

Corollary 1 *If, in Eq. (8), $|u(1)| \neq 0$ then, at high SNR, an asymptotic average PEP of the AF single relay system is*

$$P_e(\mathbf{s} \to \mathbf{s}') = C_1\rho^{-2}\ln\rho + C_2\rho^{-2} + O\Big(\frac{1}{|u(1)|^4}\rho^{-3}\Big)$$

where $C_1 = \frac{12}{|u(1)|^4}$, $C_2 = \frac{12}{|u(1)|^4}\Big(4\ln 2 + \frac{5}{12} - \gamma - 2\ln\big(\frac{||\mathbf{u}||}{|u(1)|}\big)\Big)$ and γ is the Euler constant. The terms $G = \frac{1}{12}|u(1)|^4$ and $(\rho^{-2}\ln\rho)$ are the corresponding coding gain and (full) diversity gain function, respectively. ■

Comparing Corollary 1 with the asymptotic PEP for a conventional multiple input multiple output (MIMO) system [13], the following observations are noted:

- The diversity gain for the AF single relay system has an extra factor of the logarithm of SNR. This is because the channel matrix contains a term of the product of two independent channel gains which, unlike in the case for a MIMO system, is no longer IID Gaussian.
- Similar to the design of space time block codes for a MIMO system, a precoder for the AF system can be designed based on the following two criteria.
    1. Rank criterion: In order to achieve the full diversity gain function $(\rho^{-2}\ln\rho)$ derived in Corollary 1, the denominator of C_1 must be non-zero; i.e., $|u(1)|^4 \neq 0$.
    2. Coding gain criterion: In order to obtain maximum advantage of the coding gain, the minimum of coding gain taken all over non-zero error vectors, i.e., $\mathbf{e} \neq \mathbf{0}$ must be maximized.

An efficient method has been proposed [12] for the design of a full diversity precoder using the rank criterion so that the sufficient condition of $|u(1)|^4 \neq 0$ is ensured. On the other hand, the issue of precoder design achieving maximum coding gain will be addressed in the next section.

4. OPTIMAL PRECODER DESIGN

For a data block length of 2, we seek an optimum precoder matrix $\mathbf{F}$ base on the coding gain criterion. Since we desire that the ergodic channel capacity remains unchanged after the transmitted symbols are precoded, we confine our consideration to *orthonormal* precoders. i.e., $\mathbf{F}\mathbf{F}^H = \mathbf{I}$. In the following, we design $\mathbf{F}$ such that the minimum coding gain is maximized by exploiting the properties of the *Farey sequence* whose definition is given below [14].

Definition 1 *The Farey sequence $\mathcal{F}_n$ for any positive integer n is the set of irreducible rational numbers $\frac{p}{q}$ arranged in increasing order, where $0 \le p \le q \le n$ and p and q have no common factors.* ■

For example, when $n = 4$, $\mathcal{F}_4 = \{\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{4}\}$. The Farey sequence $\mathcal{F}_n$ has the following interesting property [14]:

Lemma 1 *If $\frac{p_k}{q_k}$ and $\frac{p_{k+1}}{q_{k+1}}$ are consecutive terms in $\mathcal{F}_n$, then,$p_{k+1}q_k - p_k q_{k+1} = 1$, and $q_k + q_{k+1} \ge n+1$* ■

For our design of the optimal coding gain precoder, we introduce another property of the Farey sequence, whose proof is omitted here:

Lemma 2 *If $\frac{p_k}{q_k}$ and $\frac{p_{k+1}}{q_{k+1}}$ are consecutive terms in $\mathcal{F}_n$, and $f_k(\theta) = (-p_k \cos\theta + q_k \sin\theta)^2$, then, $f_k(\theta)$ is an increasing function and $f_{k+1}(\theta)$ is a decreasing function of θ for $\theta \in [\tan^{-1}(p_k/q_k), \tan^{-1}(p_{k+1}/q_{k+1})]$.* ■

