ENERGY EFFICIENT RELAYING AND COALITION-FORMING IN RELAY NETWORKS

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ABSTRACT

Relaying is often advocated for improving system performance by enhancing spatial diversity in wireless networks. In this paper, we address the issue of energy tradeoff made by relay nodes between transmitting their own data and forwarding other nodes' information in fading channels. We first propose a power control policy in a two-node relay network under which total energy consumption across both nodes is minimized while meeting both outage probability requirements. Based on this power control algorithm, we consider the problem of forming optimal partial coalitions of relays in an N node system subject to selfish constraints: A node participates in a relay pair (or chain) if and only if the energy cost of relaying is lower than the cost of direct transmission by the node to the destination itself. We develop a simple (1,2)-polynomial time bi-criteria approximation for this NP-Hard problem. The energy cost provided by the approximation is at most that of the optimal relay pairing, while the constraints are violated by at most a factor of two. The running time of the approximation algorithm is polynomial, as it requires the solution of a relaxed linear programming instance of the original integer programming problem.

Index Terms— Relay Channel, Outage Probability, Power Control, Algorithm, Polynomial Approximation

1. INTRODUCTION

Exploiting CSI in relay channels has attracted attention lately to further improve reliability and energy efficiency while exploiting cooperative diversities [1, 2]. The preceding cited works share one common feature in that all of them assume a set of relay nodes is already selected and the remaining issue is to determine power allocations across all transmitting nodes without considering the data originated from relay nodes themselves. No consideration is given toward the relay's own needs other than its function as relaying.

In our model, assuming that all nodes have their own data to transmit and their individual quality of service requirement, e.g. outage probability P_{out} as a good approximation for frame error rate (FER), each node divides its entire energy budget into two parts. One is for transmitting its own data, the other is devoted to relaying information. As a partner relationship is established between two nodes such that each of them helps the other forward/relay information, we are interested in a fundamental question regarding what power allocation policy should be adopted for each node in order to save its own energy to the largest extent while meeting the outage probability constraints and complying with its obligation as a relay.

In [2], perfect CSI is assumed available to all nodes in a network where central controller is present. There is *no power control issue* in this work where users' power is assumed fixed. Both centralized and distributed partner selection protocols based on CSI are developed. In [1], the authors assume fixed power for each user and adopt frame error rate (FER) and pair-wise error rate (PER) as performance metrics. The FER ratio of a cooperative block fading channel to a non-cooperative quasi-static fading channel for a user indicates whether it benefits from relaying. The system cooperation gain introduced there cannot, however, reflect the benefits to all users as the ratio of the summation of non-cooperative FER over the summation of cooperative FER can overshadow some user under consideration.

In this paper, we first present results where we optimize energy related objective functions under the assumption that nodes have perfect CSI in a network of two transmitting nodes and one central station. In particular, we develop the optimal power control algorithm based on *complete collaboration*, where relay nodes aim at minimizing *total* energy expenditure.

We then consider the issue of constructing relay node partnership for a $N > 2$-node relay network. Instead of restricting relay partnership be reciprocal i.e. node A helps node B if node B helps node A, we allow the existence of relay chains, e.g. node A helps node B and node B helps node C and node C helps node A. A novel pairing problem is formulated in which total transmission energy is minimized by finding the optimal relay partnerships in a $N > 2$ relay network. In addition, our proposed metric allows each node to compare how much it spends in a coalition with how much it takes to send its data by working alone. If the comparison turns out to be unfavorable it will prefer to work alone. We show this problem

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is NP-hard and develop a simple (1,2)-polynomial time bi-criteria approximation. The energy cost provided by the approximation is at most that of the optimal relay pairing, while the constraints are violated by at most a factor of two. The running time of the approximation algorithm is polynomial, as it requires the solution of a relaxed linear programming instance of the original integer programming problem. We also provide a heuristic for finding a $(1+\epsilon, 2-\beta)$-polynomial time approximation of the selfishly constrained optimal relay pairing problem.

