ZF V-BLAST FOR IMPERFECT MIMO CHANNELS USING AVERAGE PERFORMANCE OPTIMIZATION

Jiansong Chen and Xiaoli Yu

Communication Science Institute, Dept. of Electrical Engineering
University of Southern California, Los Angeles 90089-2565
{jiansonc,xiaoliyu}@usc.edu

ABSTRACT

In practice, channel estimation is often imperfect due to the noise and interference. This paper extends the V-BLAST system to the imperfect channel state information by dividing the channel matrix into two parts. Upon an assumption that the channel estimation error vectors are independent complex Gaussian with zero mean and known second-order statistics, a symbol detection ordering criteria for ZF V-BLAST system is proposed which aims at providing, in each layer, the maximum average signal to interference-plus-noise ratio (SINR), over the random channel errors. This robust V-BLAST ordering rule takes the imperfect CSI into account, while maintaining the simple implementation of the V-BLAST structure. It is shown that the new ordering criterion is capable of achieving global performance optimization and outperforms the standard ZF V-BLAST when estimation errors exist.

Index Terms–MIMO systems, Signal detection.

1. INTRODUCTION

In recently years, a lot of effort has been devoted to studying V-BLAST algorithm under the assumption that the channel state information (CSI) is perfectly known at the receiver. In practical scenario, however, the channel knowledge is generally imperfect due to noise and temporal variation of the channel. Based on this fact, it is necessary to study the V-BLAST systems taking into account explicitly the errors in the channel estimate.

Previous work has utilized average performance to carry out performance evaluation and error propagation analysis of V-BLAST without optimal ordering. In this paper, we extend the V-BLAST receiver design and average performance analysis to the imperfect MIMO CSI, and propose a robust design which is less sensitive to estimation errors. We model the channel matrix $\mathbf{H}$ as the summation of two parts, such that $\mathbf{H} = \hat{\mathbf{H}} + \triangle\mathbf{H}$. Upon an assumption that the column vector $\triangle\mathbf{h}_i$, $i = 1, 2, \ldots M$, are independent complex Gaussian with zero mean and known second-order statistics, a symbol detection ordering criterion for ZF V-BLAST system is proposed. In each layer, our approach optimizes the average post-detection SINR over the channel estimation errors and organizes the detection order by decoding the symbol corresponding to the best average SINR first. It has been shown that the proposed ordering method achieves global average (SINR) performance optimization.

The paper is organized as follows. We begin with the V-BLAST system models in both perfect and imperfect CSI environments in Section II. Sections III presents the proposed ordering method based on average SINR. In Section IV, this new ordering rule is proved to equal global optimization. Conclusions are drawn in Section V.

2. SYSTEM MODEL

Consider a V-BLAST system with M transmitting and N receiving antennas where $M \leq N$. Let $\mathbf{q} = [q_1, q_2, \ldots, q_M]^T$ denote the $M \times 1$ transmitted symbol vector with $E\{|q_j|^2\} = 1 (j = 1, \ldots, M)$, then the corresponding $N \times 1$ received vector $\mathbf{r}$ is given by,

$$\mathbf{r} = \mathbf{Hq} + \mathbf{n} = \sum_{j=1}^{M} \mathbf{h}_j q_j + \mathbf{n} \qquad (1)$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M]$ is the flat fading channel matrix and $\mathbf{n}$ is complex AWGN with variance σ_n^2.

In practical scenarios, the channel state information (CSI) has to be estimated from the observations, which is generally imperfect due to noise and temporal variation of the channel. Therefore, we model the inherent uncertainty in channel estimation by dividing the channel vector, $\mathbf{h}_j$, into two parts

$$\mathbf{h}_j = \hat{\mathbf{h}}_j + \triangle\mathbf{h}_j \qquad (2)$$

where $\hat{\mathbf{h}}_j$ is considered to be the part of the channel that is known, the estimate, and $\triangle\mathbf{h}_j$ is considered to be the part of the channel that is unknown, the estimation error [1]. We assume that $\triangle\mathbf{h}_j$ is composed of complex gaussian random

THIS WORK WAS SUPPORTED IN PART BY THE NATIONAL SCIENCE FOUNDATION CCF-0515358

variables (CGRVs) with zeros mean and known variance ε_j^2, $1 \le j \le M$. Another reasonably made assumption is that the noise vector $\mathbf{n}$ and the channel estimation error vectors $\triangle\mathbf{h}_j$s are statistically independent [2].

