OPTIMAL TRANSMIT STRATEGIES IN MULTI-ANTENNA BIDIRECTIONAL RELAYING

Tobias J. Oechtering and Holger Boche

Technical University of Berlin, Heinrich-Hertz-Chair for Mobile Communications,
Einsteinufer 25, 10587 Berlin, Germany

ABSTRACT

We study the transmit strategies in a MIMO bidirectional relaying scenario with individual power constraints. In two phases a half-duplex relay node decodes-and-forwards the signals of two nodes. Each node has multiple antennas. We deduce the transmit strategy in the first phase from the general Gaussian MIMO-MAC. Since each node knows a priori the interference of its own message in the second phase interference-free reception is achieved. Therefore, the optimal relay transmit strategy is given by two point-to-point water-filling solutions which are coupled by the relay power distribution. In the large SNR, the sum of any bidirectional rate tuple on the boundary of the rate region is asymptotically proportional with the minimum spatial degree of both MIMO channels.

Index Terms— Mobile communication, Bidirectional relaying, Decode-and-forward, MIMO system, Optimal transmit strategy

1. INTRODUCTION

Future cellular and ad hoc wireless systems should offer connectivity almost everywhere, which is great engineering challenge in scenarios where the direct link does not have the desired quality, e.g. due to shadowing or distance. Therefore, recently there has been an increasing interest in cooperative protocols for wireless systems where one or more relay nodes realize range extension by multi-hop communication. On the other hand, already in 1961 Shannon introduced the problem of the two-way communication channel, which is nowadays regarded as the first network information theory problem [1].

We consider a scenario where bidirectional communication between two nodes is established by one half-duplex relay node, i.e. the relay cannot transmit or receive at the same time using the same frequency. At the latest after [2] it is well known that MIMO communication systems have the ability to reach higher transmission rates than single-antenna scenarios. Therefore, in this work we extend a spectral efficient protocol, which was first proposed in [3], to the multi-antenna processing. In [4], we studied the combinatorial properties of the achievable rate region for nodes with one antenna element. Unfortunately, due to the complicated structure of the rate regions in the MIMO case the combinatoric cannot be given in closed form. Nevertheless, with the results presented here it is possible to characterize the optimal transmit strategies for any rate tuple using standard methods for convex optimization. Furthermore, we will see that the asymptotic behavior in the large SNR regime is determined by the minimum spatial degree of both MIMO channels.

In [5], an iterative water-filling algorithm is proposed which finds an optimal input distribution for the MIMO-MAC under the sum-rate criterion and individual power constraints. [6] is a helpful study on the optimal transmit covariance matrices for any rate tuple on the boundary of the MIMO-MAC region. Another way to realize spatial multiplexing gains in MIMO relay networks is shown in [7], where the equivalent MIMO channel orthogonalize in the large relay node limit by simple matched-filtering at the relay nodes.

In the following we present the optimal transmit strategies for the bidirectional MIMO relaying, which are derived from the Gaussian MIMO-MAC and point-to-point Gaussian MIMO channel.

Notation: Bold and calligraphic letters denote matrices or vectors and sets respectively; $\mathbb{R}_+$ specifies the set of non-negative real numbers; $||\cdot||_1$ denotes the L^1 vector norm; any $\log$ is the logarithm of basis two.

2. ACHIEVABLE RATE REGION

We assume a perfectly synchronized three node network, where bidirectional communication is realized by a decode-and-forward half duplex relay node with N_R antennas. Accordingly, node 1 and 2 are equipped with N_1 and N_2 antennas respectively. The communication is performed slot-wise, while one time-slot is divided in two phases, namely the *multiple access* (MAC) and *broadcast* (BC) phase of equal duration. The separation in two phases with none cooperative transmitters makes the problem tractable but need not be optimal.