2. SYSTEM MODEL

To illustrate the major idea of power control across relay nodes, we first consider a simple model in which there are two nodes N_1 and N_2 transmitting to a common receiver N_D with help from each other. Narrow-band quasi-static fading channel is assumed, where channel fading coefficients remain fixed during the transmission of a whole packet, but are independent from node to node. The complex channel coefficient $h_{i,j}$ captures the effects of both pathloss and the quasi-static fading on transmissions from node N_i to node N_j, where $i \in \{1,2\}$, and $j \in \{2,1,D\}$. Statistically, $h_{i,j}$ are modeled as zero mean, mutually independent proper complex Gaussian random variables with variances: $E|h_{i,j}|^2 = 2\sigma_{i,j}^2$. We first assume a non-causal system model in which amplitudes $|h_{i,j}|$ are available to all transmitters and receivers at the beginning of transmissions. In a quasi-static fading channel, CSI can be obtained by exploiting training sequences sent by transmitters [3].

Consider a time-division (TD) multiple access scheme in which an entire time period is divided into 4 slots [4, Fig. 2]. A repetition coding-based decode-and-forward strategy (R-DF)is assumed at N_j, $j = 1,2$, where relay node transmits the same codeword as what source sends if its decoding is successful. The cooperative communication protocol can be described as follows: Based on the available CSI, N_1 can determine whether relaying from N_2 is needed or not, as explained in the power control algorithms below. If such collaboration is sought, N_1 transmits as a source to N_D in the first slot and then in the second slot N_2 forwards its decoded messages to the destination. If N_2 is not asked for relaying, N_1 transmits in the first 2 slots of on its own. Over the last two slots, N_1 and N_2 exchange their roles as a source and relay.

The mathematical characterization of the whole process is:

$$Y_{1,D}[k] = h_{1,D}S_1[k]+W_{1,D}[k], Y_{1,2}[k] = h_{1,2}S_1[k]+W_{1,2}[k]$$

for $k \in [0, N/4]$; and

$$Y_{2,R}[k] = h_{2,D}\tilde{S}_1[k] + W_{2,R}[k]$$

for $k \in (N/4, N/2]$, if relay N_2 is needed and decoding is successful. The figure N is the total number of degrees of freedom available over the entire transmission period, and $W_{i,j}$ are independent complex white Gaussian noise with two-sided power spectral density $\mathcal{N}_0 = 1$. For R-DF schemes, $\tilde{S}_j[k]$ are scaled versions of the transmitted Gaussian codewords $S_j[k]$. Over the last two slots, similar models can be set up for node 2 based on symmetry over $k \in (N/2, N]$.

Given CSI on $|h_{i,j}|$, transmission powers over various periods are denoted as: $E|S_1[k]|^2 = P_{1,D}$, $k \in [0, N/4]$ and $E|\tilde{S}_1[k]|^2 = P_{2,R}$, $k \in (N/4, N/2]$ if N_2 is needed and decoding is successful; $E|S_1[k]|^2 = P_{1,D}$, $k \in [0, N/2]$ and $E|\tilde{S}_1[k]|^2 = 0$, $k \in [0, N/2]$, if N_2 is not needed. Similarly, we define $E|S_2[k]|^2 = P_{2,D}$, $k \in (N/2, \frac{3}{4}N]$ and $E|\tilde{S}_2[k]|^2 = P_{1,R}$, $k \in (\frac{3}{4}N, N]$ if N_1 is needed and decoding is successful; $E|S_2[k]|^2 = P_{2,D}$, $k \in (N/2, N]$ and $E|\tilde{S}_2[k]|^2 = 0$, $k \in (N/2, N]$ if N_1 is not needed.

3. TOTAL ENERGY MINIMIZATION FOR COLLABORATIVE RELAYING

3.1. Problem Statement and Solution

Under the constraint that each source node has an outage probability no greater than $P_{j,out}$, i.e. $\Pr[I_j < R_j] \le P_{j,out}$, where I_j is the mutual information of the overall link for transmitting node $j \in \{1,2\}$'s information, our objective is to investigate power control policies under which the total energy of these two nodes is minimized in a complete collaborative manner. This **Collaborative Relaying** problem can be formulated as below:

$$\min \sum_{j=1}^{2} E\left[P_{j,D} + P_{j,R}\right], \text{ subject to } \Pr\left[I_k < R_k\right] \le P_{k,out},$$

for $k = 1, 2$.