3. PROPOSED ORDERING METHOD FOR ZF V-BLAST WITH IMPERFECT CSI

3.1. Impact of random channel estimation errors

Let $\{i_1, i_2, \ldots, i_M\}$ denote the optimal/suboptimal ordering such that i_k denotes the index of the kth symbol to be detected. Assume the data symbols $\{q_{i_1}, q_{i_2}, \ldots q_{i_{k-1}}\}$ are detected in the first $(k-1)$ layers and there exists no error propagation from the decision feedback. Then, the received signal after appropriate interference cancellation in the kth layer is

$$\mathbf{r}^{(k)} = \mathbf{r} - \sum_{l=i_1}^{i_{k-1}} \hat{\mathbf{h}}_l q_l = \sum_{l \notin \{i_1, \ldots i_{k-1}\}} \mathbf{h}_l q_l + \sum_{l=i_1}^{i_{k-1}} \triangle\mathbf{h}_l q_l + \mathbf{n} \quad (3)$$

where the superscript $^{(k)}$ denotes the kth layer.

Based on the knowledge of the channel vector (2), the interference from yet-to-be-detected symbol q_i can be nulled out using ZF nulling vector $\hat{\mathbf{w}}_i^{(k)}$, which is formed to satisfy the following equations:

$$(\hat{\mathbf{w}}_i^{(k)})^H \hat{\mathbf{h}}_j = \delta_{ij} \ , \quad i, j \notin \{i_1, \ldots i_{k-1}\} \quad (4)$$

$$\|\hat{\mathbf{w}}_i^{(k)}\|^2 = [(\hat{\mathbf{H}}^{(k)})^H \hat{\mathbf{H}}^{(k)}]_{ii}^{(-1)}, \quad i \notin \{i_1, \ldots, i_{k-1}\} \quad (5)$$

where $\hat{\mathbf{H}}^{(k)}$ is composed of all $\hat{\mathbf{h}}_j$s for $j \notin \{i_1, \ldots, i_{k-1}\}$.

Multiplying both sides of (3) with $\hat{\mathbf{w}}_i^{(k)}$ given in (4) yields the conditional post-processing SINR (given $\triangle\mathbf{H}$) at the kth layer,

$$\widehat{SINR}_{i|\triangle\mathbf{H}}^{(k)} = \frac{\left|1 + (\hat{\mathbf{w}}_i^{(k)})^H \triangle\mathbf{h}_i\right|^2}{\sum_{j \neq i} |(\hat{\mathbf{w}}_i^{(k)})^H \triangle\mathbf{h}_j|^2 + \sigma_n^2 \|\hat{\mathbf{w}}_i^{(k)}\|^2} \quad (6)$$

Evidently, a direct use of (6) in V-BLAST optimal ordering procedure is infeasible due to the uncertainty of the estimation error $\triangle\mathbf{h}_j$s. This motivates us to propose a new ordering criteria in the next section via statistical optimization.

3.2. Proposed ordering method based on average SINR

Recall $\mathbf{H}$ is *block* flat fading channel. It is reasonable to organize the detection order based on average system performance:

$$i_k = arg \max_{i \notin \{i_1, \ldots i_{k-1}\}} \overline{SINR}_i^{(k)} \ , \ k = 1, \ldots, M \quad (7)$$

where $\overline{SINR}_i^{(k)} \doteq E_{\triangle\mathbf{H}}\{\widehat{SINR}_{i|\triangle\mathbf{H}}^{(k)}\}$ and $E_{\triangle\mathbf{H}}\{\cdot\}$ denotes the expectation with respect to (w.r.t.) the random channel error matrix of $\triangle\mathbf{H}$.