Let $\boldsymbol{H}_1 \in \mathbb{C}^{N_\mathrm{R}\times N_1}$ and $\boldsymbol{H}_2 \in \mathbb{C}^{N_\mathrm{R}\times N_2}$ denote the discrete-time reciprocal flat-fading MIMO channels between the relay node and node 1 and two respectively. The rank $r_1 = \mathrm{rank}[\boldsymbol{H}_1] \leq \min\{N_1, N_\mathrm{R}\}$ and $r_2 = \mathrm{rank}[\boldsymbol{H}_2] \leq \min\{N_\mathrm{R}, N_2\}$ denote the number of spatial degree of the channels, i.e. each non-zero eigenmode of the channel can support a data stream. We assume perfect channel knowledge at any node. The transmit powers of each node are restricted by peak power constraints $\hat{P}_k$, $k \in \{1, 2, R\}$ respectively. Furthermore, the reception at each antenna of every node is distorted by independent additive white Gaussian noise $\boldsymbol{n}_1, \boldsymbol{n}_2$, and $\boldsymbol{n}_\mathrm{R}$ with equal covariance matrices $\sigma^2 \boldsymbol{I} = \rho^{-1}\boldsymbol{I}$. A generalization to more general noise covariance matrices is straight forward.

MAC phase: In the first phase, node 1 and 2 transmit their information for each other to the relay node. The encoding and decoding is performed as in the classical discrete-time Gaussian MIMO-MAC channel. The messages at node 1 and 2 are encoded with rate $R_{\overrightarrow{1\mathrm{R}}}$ and $R_{\overrightarrow{2\mathrm{R}}}$ and mapped onto transmit signals $\boldsymbol{x}_1 \in \mathbb{C}^{N_1}$ and $\boldsymbol{x}_2 \in \mathbb{C}^{N_2}$ with transmit covariance matrices $\boldsymbol{Q}_k = \mathbb{E}\{\boldsymbol{x}_k \boldsymbol{x}_k^H\}$ with $\mathrm{tr}[\boldsymbol{Q}_k] = P_k \leq \hat{P}_k$ for $k = 1, 2$ respectively. For a given time-instant, the relay receives an additive noise distorted superposition of the transmitted signals

$$\boldsymbol{y}_\mathrm{R} = \boldsymbol{H}_1 \boldsymbol{x}_1 + \boldsymbol{H}_2 \boldsymbol{x}_2 + \boldsymbol{n}_\mathrm{R}.$$

If the rate tuple $\boldsymbol{R}_\mathrm{MAC} = [R_{\overrightarrow{1\mathrm{R}}}, R_{\overrightarrow{2\mathrm{R}}}]$ of node 1 and 2 is within the MIMO-MAC rate region

$$\mathcal{R}_\mathrm{MAC} = \Big\{[R_{\overrightarrow{1\mathrm{R}}}, R_{\overrightarrow{2\mathrm{R}}}] \in \mathbb{R}_+^2 : R_{\overrightarrow{1\mathrm{R}}} \leq \hat{R}_{\overrightarrow{1\mathrm{R}}}, \\ R_{\overrightarrow{2\mathrm{R}}} \leq \hat{R}_{\overrightarrow{2\mathrm{R}}}, R_{\overrightarrow{1\mathrm{R}}} + R_{\overrightarrow{2\mathrm{R}}} \leq \hat{R}_\Sigma) \Big\}$$

with well-known MIMO-MAC boundaries

$$\hat{R}_{\overrightarrow{1\mathrm{R}}} = \log\det\big[\boldsymbol{I}_{N_\mathrm{R}} + \rho \boldsymbol{H}_1 \boldsymbol{Q}_1 \boldsymbol{H}_1^H\big], \tag{1a}$$
$$\hat{R}_{\overrightarrow{2\mathrm{R}}} = \log\det\big[\boldsymbol{I}_{N_\mathrm{R}} + \rho \boldsymbol{H}_2 \boldsymbol{Q}_2 \boldsymbol{H}_2^H\big], \tag{1b}$$
$$\hat{R}_\Sigma = \log\det\big[\boldsymbol{I}_{N_\mathrm{R}} + \rho \boldsymbol{H}_1 \boldsymbol{Q}_1 \boldsymbol{H}_1^H + \rho \boldsymbol{H}_2 \boldsymbol{Q}_2 \boldsymbol{H}_2^H\big] \tag{1c}$$

it is assumed that the relay node can perfectly decode the transmitted messages from node 1 and 2.