Under the collaborative relaying approach, the optimal power allocation policy $[P_{i,D}^J, P_{j,R}^J]$ to solving problem (1) can be characterized by the following Theorem.

Theorem 1 *The optimal power allocation vector $[P_{i,D}^J, P_{j,R}^J]$ depends on channel strength ratios captured by $|h_{i,D}|/|h_{j,D}|$ and $|h_{i,D}|/|h_{i,j}|$ for $i \neq j$ and $i, j \in \{1,2\}$. The resulting solutions are:*

$P_{i,D}^J = \hat{P}_{i,D}$, $P_{j,R}^J = \hat{P}_{j,R}$ if $h_{i,j}$ are in the set

$$A_i = \Big\{|h_{i,j}| : \frac{|h_{i,D}|^2}{|h_{i,j}|^2} < \frac{2}{2^{R_i}+1} \text{ and } \frac{|h_{i,D}|^2}{|h_{i,j}|^2} + \frac{|h_{i,D}|^2}{|h_{j,D}|^2}\left(1 - \frac{|h_{i,D}|^2}{|h_{i,j}|^2}\right) \le \frac{2}{2^{R_i}+1}\Big\} \tag{1}$$

and $\hat{P}_{i,D} + \hat{P}_{j,R} \le s_{i,J}^*$. Otherwise if $h_{i,j} \in A_i^c$, the complimentary set of A_i, i.e.

$$A_i^c = \Big\{|h_{i,j}| : \frac{|h_{i,D}|^2}{|h_{i,j}|^2} \ge \frac{2}{2^{R_i}+1} \text{ or } \frac{|h_{i,D}|^2}{|h_{i,j}|^2} + \frac{|h_{i,D}|^2}{|h_{j,D}|^2}\left(1 - \frac{|h_{i,D}|^2}{|h_{i,j}|^2}\right) > \frac{2}{2^{R_i}+1}\Big\} \tag{2}$$

and $2\tilde{P}_{i,D} \leq s^*_{i,J}$, the solution is $P^J_{i,D} = \tilde{P}_{i,D}$, $P^J_{j,R} = 0$. For all other cases, transmission powers are all set to zero $P^J_{i,D} = P^J_{j,R} = 0$.
Transmission power functions are defined as follows:

$$\tilde{P}_{i,D} \triangleq (2^{R_i} - 1)(|h_{i,D}|^2), \;\; \hat{P}_{i,D} \triangleq (2^{2R_i} - 1)(|h_{i,j}|^2),$$
$$\hat{P}_{i,R} \triangleq \frac{2^{2R_j} - 1}{|h_{i,D}|^2}\left(1 - \frac{|h_{j,D}|^2}{|h_{j,i}|^2}\right).$$

The thresholds $s^*_{i,J}, i = 1,2$ are determined by solving the following equations to meet outage probability constraints:

$$1 - P_{i,out} = Pr\left\{2\tilde{P}_{i,D} < s^*_{i,J}, \textit{for } \left(\frac{|h_{i,D}|^2}{|h_{i,j}|^2}, \frac{|h_{i,D}|^2}{|h_{j,D}|^2}\right) \in A^c_i\right\}$$
$$+ Pr\left\{\hat{P}_{i,D} + \hat{P}_{j,R} < s^*_{i,J}, \textit{for } \left(\frac{|h_{i,D}|^2}{|h_{i,j}|^2}, \frac{|h_{i,D}|^2}{|h_{j,D}|^2}\right) \in A_i\right\}.$$

Proof: See [5]. ■

3.2. Power Control without Relaying and Relaying without Power Control

To reveal energy savings through power control and relaying, we will compare schemes proposed in Section 3.1 with two other possible approaches. One is cooperative diversity scheme without power control at an absence of CSI on $|h_{i,j}|$. The other one is power control without relaying as studied. The purpose of this comparison is to illustrate the impact of relaying, as well as CSI on energy consumption.