Then, depending on the model of the second-order statistics of $\triangle\mathbf{h}_j$s, we consider two cases in the following.

(1) When $\varepsilon_j^2 \neq \varepsilon_k^2$ with $j \neq k, (j, k \in \{1, \ldots, M\})$, it is shown in Appendix A that

$$\begin{aligned} \overline{SINR}_i^{(k)} =& E_{\triangle\mathbf{h}_i}\{|1 + (\hat{\mathbf{w}}_i^{(k)})^H \triangle\mathbf{h}_i|^2\} \\ & \cdot E_{\substack{\triangle\mathbf{h}_j \\ j \neq i}} \{\frac{1}{\sum_{j \neq i} |(\hat{\mathbf{w}}_i^{(k)})^H \triangle\mathbf{h}_j|^2 + \sigma_n^2 \|\hat{\mathbf{w}}_i^{(k)}\|^2}\} \\ =& (\|\hat{\mathbf{w}}_i^{(k)}\|^{-2} + \varepsilon_i^2) \sum_{j \neq i}^{M} \frac{\varepsilon_j^{2M-6} e^{\frac{\sigma_n^2}{\varepsilon_j^2}} [-Ei(-\frac{\sigma_n^2}{\varepsilon_j^2})]}{\prod_{k \neq i,j}^{M} (\varepsilon_j^2 - \varepsilon_k^2)} \end{aligned} \quad (8)$$

where $Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$ $(x < 0)$ is the exponential integral function which can be computed in advance.

(2) When $\varepsilon_1 = \varepsilon_2 = \ldots\varepsilon_M = \varepsilon$, following the similar derivation in (8), we conclude that

$$\overline{SINR}_i^{(k)} = (\|\hat{\mathbf{w}}_i^{(k)}\|^{-2} + \varepsilon^2) \cdot \beta \quad (9)$$

Note that $\beta \triangleq E\{\frac{1}{\frac{1}{2}\varepsilon \cdot \chi_{2(M-1)}^2 + \sigma_n^2}\}$ is independent of i and k, where $\chi_{2(M-1)}^2$ is a chi-squared variate with $2(M-1)$ degrees of freedom.

Fig 1 compares the Monte Carlo simulated average total error rate (TBER) between the proposed ordering method and the standard ZF V-BLAST technique which simply ignores the estimation errors. In both cases, the asterisk and circle lines indicate the TBER obtained by the proposed method and the standard V-BLAST ordering scheme, respectively. It shows that our approach outperforms the standard ZF V-BLAST when channel estimation errors exist.

4. PROPOSED MYOPIC OPTIMIZATION EQUALS GLOBAL OPTIMIZATION

Define a set Ω, which contains all the possible orderings to detect $\{q_1, \ldots q_M\}$. For any given detection ordering $L \subset \Omega$, $L \triangleq \{l_1, l_2, \ldots, l_M\}$, define the *constraint set* of l_k to be the set $\{l_{k+1}, l_{k+2}, \ldots, l_M\}$. The constraint set is just those components of L which have not yet been detected and cancelled.

The myopic optimization means that starting at the first layer and continuing iteratively to the Mth layer, always use (7) to organize the ZF V-BLAST detection order. We next follow the idea of [3] to prove that it is in fact equivalent to the following global performance optimization:

$$\max_{L \subset \Omega} \min_{k \in \{1, \ldots M\}} \overline{SINR}_{l_k}^{(k)} \quad (10)$$

Lemma 1: Let A and B be two distinct orderings. If a_k=b_k which means that A and B detect the same symbol in