Consider a square constellation of M-ary QAM signals having its m-th member given by $s_m = a_{mR} + ja_{mI}$; $m = 1, \cdots, M$, where $a_{mR}, a_{mI} \in \{\pm 1, \pm 3, \cdots, \pm(\sqrt{M}-1)\}$. Let the difference between two of these members be represented by $\epsilon = \epsilon_R + j\epsilon_I$ where ϵ_R, ϵ_I are even integers such that $-2m_0 \le \epsilon_R, \epsilon_I \le 2m_0$ with $m_0 = \sqrt{M} - 1$. Thus, the error vector in Eq. (6) is composed of the error quantities at two different time slots such that $\mathbf{e} = [e(1) \;\; e(2)]^T \in \mathcal{S}_{\rm d}^2$ with $\mathcal{S}_{\rm d} = \{\epsilon_R + j\epsilon_I\}$. Now, let a 2×2 orthonormal precoder matrix $\mathbf{F}$ be given by

$$\mathbf{F} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \tag{10}$$

We can re-write the coding gain in Corollary 1 as

$$G = \frac{|u(1)|^4}{12} = \frac{1}{12}|e(1)\cos\theta + e(2)\sin\theta|^4 \tag{11}$$

To find an optimal $\mathbf{F}$ satisfying the coding gain criterion, i.e., $\max_{\mathbf{F}} \min_{\mathbf{e}\neq\mathbf{0}} G$, we set the following problem:

Let $J(\mathbf{e},\theta) = |e(1)\cos\theta + e(2)\sin\theta|^2$. Find a $\theta_{\rm op}$ such that

$$\theta_{\rm op} = \arg \max_{\theta\in[0,\,2\pi]} \min_{\mathbf{e}\in\mathcal{S}_d^2, \mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta) \tag{12}$$

A solution to this problem is provided by the following theorem:

Theorem 1 *For the problem in Eq. (12), we have*

$$\theta_{\rm op} = \tan^{-1}(1/\sqrt{M}) \tag{13a}$$

$$J_{\rm op} = \max_{\theta\in[0,\,2\pi]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta) = 4/(1+M) \tag{13b}$$

The corresponding optimal coding gain is

$$G_{\rm op} = \frac{1}{12}\left(\frac{4}{1+M}\right)^2 \tag{14}$$

Note:- *By choosing $\theta_{\rm op}$ as in Eq. (13a), the rank criterion that $|u(1)|^4 \neq 0,\ \forall\ \mathbf{e} \neq \mathbf{0}$ is automatically satisfied.*

To prove Theorem 1, we first show that the feasible set $\theta \in [0, 2\pi]$ in Eq. (12) can be reduced to $\theta \in [0, \frac{\pi}{4}]$ by letting $Z = \max_{\theta\in[0,2\pi]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta)$. We notice that

$$Z = \max\{ \max_{\theta\in[\frac{\ell\pi}{2}, \frac{(\ell+1)\pi}{2}]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta)\},\ \ell = 0, \cdots, 3$$

On the other hand, we note that for $\mathbf{e} \in \mathcal{S}_d^2, \mathbf{e} \neq \mathbf{0}$,

$$\begin{aligned} &|e(1)\cos\bar\theta + e(2)\sin\bar\theta|^2\Big|_{\bar\theta\in[\frac{(\ell+1)\pi}{2}, \frac{(\ell+2)\pi}{2}]} \\ &= |-e(1)\sin\theta + e(2)\cos\theta|^2\Big|_{\theta\in[\frac{\ell\pi}{2}, \frac{(\ell+1)\pi}{2}]} \end{aligned} \tag{15}$$

Since both $[e(1) \;\; e(2)]^T$ and $[-e(1) \;\; e(2)]^T$ cover $\mathcal{S}_d^2$, therefore, for $\ell = 0, \cdots, 3$, we have

$$\max_{\theta\in[\frac{(\ell+1)\pi}{2}, \frac{(\ell+2)\pi}{2}]} \min_{\substack{\mathbf{e}\in\mathcal{S}_d^2 \\ \mathbf{e}\neq\mathbf{0}}} J(\mathbf{e},\theta) = \max_{\theta\in[\frac{\ell\pi}{2}, \frac{(\ell+1)\pi}{2}]} \min_{\substack{\mathbf{e}\in\mathcal{S}_d^2 \\ \mathbf{e}\neq\mathbf{0}}} J(\mathbf{e},\theta).$$

Hence we can obtain $Z = \max_{\theta\in[0,\frac{\pi}{2}]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta)$. Using the same argument, the feasible set $\theta \in [0, \frac{\pi}{2}]$ can be further reduced to $\theta \in [0, \frac{\pi}{4}]$. Now, to prove Theorem 1, we take the following two steps:

- Step 1: Show that $J_{\rm op}$ in Eq. (13b) is the upper bound of Z, i.e., $Z = \max_{\theta\in[0,\frac{\pi}{4}]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta) \le J_{\rm op}$. We segment $[0, \frac{\pi}{4}]$ into $K-1$ non-overlapping sub-intervals, i.e., $[0, \frac{\pi}{4}] = \cup_{k=1}^{K-1}\Upsilon_k$, where $\Upsilon_k = [\tan^{-1}\frac{p_k}{q_k}, \tan^{-1}\frac{p_{k+1}}{q_{k+1}}]$, with $\{\frac{p_k}{q_k}\}$ being the Farey sequence $\mathcal{F}_{m_0}$ such that $\frac{0}{1} = \frac{p_1}{q_1} < \frac{p_2}{q_2} < \cdots < \frac{p_K}{q_K} = \frac{m_0}{m_0} = 1$, and K being the total number of elements in $\mathcal{F}_{m_0}$. The properties in Lemma 1 and Lemma 2 are then used to prove Step 1.
- Step 2: Show that $J_{\rm op}$ is also a lower bound of Z, i.e., $Z = \max_{\theta\in[0,\frac{\pi}{4}]} \min_{\mathbf{e}\in\mathcal{S}_d^2,\mathbf{e}\neq\mathbf{0}} J(\mathbf{e},\theta) \ge J_{\rm op}$. A lower bound of Z is $Z \ge \frac{1}{M+1}|\sqrt{M}e(1) + e(2)|^2$. Since the minimum value of non-zero $|e(i)|$ for $i = 1, 2$ is 2, Step 2 holds.

5. NUMERICAL EXPERIMENTS

To demonstrate the performance of the optimal rotation matrix, we evaluate the bit error rate (BER) by computer simulations. The transmitted signals, the channel gains, and the noise are all of the forms as described in Section 2. At the destination, the data is received by a maximum likelihood detector. We compare the BER of the relay system: 1) with $\mathbf{F} = \mathbf{I}$, i.e., without a precoder; 2) with a precoder designed for the rank criterion [12]; and 3) with the optimal precoder given in Theorem 1 in the previous Section. Fig. 2 shows the comparison of the performance when the transmitted signals are from a 4-QAM constellation. As can be observed, at BER of 10^{-4}, the system with optimal coding gain precoder is 8dB and 2dB lower in SNR than the systems with no precoder and with the precoder designed for the rank criterion respectively.

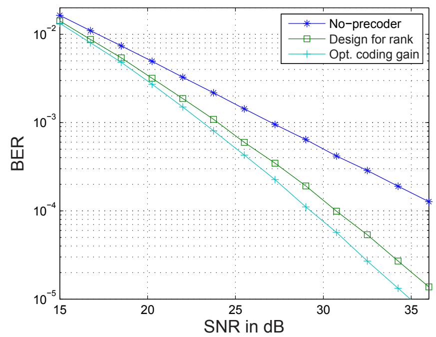

Fig. 2. Performance Comparison for 4-QAM Signals

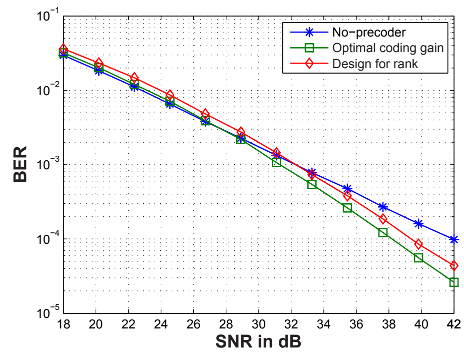

Fig. 3. Performance Comparison for 16-QAM Signals

Fig. 3 shows similar comparison of the BERs when the transmitted signals are from a 16-QAM constellation. Again, the maximum coding gain precoder is superior in performance not only to the system with no precoder but also to the system with a precoder designed for rank criterion. In this case, at BER of 10^{-4}, its performance is about 4dB better than the system without a precoder and is about 1.5dB better than the system with a precoder designed for the rank criterion.

6. CONCLUSION

In this paper, we have analyzed the pairwise error performance of the AF half-duplex single relay transmission system with transmitting data blocks of length 2. By extracting the two dominant terms in the pairwise error probability, a more accurate expression has been arrived at from which the coding gain and maximum diversity have been identified. From these results, employing some interesting properties of the Farey sequence in number theory, a closed form optimal precoder has been designed to achieve the optimal coding gain for square QAM signals. It is also noted that such an optimum precoder satisfies full diversity as well. Simulation results show that the proposed design significantly improves the BER performance of the relay system.

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