For relaying with fixed power, [6] derived outage probability of R-DF schemes. Therefore, transmission powers of two users under given outage probabilities can be determined from those two non-linear equations resulting from two outage probability expressions.

While for the case when each node employs power control strategy to transmit its data without relaying, the outage probability is

$$P_{i,out} = \Pr\left\{\log_2\left(1 + P_{i,Nr}|h_{i,D}|^2\right) < R_i\right\}, i = 1,2 \quad (3)$$

where $P_{i,Nr}(|h_{i,D}|), i = 1,2$ can be determined using the similar techniques developed in [3]:

$$P_{i,Nr} = \begin{cases} \frac{2^{R_i}-1}{|h_{i,D}|^2} & \text{if } 2\frac{2^{R_i}-1}{|h_{i,D}|^2} < s^*_{(i,Nr)} \\ 0 & \text{Otherwise} \end{cases}, \quad (4)$$

where the threshold $s^*_{(i,Nr)}$ is the solution to
$\Pr\left[2\frac{2^{R_i}-1}{|h_{i,D}|^2} < s^*_{(i,Nr)}\right] = 1 - P_{i,out}$.

4. A $(1,2)$-POLYNOMIAL APPROXIMATION FOR OPTIMAL RELAY PAIRING UNDER SELFISH CONSTRAINTS

When we have $N > 2$ nodes in a wireless network, we consider the problem of finding relaying partnerships in the network. As stressed in [2], it is not necessary to require partnership be reciprocal in a relay network and the authors provide a centralized algorithm to construct partnerships, where each node is allowed to have at most one relay node to help forward its information. Although perfect CSI is assumed available at a central controller, in [2], transmission power is assumed fixed without performing power control. As a contrast, we propose a centralized power-control-based partnership construction algorithm suitable for wireless networks with infrastructure to exploit. Specifically, we look at finding the optimal arrangement of relay pairs (or chains) when each node is subject to an additional 'selfish' constraint: A node participates in a relay pair (or chain) if and only if the energy cost of relaying is lower than the cost of direct transmission by the node to the destination (i.e. the NRP approach). The problem can be formally stated as follows: Find the optimal partial coalition of K relay nodes ($K \subseteq N$) that form relaying pairs/chains with each other while the remaining $N - K$ nodes work on their own following NRP, such that the total energy consumption over N nodes is minimized and no nodes energy cost in the coalition exceeds its cost through NRP.

We model the problem as a constrained minimum cost matching problem on a complete bipartite graph $B = (V, U, E)$, where $|V| = |U| = N$, V represents the N nodes as transmitters, U represents the same nodes as potential relays (receivers) receiving information from the transmitters to be relayed. Consider edge (i, j) in the graph, where $i \neq j$. (i, j) represents a potential relay pairing with i as a transmitter and j acting as its relay (but not necessarily vice versa), in other words the nodes form a relaying chain $i \to j \to k \to \ldots \to i$. We assume that time is divided into N slots (or equivalently communication over N orthogonal frequencies), one slot for each node as a source. Each such slot is further divided into two half-slots, during the first half-slot i transmits to j and the destination while during the second slot j relays the received message of the first half-slot to the destination. The power control algorithm for nodes i and j will be exactly as characterized in Theorem 1 with the modification that since i is not necessarily j's relay, we don't consider the outage probability and power control results for j's information.

Thus each edge (i, j) is parameterized by two costs: c^T_{ij} and c^R_{ij}. c^T_{ij} represents the transmission energy cost to node i of choosing node j as a relay and transmitting its information to j during the first half-slot while c^R_{ij} represents the energy cost to j for receiving and relaying node i's information. Finally, edges of the type (i, i) in the graph represent nodes not included in the relaying coalition. Thus we set $2c^T_{ii} = 2c^R_{ii} = c_{ii} = E\left[2\tilde{P}_{i,D}\right]$, which is the average energy cost to meet the outage probability requirement $P_{i,out}$ for the non-relaying case (i.e. NRP). Given CSI and statistics of $h_{i,j}$, the values of c^T_{ij} and c^R_{ij} are obtained off-line by using the power-control algorithm of Theorem 1.

The solution to the optimal relay pairing with selfish constraints problem is the perfect matching on the bipartite graph G that satisfies the integer programming formulation:

$$\min \sum_i \sum_j x_{ij} \left(c_{ij}^T + c_{ij}^R\right), \quad i,j = 1,2,\ldots,N \quad (5)$$

$$\text{s.t} \sum_{j \neq i} x_{ij} c_{ij}^T + \sum_{k \neq i} x_{ki} c_{ki}^R \leq c_{ii} \quad (6)$$

$$\sum_i x_{ij} = 1 \quad (7)$$

$$\sum_j x_{ij} = 1 \quad (8)$$

$$x_{ij} \in \{0,1\}, \qquad \forall i,j \quad (9)$$

The x_{ij} represent the $0-1$ integer variables and Eq. 6 represents the selfish constraint: a node will not participate in relaying unless the total cost of relaying is lower than the cost of the individual (NRP) alternative. M_{opt} represents the perfect matching obtained as the solution to the above integer programming problem. Adding this constraint makes the problem of finding M_{opt} much harder computationally, as compared to the unconstrained optimal relay pairing case. It can be shown that the selfishly constrained version (and its variants, where the right hand side of Equation 6 is replaced by αc_{ii}, where $\alpha > 0$ is any positive constant) is NP-Hard. While branch-and-bound and other Integer Linear Programming (ILP) techniques can be used to obtain the optimal pairing solution, the running time is exponential in the worst case. Therefore, in this paper, we are interested in finding polynomial-time approximation algorithms for Eq. 5. Such approximations are bi-criteria approximations that optimize the energy cost of relaying while violating the selfishness constraint by as small a factor as possible. We describe a simple (1,2) bi-criteria approximation below. The energy cost provided by the approximation is at most that of the optimal relay pairing, while the constraints are violated by at most a factor of two. The running time of the approximation algorithm is polynomial, as it requires the solution of a relaxed linear programming instance of the original integer programming problem.

Consider the following Linear Programming relaxation of Eq. 5-Eq. 9.

$$\min \sum_i \sum_j x_{ij} \left(c_{ij}^T + c_{ij}^R\right), \quad i,j = 1,2,\ldots,N$$

$$\text{s.t} \sum_{j \neq i} x_{ij} c_{ij}^T + \sum_{k \neq i} x_{ki} c_{ki}^R + (x_{ii} - 1) c_{ii} \leq 0$$

$$\sum_i x_{ij} = 1$$

$$\sum_j x_{ij} = 1$$

$$0 \leq x_{ij} \leq 1, \qquad \forall i,j$$

$$x_{ij} = 0 \qquad \text{if } c_{ij}^T > c_{ii}$$

$$x_{ij} = 0 \qquad \text{if } c_{ij}^R > c_{jj}$$

Let C be the objective function value returned by the Linear Program (LP) in Eq. 10. Clearly $C \leq C_{opt}$ where C_{opt} is the cost of the optimal relay pairing defined by M_{opt}, since the LP of Eq. 10 includes integer values of x_{ij} in its feasible set and M_{opt} cannot include edges with $c_{ij}^T > c_{ii}$ or $c_{ij}^R > c_{jj}$. Furthermore the solution to the LP forms an exact fractional matching on the nodes of the bipartite graph B (i.e. the sum of the x_{ij}'s exactly add to 1 at each node in the graph). As shown in [7], this implies there exists an integral matching M in B of cost at most $C \leq C_{opt}$. Since M consists of edges whose c^T and c^R costs are at most that of the corresponding NRP cost (i.e. c_{ii}), therefore we must have $c_{ij}^T + c_{ki}^R \leq 2c_{ii}$ for all edges (k,i) and (i,j) in the matching M. Hence M is a (1,2)-approximation to the optimal relay pairing with selfish constraints problem, i.e. the energy cost of the approximation is no more than that of the optimal while violating the constraints by at most a factor of 2. For details on the exact algorithm to find the integral matching, see [7].

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