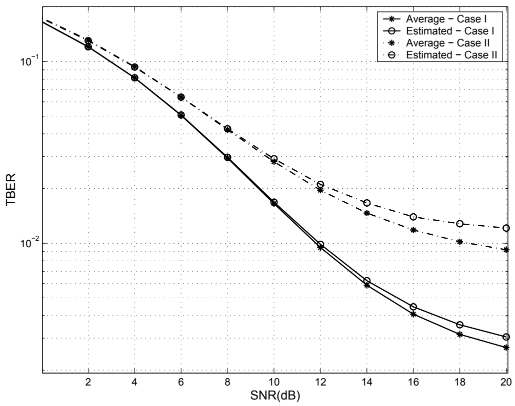

Fig. 1. 4x4 V-BLAST system. BPSK modulation. $\varepsilon_j^2 = 0.01{*}j$ in case I and $\varepsilon_j^2 = 0.01{*}j^2$ in case II with $j = 1, \ldots, 4$

the kth layer and if the constraint sets of a_k and b_k consist of identical elements (regardless of their order), then $\overline{SINR}_{a_k}^{(k)} = \overline{SINR}_{b_k}^{(k)}$.

Proof: Since $a_k = b_k$ and their constraint sets consist of identical elements, we have $\hat{\mathbf{H}}_A^{(k)} = \hat{\mathbf{H}}_B^{(k)}$, where $\hat{\mathbf{H}}_A^{(k)}$ and $\hat{\mathbf{H}}_B^{(k)}$ denote the $N \times (M - k + 1)$ matrices obtained by zeroing $(k-1)$ columns of $\hat{\mathbf{H}}$ corresponding with ordering A and ordering B, respectively. With (5) , it follows that ZF nulling vectors $\hat{\mathbf{w}}_i^{(k)}$ of ordering A and ordering B have the same vector norm in the kth layer

$$\hat{\mathbf{w}}_{a_k}^{(k)} = \hat{\mathbf{w}}_{b_k}^{(k)} \tag{11}$$

Considering ordering A and B are detecting the same symbol in the kth layer ($a_k = b_k$), it is easy to see that $\overline{SINR}_{a_k}^{(k)} = \overline{SINR}_{b_k}^{(k)}$ for both (8) and (9).

Lemma 2: Let A and B be two distinct orderings. If $a_k = b_{k+m}$ $(m > 0)$, which means that the symbol detected in the kth layer corresponding with ordering A is the same as that detected in the $(k+m)th$ layer corresponding with ordering B, and if the constraint set of b_{k+m} is a subset of the constraint set of a_k, then $\overline{SINR}_{a_k}^{(k)} \leq \overline{SINR}_{b_{k+m}}^{(k+m)}$.

Proof: It is shown in Appendix B that, if $a_k = b_{k+m}$ $(m > 0)$, we have

$$\hat{\mathbf{w}}_{a_k}^{(k)} \geq \hat{\mathbf{w}}_{b_{k+m}}^{(k+m)} \tag{12}$$

Therefore, we conclude that $\overline{SINR}_{a_k}^{(k)} \leq \overline{SINR}_{b_{k+m}}^{(k+m)}$ with $a_k = b_{k+m}$ $(m > 0)$ for both (8) and (9).

With Lemma 1 and Lemma2, we will show that the proposed myopic optimization equals global optimization, which is given in the following theorem.

Theorem1: For ZF V-BLAST system with imperfect CSI (2), the myopic optimization is equivalent to the global optimization (10), which means that using the proposed detection ordering criterion (7) in each layer maximizes the minimum average SINR of all layers over all possible orderings.

Proof : Let $I \triangleq \{i_1, i_2, \ldots, i_M\}$ be the order obtained using myopic optimization (7) in each layer. $L \triangleq \{l_1, l_2, \ldots, l_M\}$ denotes an arbitrary ordering distinct from I. Let m be the index of the first (leftmost) element for which I and L differ. n indicates the index of the layer in which $l_n = i_m$ $(m < n)$.

Define another ordering L' which is a perturbation of L obtained by moving l_n from the nth layer to the mth layer so that the element of L' are

$$\begin{aligned} L' &\triangleq \{l'_1, l'_2, \ldots, l'_M\} \\ &= \{l_1, \ldots l_{m-1}, l_n, l_m, \ldots l_{n-1}, l_{n+1}, \ldots l_M\} \end{aligned} \tag{13}$$

From Lemma 1,

$$\overline{SINR}_{l'_k}^{(k)} = \overline{SINR}_{l_k}^{(k)}, \;\; 1 \leq k \leq m-1 \text{ and } n+1 \leq k \leq M \tag{14}$$

By using Lemma 2,

$$\overline{SINR}_{l'_k}^{(k)} \geq \overline{SINR}_{l_{k-1}}^{(k-1)}, \;\; m+1 \leq k \leq n \tag{15}$$

Note that $\overline{SINR}_{l'_m}^{(m)} = \overline{SINR}_{i_m}^{(m)}$ and $\overline{SINR}_{i_m}^{(m)}$ is, by virtue of the myopic optimization procedure (7), the largest among any other choice in this layer. Thus, we have

$$\overline{SINR}_{l'_m}^{(m)} \geq \overline{SINR}_{l_m}^{(m)} \tag{16}$$

and (14)-(16) can be summarized as

$$\min_{k \in \{1, \ldots M\}} \overline{SINR}_{l'_k}^{(k)} \geq \min_{k \in \{1, \ldots M\}} \overline{SINR}_{l_k}^{(k)} \tag{17}$$

Continuing the similar perturbation as (13), L' can finally be transformed into I while maintaining an inequality analogous to (17) at each perturbation. Therefore, we conclude that

$$\min_{k \in \{1, \ldots M\}} \overline{SINR}_{i_k}^{(k)} \geq \min_{k \in \{1, \ldots M\}} \overline{SINR}_{l_k}^{(k)} \tag{18}$$

Since L is any arbitrary ordering distinct from I, it turns out that the myopic optimization (7) achieves global optimization (10).

5. CONCLUSION

Under the assumption that the random channel estimation errors are independent complex Gaussian with zero mean and known second-order statistics, a novel ZF V-BLAST ordering algorithm is developed. In each layer, the proposed approach optimizes the average post-detection SINR over the channel estimation errors and indexes the detection order by decoding the symbol corresponding to the best average SINR first. We have shown that the proposed algorithm is capable of achieving global performance optimization. Simulation results validate the analytical results.

APPENDIX A – Proof of equation (8)

Let g_j denote a chi-squared variate with two degrees of freedom, i.e., $g_j \sim \chi_2^2$. We then express $|(\hat{\mathbf{w}}_i^{(k)})^H \triangle \mathbf{h}_j|^2$ in terms of g_j as $|(\hat{\mathbf{w}}_i^{(k)})^H \triangle \mathbf{h}_j|^2 \sim \frac{1}{2}(\|\hat{\mathbf{w}}_i^{(k)}\|^2 \varepsilon_j^2) g_j$. Thus, for $j = 1, \ldots, M$ and $j \neq i$, we have

$$\sum_{j \neq i} |(\hat{\mathbf{w}}_i^{(k)})^H \triangle \mathbf{h}_j|^2 = \|\hat{\mathbf{w}}_i^{(k)}\|^2 \sum_{j \neq i} \frac{1}{2} \varepsilon_j^2 g_j \qquad (19)$$

and the characteristic function of $z \stackrel{\triangle}{=} \sum_{j \neq i} \frac{1}{2} \varepsilon_j^2 g_j$ is

$$\psi_z(jv) = \prod_{j \neq i}^{M} \frac{1}{1 - jv\varepsilon_j^2} \qquad (20)$$

Applying a partial fraction expansion to (20), the density function of z is then obtained by the inverse Fourier transform as

$$p_z(x) = \sum_{j \neq i}^{M} \frac{\varepsilon_j^{2(M-3)}}{\prod_{k \neq i,j}^{M} (\varepsilon_j^2 - \varepsilon_k^2)} e^{-\frac{x}{\varepsilon_j^2}} \qquad (21)$$

It follows that

$$\begin{aligned} & E_{\substack{\triangle \mathbf{h}_j \\ j \neq i}} \{ \frac{1}{\sum_{j \neq i} |(\hat{\mathbf{w}}_i^{(k)})^H \triangle \mathbf{h}_j|^2 + \sigma_n^2 \|\hat{\mathbf{w}}_i^{(k)}\|^2} \} \\ =& \|\hat{\mathbf{w}}_i^{(k)}\|^{-2} \sum_{j \neq i}^{M} \frac{\varepsilon_j^{2(M-3)}}{\prod_{k \neq i,j}^{M} (\varepsilon_j^2 - \varepsilon_k^2)} \int_0^\infty \frac{1}{x + \sigma_n^2} e^{-\frac{x}{\varepsilon_j^2}} dx \\ =& \|\hat{\mathbf{w}}_i^{(k)}\|^{-2} \sum_{j \neq i}^{M} \frac{\varepsilon_j^{2M-6} e^{\frac{\sigma_n^2}{\varepsilon_j^2}} [-Ei(-\frac{\sigma_n^2}{\varepsilon_j^2})]}{\prod_{k \neq i,j}^{M} (\varepsilon_j^2 - \varepsilon_k^2)} \end{aligned} \qquad (22)$$

Since $E_{\triangle \mathbf{h}_i}\{|1+(\hat{\mathbf{w}}_i^{(k)})^H \triangle \mathbf{h}_i|^2\}=1+\varepsilon_i^2 \|\hat{\mathbf{w}}_i^{(k)}\|^2$, it completes the proof of (8).

APPENDIX B – Proof of equation (12)

By the induction method, we first discuss the simplest case with $m = 1$. In this case, $\hat{\mathbf{H}}_A^{(k)}$ differs to $\hat{\mathbf{H}}_B^{(k+1)}$ by only one column. Assume the pth column of $\hat{\mathbf{H}}_A^{(k)}$, h_p, does not contain in $\hat{\mathbf{H}}_B^{(k+1)}$. By introducing a permutation matrix $\mathbf{P}_{i,j}$ which interchange the ith and jth rows or columns of a matrix, we have that $\hat{\mathbf{H}}_A^{(k)} \mathbf{P} = \left[\hat{\mathbf{H}}_B^{(k+1)}; h_p\right]$, where $\mathbf{P} = \mathbf{P}_{p,(p+1)} \mathbf{P}_{(p+1),(p+2)} \ldots \mathbf{P}_{(M-k),(M-k+1)}$.

It follows that

$$\mathbf{P}^H \mathbf{R}_A \mathbf{P} = \left[\hat{\mathbf{H}}_B^{(k+1)}; h_p\right]^H \left[\hat{\mathbf{H}}_B^{(k+1)}; h_p\right] = \begin{bmatrix} \mathbf{R}_B & \mathbf{v} \\ \mathbf{v}^H & \beta_p \end{bmatrix} \qquad (23)$$

where $\mathbf{R}_A$=$[(\hat{\mathbf{H}}_A^{(k)})^H \hat{\mathbf{H}}_A^{(k)}]$, $\mathbf{R}_B$=$[(\hat{\mathbf{H}}_B^{(k+1)})^H \hat{\mathbf{H}}_B^{(k+1)}]$ and $\mathbf{v} = (\hat{\mathbf{H}}_B^{(k+1)})^H h_p$, $\beta_p = h_p^H h_p$

Since $\mathbf{P}$ is an orthogonal matrix,

$$\begin{aligned} [\mathbf{P}^H \mathbf{R}_A \mathbf{P}]^{-1} = \mathbf{P}^H \mathbf{R}_A^{-1} \mathbf{P} &= \begin{bmatrix} \mathbf{R}_B & \mathbf{v} \\ \mathbf{v}^H & \beta_p \end{bmatrix}^{-1} \\ &= \begin{bmatrix} \mathbf{T}^{-1} & -\frac{\mathbf{T}^{-1}\mathbf{v}}{\beta_p} \\ -\frac{\mathbf{v}^H \mathbf{T}^{-1}}{\beta_p} & \beta_p^{-1} + \frac{\mathbf{v}^H \mathbf{T}^{-)} \mathbf{v}}{\beta_p^2} \end{bmatrix} \end{aligned} \qquad (24)$$

where $\mathbf{T} = \mathbf{R}_B - \frac{\mathbf{v}\mathbf{v}^H}{\beta_p}$ is the Schur complement of β_p.

Using matrix inversion lemma, it follows that

$$\mathbf{R}_B^{-1} = [\mathbf{T} + \frac{\mathbf{v}\mathbf{v}^H}{\beta_p}]^{-1} = \mathbf{T}^{-1} - \frac{\mathbf{T}^{-1}\mathbf{v}\mathbf{v}^H\mathbf{T}^{-1}}{\beta_p + \mathbf{v}^H \mathbf{T}^{-1} \mathbf{v}} \qquad (25)$$

Note that $\mathbf{P}^H \mathbf{R}_A^{-1} \mathbf{P}$ and its principle sub-matrix $\mathbf{T}^{-1}$ are non-negative since $\mathbf{R}_A$ is a non-negative definite matrix. Thus, the non-negative diagonal elements of the last term in (25) lead to the fact that the diagonal elements of $\mathbf{R}_B^{-1}$ are not larger than the corresponding diagonal elements of $\mathbf{T}^{-1}$. It follows that the diagonal elements of $\mathbf{R}_B^{-1}$ are not larger than the corresponding diagonal elements of $\mathbf{P}^H \mathbf{R}_A^{-1} \mathbf{P}$. Note that $\mathbf{R}_A^{-1}$ and $\mathbf{P}^H \mathbf{R}_A^{-1} \mathbf{P}$ should have the same diagonal elements, although they may be in the different order. Recall (5) and $a_k = b_{k+m}$ in Lemma 2. It turns out that $\hat{\mathbf{w}}_{a_k}^{(k)} \geq \hat{\mathbf{w}}_{b_{k+m}}^{(k+m)}$ for $m = 1$.

Next, assume Lemma 2 is valid for the case of m=$n > 1$. We then have to prove it holds for $m = n + 1$ in order to complete the induction approach.

Let C denote an order such that a_k=c_{k+n} and the constraint set of c_{k+n} is a subset of the constraint set of a_k. It follows that $c_{k+n} = b_{k+n+1}$ or $c_{k'} = b_{k'+1}$ by denoting $n + k = k'$ and the constraint set of $b_{k'+1}$ is a subset of the constraint set of $c_{k'}$. Using the result of m=1, we have that $\hat{\mathbf{w}}_{c_k'}^{(k')} \geq \hat{\mathbf{w}}_{b_{k'+1}}^{(k'+1)}$, i.e., $\hat{\mathbf{w}}_{c_{k+n}}^{(k+n)} \geq \hat{\mathbf{w}}_{b_{k+n+1}}^{(k+n+1)}$. By the assumption of $\hat{\mathbf{w}}_{a_k}^{(k)} \geq \hat{\mathbf{w}}_{c_{k+n}}^{(k+n)}$ for the case of m=n, we conclude that $\hat{\mathbf{w}}_{a_k}^{(k)} \geq \hat{\mathbf{w}}_{b_{k+n+1}}^{(k+n+1)}$ and it completes the proof of (12).

6. REFERENCES

[1] M.Medard, "The effect upon channel capacity in wireless communications of perfect and impefect knowledge of the channel," *IEEE Trans. on Information Theory,* vol.46, no.3, pp.933-946, May 2000.

[2] T.Weber, A.Sklavos and M.Meurer, "Imperfect Channel-State Information in MIMO Transmission," *IEEE Trans. Commun.*, vol. 54, no. 3, March 2006

[3] Golden, G.D., et al., "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel". *Proc. 1998 Int. Symp. on Advanced Radio Technologies,* Boulder, