# JOINT ENERGY AND LOCATION OPTIMIZATION FOR RELAY NETWORKS WITH DIFFERENTIAL MODULATION

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# ABSTRACT

The optimum resource allocation in communication systems is critical to enhance their performance and efficiency. In wireless networks, relay transmissions can enable cooperative diversity by forming virtual antenna arrays. In this paper, we consider resource allocation which minimizes the average system error rate not only by the power optimization, but also by the location optimization for systems with arbitrary number of relays. Differential modulation which bypasses the channel estimation at the receiver is investigated using the decode-and-forward protocol. Analytical and simulated comparisons confirm that the optimized systems provide considerable improvement over un-optimized ones, and that the minimum error rate can be achieved via joint energy-location optimization.

*Index Terms*— Cooperative systems, Differential phase shift keying, Resource management

## 1. INTRODUCTION

Virtual antenna arrays formed by distributed wireless network nodes can provide cooperative diversity gain without imposing antenna packing limitations [5,9]. Majority of existing works on relay networks focuses on coherent demodulation based on the availability of the channel state information (CSI) at both the relays and the destination node (see e.g., [5,9]). However, accurate CSI estimation can induce considerable communication overhead and transceiver complexity as the number of relay nodes increases. In addition, CSI estimation may not be feasible when the channel is rapidly time-varying. To bypass channel estimation, cooperative diversity schemes obviating CSI have been recently introduced. These relay networks rely on noncoherent or differential modulations, including conventional frequency-shift keying (FSK) and differential phase-shift keying (DPSK) [4, 11], as well as space-time coding (STC-)based ones [6, 10]. Performance of coherent and differential schemes is compared in [12].

To improve the error performance and to enhance the energy efficiency of relay networks, optimum resource allocation strategies recently emerge as an important problem attracting increasing research interests (see e.g., [1-3, 8]). These works are based on different relaying protocols, under various optimization criteria, and with different levels of CSI. However, all of them only consider the power allocation.

In this paper, we consider a relay network with arbitrary number of relays. More importantly, we treat the resource allocation as a two-dimensional optimization problem: the optimization of the energy (power) distribution and the optimization of the relay location. Equally attractive is that our analysis is tailored for relay systems with differential modulation, which can reduce the receiver complexity by bypassing channel estimation. To enable the resource optimization, we first derive an upper bound of the overall symbol error rate (SER) performance for relay networks employing the decode-and-forward (DF) protocol. The energy and location optimization will then be carried out based on this performance bound. We show that under the constraints of the total energy per symbol and the source-destination distance, the optimum SER performance can be achieved through the joint energy and location optimization.

The rest of this paper is organized as follows. The system model is introduced in Section 2. In Section 3, an SER upper bound is established for a setup with arbitrary number of relays. The derivation of the optimum energy and distance allocation, joint optimization, and their discussions are presented in Section 4. Summarizing remarks are given in Section 5.

*Notation:* We use  $(\cdot)^*$  for conjugate,  $\mathbb{E}[\cdot]$  for expectation,  $\Re\{\cdot\}$  for the real part, and := for "is defined as".  $\mathcal{CN}(\mu, \sigma^2)$  represents the complex Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ .

## 2. SYSTEM MODEL

Consider a network setup with one source node s, L relay nodes  $\{r_k\}_{k=1}^{L}$  and one destination node d. Each node is equipped with a switch that controls its transmit/receive mode to enable half-duplex communications.

# 2.1. Relaying Protocol and Channel Modeling

We consider the decode-and-forward (DF) relaying protocol, in which the relay nodes demodulate the signal from the source node, then remodulate and forward to the destination node. During the first segment of each symbol duration, the source node broadcasts the first symbol to all relay nodes. Next, each relay transmits the remodulated signal to the destination during their distinct segments within the rest of the symbol duration. This is essentially a time-division multiplexing (TDM) scheme, but the analysis and results are readily applicable to frequency-division multiplexing (FDM) and codedivision multiplexing (CDM).