BC phase: In the second phase, the relay broadcasts the previously received messages. Since each node knows its own message the relay node encode the independent messages from node 1 and 2 like in a classical Gaussian MIMO point-to-point link with rate $R_{\overrightarrow{\mathrm{R}2}}$ and rate $R_{\overrightarrow{\mathrm{R}1}}$ respectively. It therefore maps the message from node 1 onto the transmit signal $\boldsymbol{w}_1$ with transmit covariance matrix $\boldsymbol{Q}_{R,1}^T = \mathbb{E}\{\boldsymbol{w}_1 \boldsymbol{w}_1^H\}$ and the message from node 2 onto the transmit signal $\boldsymbol{w}_2$ with transmit covariance matrix $\boldsymbol{Q}_{R,2}^T = \mathbb{E}\{\boldsymbol{w}_1 \boldsymbol{w}_1^H\}$. Then the relay transmits the superposition of both, i.e. $x_\mathrm{R} = \boldsymbol{w}_1 + \boldsymbol{w}_2$. Finally, let $\beta \in [0, 1]$ denote the relay power distribution with $\mathrm{tr}[\boldsymbol{Q}_{R,1}] \leq \beta \hat{P}_\mathrm{R}$ and $\mathrm{tr}[\boldsymbol{Q}_{R,2}] \leq (1 - \beta)\hat{P}_\mathrm{R}$.

We assume reciprocal channels H_1 and H_2. Therefore, the received signal at node 1 and 2 are given by

$$\boldsymbol{y}_k = \boldsymbol{H}_k^T \boldsymbol{x}_\mathrm{R} + \boldsymbol{n}_k, \quad k = 1, 2.$$

Each node receives its own message as interference. Before decoding the unknown message, each node subtracts the interference caused by its own message, i.e. effectively, we achieve interference-free reception. Accordingly, we assume error-free decoding if the rate tuple $\boldsymbol{R}_\mathrm{BC} = [R_{\overrightarrow{\mathrm{R}2}}, R_{\overrightarrow{\mathrm{R}1}}]$ is within the following MIMO-BC rate region of the BC phase

$$\mathcal{R}_\mathrm{BC} = \Big\{[R_{\overrightarrow{\mathrm{R}2}}, R_{\overrightarrow{\mathrm{R}1}}] \in \mathbb{R}_+^2 : R_{\overrightarrow{\mathrm{R}2}} \leq \hat{R}_{\overrightarrow{\mathrm{R}2}}(\beta), \\ R_{\overrightarrow{\mathrm{R}1}} \leq \hat{R}_{\overrightarrow{\mathrm{R}1}}(\beta),\, \beta \in [0, 1] \Big\}$$

with the parametrized boundary

$$\hat{R}_{\overrightarrow{\mathrm{R}2}}(\beta) = \log\det\big[\boldsymbol{I}_{N_2} + \rho \boldsymbol{H}_2^H \boldsymbol{Q}_{R,1} \boldsymbol{H}_2\big] \tag{2a}$$
$$\hat{R}_{\overrightarrow{\mathrm{R}1}}(\beta) = \log\det\big[\boldsymbol{I}_{N_1} + \rho \boldsymbol{H}_1^H \boldsymbol{Q}_{R,2} \boldsymbol{H}_1\big] \tag{2b}$$

We assume that no information will be stored at the relay node. This means that any information received in the MAC phase has to be forwarded in the BC phase immediately. Therefore, the bidirectional achievable rates for equal time division between the MAC and BC phase are given by

$$R_1 = \min\big\{R_{\overrightarrow{1\mathrm{R}}}, R_{\overrightarrow{\mathrm{R}2}}\big\}/2, \tag{3a}$$
$$R_2 = \min\big\{R_{\overrightarrow{2\mathrm{R}}}, R_{\overrightarrow{\mathrm{R}1}}\big\}/2 \tag{3b}$$

with $[R_{\overrightarrow{1\mathrm{R}}}, R_{\overrightarrow{2\mathrm{R}}}] \in \mathcal{R}_\mathrm{MAC}$ and $[R_{\overrightarrow{\mathrm{R}2}}, R_{\overrightarrow{\mathrm{R}1}}] \in \mathcal{R}_\mathrm{BC}$, while R_1 and R_2 denote the unidirectional rates using the relay for communication between node 1 and 2 and vice versa respectively.

In accordance, the achievable rate region of the bidirectional relaying scheme is given by the intersection of the scaled rate regions of the MAC and BC phase.

$$\mathcal{R}_\mathrm{BIR} = \tfrac{1}{2}\left(\mathcal{R}_\mathrm{MAC} \cap \mathcal{R}_\mathrm{BC}\right). \tag{4}$$

In the next section, we characterize the optimal transmit strategy for any rate tuple $[R_1, R_2] \in \mathcal{R}_\mathrm{BIR}$, i.e. the optimal power allocation and transmit covariance matrix.

3. OPTIMAL TRANSMIT STRATEGIES

Due to the intersection and the difficult characterization of the MIMO-MAC boundary the boundary of the MIMO-BIR can not be characterized in closed form as it is possible in the SISO case [4]. The optimal transmit strategies in both phases depend on the desired rate pair $\boldsymbol{R} \in \mathcal{R}_\mathrm{BIR}$. Since any rate pair in the interior can be achieved by time-sharing the most interesting operating rate pairs are those on the boundary of the rate regions $\mathcal{R}_\mathrm{BC}$ and $\mathcal{R}_\mathrm{MAC}$. In the following we present the optimal transmit strategies, which are adapted from the MIMO-MAC and point-to-point MIMO channel.

3.1. MIMO-BC

For any relay power distribution factor β the maximal achievable rates $\hat{R}_{\overrightarrow{\mathrm{R}2}}(\beta)$ and $\hat{R}_{\overrightarrow{\mathrm{R}1}}(\beta)$ are given by (2a) and (2b) respectively. Thus, for a fixed β the optimal transmit strategies $\boldsymbol{Q}_{R,1}$ and $\boldsymbol{Q}_{R,2}$ are decoupled and follow from the classical water-filling solution for the point-to-point MIMO channel, e.g. [2]. In the following, we briefly characterize the optimal transmit covariance matrices.

Let $\boldsymbol{H}_2\boldsymbol{H}_2^H = \boldsymbol{V}_2\boldsymbol{\Sigma}_2\boldsymbol{V}_2^H$ be the eigenvalue decomposition with $\boldsymbol{\Sigma}_2 = \operatorname{diag}_{1\leq i\leq N_\mathrm{R}}\{\lambda_{2,i}\}$ and $\lambda_{2,1} \geq \cdots \geq \lambda_{2,N_\mathrm{R}} \geq 0$ sorted in decreasing order. Then the optimal transmit covariance matrix $\boldsymbol{Q}_{R,1} = \boldsymbol{V}_2^H \operatorname{diag}_{1\leq i\leq N_\mathrm{R}}\{\xi_i\}\boldsymbol{V}_2^H$ with $\xi_i = \max\{\nu - 1/\lambda_{2,i}, 0\}$ and water-level condition $\beta\hat{P}_\mathrm{R}/\sigma^2 = \sum_{i=1}^{N_\mathrm{R}}\xi_i$. For this procedure, we introduce the short notation $\boldsymbol{Q}_{R,1} = \boldsymbol{Q}(\boldsymbol{H}_2^H, \beta\hat{P}_\mathrm{R}/\sigma^2)$. With this, the optimal transmit covariance matrix $\boldsymbol{Q}_{R,2} = \boldsymbol{Q}(\boldsymbol{H}_1^H, (1-\beta)\hat{P}_\mathrm{R}/\sigma^2)$ follows accordingly.

From the water-filling solution we can also specify the beamforming optimality range, where only $\xi_1 \neq 0$ and $\xi_i = 0\ \forall i \geq 2$. This is case for (2a) if $\beta\hat{P}_\mathrm{R}/\sigma^2 \leq (1/\lambda_{2,2} - 1/\lambda_{2,1})$. More generally, it is optimal to use $K \leq r_2$ channel eigenmodes if we have

$$\frac{\beta\hat{P}_\mathrm{R}}{\sigma^2} \leq \sum_{k=1}^{K}\Big(\frac{1}{\lambda_{2,K}} - \frac{1}{\lambda_{2,k}}\Big).$$

Obviously, with increasing transmit power more channel modes are used until every eigenmode greater than zero is used (high SNR case). Similar threshold values can be formulated for any water-filling solution.

3.2. MIMO-MAC

In the following we will characterize the boundary of the MIMO-MAC region. Some rate pairs can only be reached if time-sharing is applied, others require a certain decoding order. Therefore, let π_1 denote the decoding order where the message from node 2 is decoded first and therefore sees the signal from node 1 as additional interference. After decoding the message from node 2 its interference is canceled and interference-free decoding of the message from node 1 is possible (successive interference cancellation). Accordingly, the achievable rates are given by

$$R_{\overrightarrow{1\mathrm{R}}}^{\pi_1} = \tfrac{1}{2}\log\det\left[\boldsymbol{I}_{N_\mathrm{R}} + \rho\boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H\right] \tag{5a}$$

$$R_{\overrightarrow{2\mathrm{R}}}^{\pi_1} = \tfrac{1}{2}\log\frac{\det\left[\boldsymbol{I}_{N_\mathrm{R}} + \rho\boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H + \rho\boldsymbol{H}_2\boldsymbol{Q}_2\boldsymbol{H}_2^H\right]}{\det\left[\boldsymbol{I}_{N_\mathrm{R}} + \rho\boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H\right]}. \tag{5b}$$

This allows us to derive the first characteristic rate pair, which we denote $\boldsymbol{E}_1 = [R_1^{\mathrm{E}_1}, R_2^{\mathrm{E}_1}]$. From (5a) we see that the maximal unidirectional rate $R_1^{\mathrm{E}_1} = \hat{R}_{\overrightarrow{1\mathrm{R}}}/2$ is achieved if we choose $\boldsymbol{Q}_1$ according to the classical water-filling solution $\boldsymbol{Q}(\boldsymbol{H}_1, \hat{P}_1/\sigma^2)$. The rate remains unchanged for any transmit strategy $\boldsymbol{Q}_2$. Then, $R_2^{\mathrm{E}_1}$ follows from the optimal transmit covariance matrix $\boldsymbol{Q}_2$ under the constraint $R_1^{\mathrm{E}_1} = \hat{R}_{\overrightarrow{1\mathrm{R}}}/2$. The optimal transmit strategy is given by the water-filling solution $\boldsymbol{Q}(\tilde{\boldsymbol{H}}_2, \hat{P}_1/\sigma^2)$ with an equivalent channel $\tilde{\boldsymbol{H}}_2$ which satisfies the decomposition $\tilde{\boldsymbol{H}}_2^H\tilde{\boldsymbol{H}}_2 = \boldsymbol{H}_2^H(\boldsymbol{I}_{N_\mathrm{R}} + \rho\boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H)^{-1}\boldsymbol{H}_2$ (Cholesky). With this, we are able to define the first section $\mathcal{E}_1 = \{[\hat{R}_{\overrightarrow{1\mathrm{R}}}/2, R_{\overrightarrow{2\mathrm{R}}}] : 0 \leq R_{\overrightarrow{2\mathrm{R}}} \leq R_2^{\mathrm{E}_1}\}$ of the boundary of $\mathcal{R}_\mathrm{MAC}/2$.

Accordingly, let π_2 denote the decoding order where the message from node 1 is decoded first. By interchanging the indices 1 and 2 in (5a) and (5b) and following the same calculation we get the rate pair $\boldsymbol{E}_2 = [R_1^{\mathrm{E}_2}, R_2^{\mathrm{E}_2}]$ with $R_2^{\mathrm{E}_2} = \hat{R}_{\overrightarrow{2\mathrm{R}}}/2$ and accordingly the section $\mathcal{E}_2 = \{[R_{\overrightarrow{1\mathrm{R}}}, \hat{R}_{\overrightarrow{2\mathrm{R}}}/2] : 0 \leq R_{\overrightarrow{1\mathrm{R}}} \leq R_1^{\mathrm{E}_2}\}$ of the boundary of $\mathcal{R}_\mathrm{MAC}/2$.

Since the boundary of a convex set can be characterized by the weighted rate sum maxima for the next sections on the boundary we consider optimization problem [5], [6]

$$\boldsymbol{R}_\mathrm{MAC}(\boldsymbol{q}) = \operatorname*{arg\,max}_{\boldsymbol{R}\in\mathcal{R}_\mathrm{MAC}} q_1 R_{\overrightarrow{1\mathrm{R}}} + q_2 R_{\overrightarrow{2\mathrm{R}}}. \tag{6}$$

In [6] it is shown that in the case $q_1 \geq q_2$ the decoding order π_1 is optimal. In accordance, if $q_1 \leq q_2$ the decoding order π_2 is optimal. With the optimal decoding order the objective is a sum of concave functions and therefore the optimization problem (6) is convex.

In the following we will study the case $q_1 \geq q_2$. The Lagrangian function of the optimization problem is given by

$$L(\boldsymbol{Q}_1, \boldsymbol{Q}_2, \boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \mu_1, \mu_2) = -q_1 R_{\overrightarrow{1\mathrm{R}}}^{\pi_1} - q_2 R_{\overrightarrow{2\mathrm{R}}}^{\pi_1} - \sum_{k=1}^{2}\mu_k\big(P_k - \operatorname{tr}[\boldsymbol{Q}_k]\big) - \sum_{k=1}^{2}\operatorname{tr}[\boldsymbol{Q}_k\boldsymbol{\Psi}_k].$$

Therefore, the optimal transmit strategies $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ for rate pair $\boldsymbol{R}_\mathrm{MAC}(\boldsymbol{q})$ are uniquely characterized by the Karush-Kuhn-Tucker (KKT) conditions, which we present in the following for the case of $q_1 \geq q_2$

$$\mu_1\boldsymbol{I}_1 + \boldsymbol{\Psi}_1 = \boldsymbol{H}_1^H\big((q_2 - q_1)\big[\sigma^2\boldsymbol{I}_{N_\mathrm{R}} + \boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H\big]^{-1} - q_2\big[\sigma^2\boldsymbol{I}_{N_\mathrm{R}} + \boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H + \boldsymbol{H}_2\boldsymbol{Q}_2\boldsymbol{H}_2^H\big]^{-1}\big)\boldsymbol{H}_1 \tag{7a}$$

$$\mu_2\boldsymbol{I}_2 + \boldsymbol{\Psi}_2 = -q_2\boldsymbol{H}_2^H\big(\sigma^2\boldsymbol{I}_{N_\mathrm{R}} + \boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{H}_1^H + \boldsymbol{H}_2\boldsymbol{Q}_2\boldsymbol{H}_2^H\big)^{-1}\boldsymbol{H}_2 \tag{7b}$$

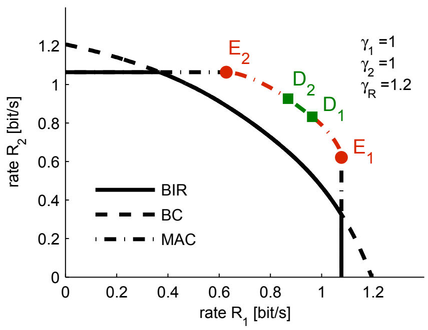
$$\operatorname{tr}[\boldsymbol{Q}_k\boldsymbol{\Psi}_k] = 0, \quad \mu_k(P_k - \operatorname{tr}[\boldsymbol{Q}_k]) = 0, \quad k = 1,2 \tag{7c}$$

$$\boldsymbol{\Psi}_k \succeq \boldsymbol{0}_{N_k}, \quad \mu_k \geq 0 \quad k = 1,2 \tag{7d}$$

$$\boldsymbol{Q}_k \succeq \boldsymbol{0}_{N_k}, \quad P_k \geq \operatorname{tr}[\boldsymbol{Q}_k] \quad k = 1,2 \tag{7e}$$

with complementary slackness, dual, and primal conditions (7c),(7d), and (7e) respectively. Since the optimization problem is convex efficient algorithms like interior-point method exist to calculate the optimal covariance matrices.

The previous discussion allows us to specify the set of rate pairs on the boundary $\mathcal{D}_1 = \{\boldsymbol{R}_\mathrm{MAC}(\boldsymbol{q}) : \boldsymbol{q} \in \mathbb{R}_+^2, q_1 \geq q_2\}$ for $q_1 \geq q_2$. Thereby, we denote by $\boldsymbol{D}_1$ the sum-rate maximum $\boldsymbol{R}_\mathrm{MAC}([1,1]) \in \mathcal{D}_1$. Accordingly, in the case $q_1 \leq q_2$ the Lagrange function and KKT conditions follow by interchanging the indices 1 and 2. This gives us the next set of rate pairs on the boundary $\mathcal{D}_2 = \{\boldsymbol{R}_\mathrm{MAC}(\boldsymbol{q}) : \boldsymbol{q} \in \mathbb{R}_+^2, q_1 \leq q_2\}$ and the sum-rate maximum $\boldsymbol{D}_2 = \boldsymbol{R}_\mathrm{MAC}([1,1]) \in \mathcal{D}_2$.



$\boldsymbol{H}_1 = \begin{bmatrix} 1-0.5j & 1-0.5j \\ -0.5 & 1.3 \end{bmatrix}$, $\boldsymbol{H}_2 = \begin{bmatrix} 0.4-0.4j & 1.2 \\ 1.6 & -0.2j \end{bmatrix}$ with $\lambda_{1,1} = 3.16$, $\lambda_{1,2} = 1.28$, $\lambda_{2,1} = 3.04$, and $\lambda_{2,2} = 1.32$.

Fig. 1. Rate Region MIMO-BIR with $N_1 = N_2 = N_\mathrm{R} = 2$

The last section $\mathcal{T} = \{\boldsymbol{R} : \boldsymbol{R} = \alpha\boldsymbol{D}_1 + (1-\alpha)\boldsymbol{D}_2, \alpha \in [0,1]\}$ is given by the connection between both sum-rate maxima and can be reached by time-sharing between the corresponding strategies of $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ only. If one is interested in sum-rate optimal rate pairs only one can also use an iterative waterfilling algorithm presented in [5].

4. HIGH SNR BEHAVIOR

We can deduce the high SNR-behavior of the sum of any rate pair $\boldsymbol{R}_\mathrm{BIR}$ on the boundary of the rate region $\mathcal{R}_\mathrm{BIR}$ from the asymptotic behavior of the maximal achievable unidirectional rates

$$R_1^\star = \min\{\hat{R}_{\overrightarrow{1\mathrm{R}}}, \hat{R}_{\overrightarrow{\mathrm{R}2}}(1)\}/2 \tag{8a}$$
$$R_2^\star = \min\{\hat{R}_{\overrightarrow{2\mathrm{R}}}, \hat{R}_{\overrightarrow{\mathrm{R}1}}(0)\}/2 \tag{8b}$$

using the following relation

$$\min\{R_1^\star, R_2^\star\} \leq ||\boldsymbol{R}_\mathrm{BIR}||_1 \leq R_1^\star + R_2^\star. \tag{9}$$

At high SNR, allocating equal amounts of power on the nonzero eigenmodes is asymptotically optimal. Thus, at high SNR we can approximate $\hat{R}_{\overrightarrow{1\mathrm{R}}}$ and $\hat{R}_{\overrightarrow{\mathrm{R}2}}(1)$ by

$$\hat{R}_{\overrightarrow{1\mathrm{R}}} \approx \sum_{j=1}^{r_1} \log\left[P\gamma_1 \tfrac{\lambda_{1,j}}{r_1}\right]$$
$$\hat{R}_{\overrightarrow{\mathrm{R}2}}(1) \approx \sum_{j=1}^{r_2} \log\left[P\gamma_\mathrm{R} \tfrac{\lambda_{2,j}}{r_2}\right]$$

with $\gamma_1 = \frac{\hat{P}_1}{P\sigma^2}$ and $\gamma_\mathrm{R} = \frac{\hat{P}_\mathrm{R}}{P\sigma^2}$. This gives us a high SNR approximation of the first unidirectional rate

$$R_1^\star \approx \min\{r_1, r_2\}\log[P] + c_1 \tag{10}$$

with a finite constant c_1. Accordingly, there exist a finite constant c_2 so that at high SNR $R_2^\star$ can be approximated by

$$R_2^\star \approx \min\{r_1, r_2\}\log[P] + c_2. \tag{11}$$

In other words, at high SNR the unidirectional rates grow with the minimum spatial degree of both channels, which is a reasonable result since every message has to be transmitted via both channels.

Furthermore, from (9) it follows that at high SNR the growth of the sum of any rate pair on the boundary of $\mathcal{R}_\mathrm{BIR}$ is asymptotically proportional with minimum spatial degree of both channels, i.e. $\min\{r_1, r_2\}\log[P]$.

5. CONCLUSION

MIMO specific difficulties make the processing of the extension of an efficient bidirectional relay communication protocol to the MIMO case more complex. In the MAC phase the optimal transmit strategy follows form the general MIMO-MAC channel. In the BC phase the optimal transmit strategies are derived from two point-to-point MIMO links which are only coupled by the distribution of the relay power.

In the high SNR, the sum of any rate pair on the boundary of the achievable bidirectional rate region grows linearly with the minimum of the spatial degrees of both MIMO channels.

Similar investigations regarding the cross-layer design or relay selection as in the SISO case are straight forward but are much more involved due to the difficult characterization of the rate regions.

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