With the *n*th phase-shift keying (PSK) symbol being denoted as  $s_n = e^{j2\pi c_n/M}$ ,  $c_n \in \{0, 1, ..., M-1\}$ , the corresponding transmitted signal from the source is  $x_n^s = x_{n-1}^s s_n$  with initial value  $x_0^s = 1$ .

The encoded signal is broadcast to all relays, and the received signal at the kth relay is given by

$$y_n^{r_k,s} = \sqrt{\mathcal{E}_s} h_n^{r_k,s} x_n^s + z_n^{r_k}, \ k = 1, 2, ..., L,$$
(1)

where  $\mathcal{E}_s$  is the energy per symbol at the source node, the fading coefficient of the channel between s and  $r_k$  during the nth symbol duration is  $h_n^{r_k,s} \sim \mathcal{CN}(0, \sigma_{h_{r_k,s}}^2)$  and the noise component is  $z_n^{r_k} \sim \mathcal{CN}(0, \mathcal{N}_{r_k})$ . The received signal is then differentially demodulated and remodulated independently at each relay  $r_k$ . Let  $x_n^{r_k}$  denote the nth transmitted symbol from the kth relay, k = 1, 2, ..., L, then the received signal at the destination corresponding to each relay node is given by

$$y_n^{d,r_k} = \sqrt{\mathcal{E}_{r_k}} h_n^{d,r_k} x_n^{r_k} + z_n^d, \ k = 1, 2, ..., L,$$
 (2)

where  $\mathcal{E}_{r_k}$  is the energy per symbol at the *k*th relay node, the fading coefficient of the channel between  $r_k$  and *d* during the *n*th symbol

duration is  $h_n^{d,r_k} \sim \mathcal{CN}(0, \sigma_{h_{d,r_k}}^2)$  and the noise component is  $z_n^d \sim \mathcal{CN}(0, \mathcal{N}_d)$ . Accordingly, we can find the received instantaneous signal-to-noise ratio (SNR) and the average received SNR between the transmitter j and the receiver i as  $\gamma_{i,j} = (|h_n^{i,j}|^2 \mathcal{E}_j)/\mathcal{N}_i$  and  $\bar{\gamma}_{i,j} = (\sigma_{h_{i,j}}^2 \mathcal{E}_j)/\mathcal{N}_i$ , respectively, where  $i, j \in \{s, r_k, d\}$ .

## 2.2. Differential Demodulation and Decision Rules

To derive the demodulation, decision and diversity combining rules, let us begin with the received signal at the relay or the destination node  $y_n = h_n x_n + z_n$ , which is extracted from Eqs. (1) and (2) by dropping the superscripts. Using the differential encoding, the received signal can be re-expressed as:

$$y_n = h_n(x_{n-1}s_n) + z_n = y_{n-1}s_n + z'_n$$
,

where  $z'_n = z_n - z_{n-1}s_n$ . For *M*-ary PSK symbols, it follows that  $\mathbb{E}[s_n^*s_n] = 1$ . Hence, the conditional distribution of  $y_n$  is complex Gaussian with mean  $y_{n-1}s_n$  and variance  $2\mathcal{N}_i$ . As a result, we obtain the log likelihood function (LLF) of  $y_n$  as:

$$l_m^{i,j}(y_n) := \ln p_{y_n|s_n}(y_n|I_m) = \Re\{(y_n)^* y_{n-1}I_m\}, \qquad (3)$$

where  $i, j \in \{s, r_k, d\}$ ,  $I_m = e^{j2\pi m/M}$  and  $m \in \{0, 1, ..., M-1\}$ .

At the *k*th relay node, the differential demodulator is then straightforward:

$$\hat{s}_n^{r_k} = e^{j2\pi m'/M} \colon m' = \arg\max_m \, \Re\{(y_n^{r_k,s})^* y_{n-1}^{r_k,s} I_m\}.$$

At the destination node, however, there are L different LLF's corresponding to the L transmitted signals from the relays:

$$l_m^{d,r_k}(y_n) = \Re\{\left(y_n^{d,r_k}\right)^* y_{n-1}^{d,r_k} I_m\}, \ k = 1, 2, \dots, L.$$
(4)

If the (full or partial) CSI is known at the relays and the destination node, then it is possible to combine the LLF by capturing the detection error at the relay node according to the so-termed transition probability (see e.g., [4]). However, keeping in mind that differential modulation is considered in the first place because of its capability of bypassing channel estimation, we will focus on the scenario where *no CSI is available*. Under this circumstance, the LLF's in Eq. (4) have to be combined with equal weights. Accordingly, the decision rule at the destination node can be readily obtained as:

$$\hat{s}_n^d = e^{j2\pi m'/M} \colon m' = \arg\max_m \sum_{k=1}^L \Re\{(y_n^{d,r_k})^* y_{n-1}^{d,r_k} I_m\}.$$

With no channel information assumed at either the relays or the destination node, this decision rule turns out to be the differential detection with postdetection equal gain combining (EGC) [14, Chapter 9].

#### 3. ERROR PERFORMANCE ANALYSIS

Let us denote the average SER at the *k*th relay node as  $P_{e,r_k}$ . For differential *M*-ary PSK (DMPSK) signaling, the  $s - r_k$  link SER  $P_{e,r_k}$  can be obtained as in [14, Chapter 8]. At the destination, the signals from the *L* relays are combined to make a decision. Conditioned on that the symbol  $s_n$  is correctly demodulated at all relay nodes, the conditional SER  $P_{e,d}$  can be obtained by applying the results for *L*-diversity branch reception of *M*-phase signals in [13, Appendix C]. For DBPSK,  $P_{e,r_k}$  and  $P_{e,d}$  can be simplified to

$$P_{e,r_k} = 1/\left[2(1+\bar{\gamma}_{r_k,s})\right] , \qquad (5)$$

and

$$P_{e,d} = \frac{1}{2} \left[ 1 - \mu \sum_{k=0}^{L-1} {\binom{2k}{k}} \left(\frac{1-\mu^2}{4}\right)^k \right],$$
 (6)

respectively, where  $\mu = \bar{\gamma}_{d,r_k} / (1 + \bar{\gamma}_{d,r_k})$ .

Using the unconditional SER  $P_{e,r_k}$  at the relays and the conditional SER  $P_{e,d}$  at the destination, we formulate an upper bound on the overall average error performance, namely the unconditional SER  $P_e$  at the destination, as follows:

**Proposition 1 :** With any given  $P_{e,r_k}$  and  $P_{e,d}$ , an upper bound on  $P_e$  can be found as :

$$P_e \le \bar{P}_e = 1 - \prod_{k=1}^{L} (1 - P_{e,r_k})(1 - P_{e,d}).$$
(7)

For detailed proof, please refer to [7]. The tightness of the error bound in Proposition 1 is determined by the gap between the true SER and its bound, i.e.,  $\Delta P_e := \bar{P}_e - P_e$ . For DBPSK with a single relay, this gap can be easily obtained. However, for  $L \ge 2$ , all possible errors have to be considered for both the s - r and r - dlinks, which renders  $\Delta P_e$  analytically untractable. Intuitively, as the L increases,  $\Delta P_e$  also increases since there is an increasing chance that detection errors at the relay nodes do not lead to a detection error at the destination node. Several simulated examples on the tightness of Eq. (7) can be found in [7].

#### 4. OPTIMUM RESOURCE ALLOCATION

In this section, we will investigate the effects of resource allocation on the SER performance. For analytical tractability, we consider an idealized *L*-relay system with all relay nodes located at the same distance from the source and the destination nodes; that is,  $D_{s,r_k} = D_{s,r}$  and  $D_{r_k,d} = D_{r,d}$ ,  $\forall k$ . It is then reasonable to assign equal energies to all relay nodes  $\mathcal{E}_{r_k} = \mathcal{E}_r$ ,  $\forall k$ . To carry out the optimization, we will also make use of the relationship between the average power of the channel fading coefficient  $\sigma_{h_{i,j}}^2$  and the inter-node distance  $D_{j,i}$  as follows:

$$\sigma_{h_{i,j}}^2 = C \cdot D_{j,i}^{-\nu}, \ i, j \in \{s, r, d\} ,$$
(8)

where  $\nu$  is the path loss exponent of the wireless channel and C is a constant which we henceforth set to 1 without loss of generality.

## 4.1. Optimum energy allocation

**Problem Statement:** For any given source, relay and destination node locations  $(D_{s,r} \text{ and } D_{r,d}, \text{ or equivalently } \sigma_{h_{r,s}}^2 \text{ and } \sigma_{h_{d,r}}^2)$ , and the total energy per symbol  $\mathcal{E}$ , determine the optimum energy allocation  $\mathcal{E}_s$  and  $\mathcal{E}_r$  which minimize  $\overline{P}_e$  in Eq. (7) while satisfying:

$$\mathcal{E}_s + \sum_{k=1}^{L} \mathcal{E}_{r_k} = \mathcal{E}_s + L \mathcal{E}_r = \mathcal{E} .$$
(9)

Without loss of generality, assuming that all noise components are independent and identically distributed (i.i.d) with  $\mathcal{N}_{r_k} = \mathcal{N}_d = \mathcal{N}_0$ . Then, by defining the total SNR,  $\rho := \mathcal{E}/\mathcal{N}_0$ , the transmit SNR at the source node  $\rho_s := \mathcal{E}_s/\mathcal{N}_0$  and the transmit SNR at the relay nodes  $\rho_r := \mathcal{E}_r/\mathcal{N}_0$ , one can re-express the energy constraint as the SNR constraint :

$$\rho = \rho_s + L\rho_r \ . \tag{10}$$

Using Eq. (8), the average received SNR at the relay and destination nodes can be expressed in terms of the transmit SNR as:

$$\bar{\gamma}_{r,s} = \rho_s \sigma_{h_{r,s}}^2 = \rho_s D_{s,r}^{-\nu} \text{ and } \bar{\gamma}_{d,r} = \rho_r \sigma_{h_{d,r}}^2 = \rho_r D_{r,d}^{-\nu}$$
 (11)

To gain some insights, we start from a single-relay setup and establish the following result:



Fig. 1. SER comparison between relay systems with and without energy optimization ( $\rho = 10$ dB,  $\nu = 4$ , DBPSK).

**Proposition 2 :** For a single-relay setup with L = 1, at given s - r and r - d distances  $D_{s,r}$  and  $D_{r,d}$ , and under the total energy constraint in Eq. (9), the optimum energy allocation  $\mathcal{E}_s$  should satisfy :

$$\rho_{s} = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \rho \Leftrightarrow \mathcal{E}_{s} = \frac{D_{r,d}^{-\nu/2}}{D_{s,r}^{-\nu/2} + D_{r,d}^{-\nu/2}} \cdot \mathcal{E} .$$
(12)

and correspondingly,  $\mathcal{E}_r = \mathcal{E} - \mathcal{E}_s$ .

This solution is achieved by solving the first order conditions under medium-to-high SNR values. Treating the SER bound  $\bar{P}_e$  as a function of  $\bar{\gamma}_{r,s}$  and  $\bar{\gamma}_{d,r}$ , we have the first order conditions for the optimum solution

$$\partial \bar{P}_e / \partial \bar{\gamma}_{r,s} - \lambda D_{s,r}^{\nu} = 0 , \partial \bar{P}_e / \partial \bar{\gamma}_{d,r} - \lambda D_{r,d}^{\nu} = 0 ,$$
(13)

$$\rho - (\bar{\gamma}_{r,s} D_{s,r}^{\nu} + \bar{\gamma}_{d,r} D_{r,d}^{\nu}) = 0 ,$$

where  $\lambda$  is the Lagrange multiplier. The detailed proof of Eq. (12) for the optimum energy allocation can be found in [7]. Interestingly, this solution coincides with the optimum power allocation obtained by minimizing the outage probability [8, (8)] and by minimizing the error probability bound of the coherent (de-)modulation [1, (8)] with a single-relay transmission. From Eq. (12), it readily follows that the energy allocation ratio between the source and the relay is

$$\frac{\mathcal{E}_s}{\mathcal{E}_r} = \left(\frac{D_{s,r}}{D_{r,d}}\right)^{\nu/2} \,. \tag{14}$$

Eq. (14) explicitly reveals that the optimum energy allocation heavily hinges upon the inter-node distances. In addition, the path loss exponent of the wireless channel,  $\nu$ , also affects the optimum energy allocation. Interestingly, the  $\mathcal{E}_s/\mathcal{E}_r$  ratio is linear in  $D_{s,r}/D_{r,d}$  only when  $\nu = 2$ .

For  $L \geq 2$ , the first order conditions in Eq. (13) obtained by differentiating the SER bound  $\bar{P}_e$  have complicated forms, which render analytical solutions impossible. Fortunately, the SER bound  $\bar{P}_e$  as in Proposition 1 still allows for a *numerical search*, as opposed to Monte Carlo simulations needed otherwise.

To verify the advantage of the energy optimization, we plot the SER of the relay system with and without energy optimization. A one-dimensional setup is considered; that is  $D_{s,r}+D_{r,d}=D_{s,d}$ . The system parameters are:  $\rho = 10$ dB,  $D_{s,d} = 1$ , and L = (1, 2, 3, 4). In the system without energy optimization, a uniform energy allocation



Fig. 2. SER comparison between relay systems with and without location optimization ( $\rho = 10$ dB,  $\nu = 4$ , DBPSK).

is employed; that is,  $\rho_s = \rho_r = \rho/(L+1)$  at any  $D_{s,r}$ . From Fig. 1, we observe that, as L increases, the SER performance does not always improve unless the energy optimization is performed, and the energy-optimized system universally outperforms the un-optimized system. Interestingly, notice that the minima of the energy-optimized SER curves almost coincide with those of the un-optimized ones. This implies that the near-optimum SER can be achieved even with the uniform energy allocation across the source and relay nodes, provided that the relay location is carefully selected.

# 4.2. Optimum distance allocation

**Problem Statement:** For any given transmit energies at the source and relay nodes ( $\mathcal{E}_s$  and  $\mathcal{E}_r$ , or equivalently  $\rho_s$  and  $\rho_r$ ), and the path loss exponent  $\nu$  of the wireless channel, determine the optimum location of the relays,  $D_{s,r}$ , which minimizes  $\bar{P}_e$  in Eq. (7) while satisfying  $0 < D_{s,r} < D_{s,d}$ .

Starting with the single-relay (L = 1) setup and applying the high-SNR approximation, we establish the following result:

**Proposition 3 :** For a single-relay setup with L = 1 and the sourcedestination distance  $D_{s,d}$ , and let  $\mathcal{E}_s$  and  $\mathcal{E}_r$  denote the prescribed transmit energy levels at the source and relay nodes, respectively, the optimum location of the relay is

$$D_{s,r} = \frac{\rho_s^{1/(\nu-1)}}{\rho_s^{1/(\nu-1)} + \rho_r^{1/(\nu-1)}} \cdot D_{s,d} , \qquad (15)$$

and accordingly,  $D_{r,d} = D_{s,d} - D_{s,r}$ .

Similar to the previous subsection, this solution is achieved by solving the first order conditions, in which we treat  $\bar{P}_e$  as a function of distances  $D_{s,r}$  and  $D_{r,d}$ . With the one-dimensional setup, Proposition 3 can be represented as

$$\frac{D_{s,r}}{D_{r,d}} = \left(\frac{\rho_s}{\rho_r}\right)^{1/(\nu-1)} = \left(\frac{\mathcal{E}_s}{\mathcal{E}_r}\right)^{1/(\nu-1)} .$$
(16)

Interestingly, Eq. (16) bears a very similar form as its counterpart for the optimum energy allocation in Eq. (14). In fact, when the path loss exponent  $\nu = 2$ , Eq. (16) is essentially identical to Eq. (14). For general values of  $\nu$ , however, these two relationships are quite different. Such a discrepancy is actually very reasonable, because Eqs. (14) and (16) result from two distinct optimization problems. With the SER bound  $\bar{P}_e$  being a two-dimensional function, the two optimizations are carried out on uncorrelated dimensions. For general L values, the path loss exponent  $\nu$  renders it impossible to derive the analytical solution to the optimum location problem, even with the high SNR approximation. One can resort to the numerical search using the SER bound in Proposition 1.

In Fig. 2, we verify the advantage of the optimum distance allocation by comparing the SER with and without location optimization. In the system without location optimization, the relays are placed at the midpoint of the source-destination link. Similar to the energy optimization case, Fig. 2 confirms that the location-optimized system universally outperforms the un-optimized system. Different from the energy optimization case, however, the SER performance always improves with L even in the un-optimized systems. The curves in Fig. 2 also exhibit more flatness compared with the ones in Fig. 1. This implies that the system SER is more sensitive to the location distribution than to the energy distribution. In addition, the minima of the location-optimized SER curves are far from those of the un-optimized ones, except for the L = 1 case (see Fig. 2). This indicates that placing the relay nodes at the midpoint *cannot* achieve the minimum SER even with careful allocation of the source and relay energies, for any L > 1.

# 4.3. Joint Optimization

So far, we have been focusing on the energy optimization and location optimization separately. Now let us consider the joint energy and location optimization. Mathematically, the joint energy and location optimization can be achieved by finding the common solution which satisfies the first order conditions for optimum energy solution and location solution simultaneously. Correspondingly, this solution provides the global optimum which minimizes the SER. For L = 1, we can readily obtain the global solution from Eqs. (14) and (16), which gives us  $D_{s,r} = D_{r,d} = 0.5$  with  $\rho_s / \rho = \rho_r / \rho = 0.5$ ,  $\forall \nu$ . For general L, the global optimization can be obtained by carrying out a two-dimensional numerical search iteratively. The searching steps are as follows :

- Step 1. For the given energy allocation, find the optimum location which is SER-minimizing. If the location differs from the original location, continue to Step 2; otherwise, stop.
- Step 2. For the given location, find the optimum energy allocation. Continue to Step 1 if the energy allocation differs from the original, and stop otherwise.

Fig. 3 shows the SER performance surface when L = 3 with  $\rho = 10$ dB and  $\nu = 4$ . The optimization in Figs. 1 and 2 can be obtained by taking the minimum value along the  $D_{s,r}$  axis and  $\rho_s/\rho$  axis from Fig. 3, respectively. Using the above steps, the global minimum value can be obtained. This value provides the joint energy and location optimization.

#### 5. CONCLUSIONS

In this paper, we investigated the optimum energy distribution and optimum location of relays in a wireless system with arbitrary number of relays employing differential demodulation. A two-dimensional optimization is studied on the basis of minimizing an upper bound on the average SER, which we derived for the decode-and-forward cooperative protocol. Our simulations and numerical examples show that both the energy and location optimizations provide remarkable SER advantages. We have shown that the minimum SER can be achieved by the joint energy-location optimization, and that the location optimization may be more critical than the energy optimization. In other words, the differential relay system with uniform energy distribution can achieve near-optimum SER by appropriately choosing the relay location; while a system with relays sitting at the midpoint between the source and the destination cannot approach the optimum SER even with optimized energy distribution.



**Fig. 3**. Performance surface versus  $\rho_s/\rho$  and  $D_{s,r}$  ( $\rho = 10$ dB,  $\nu = 4, L = 3$ , DBPSK).

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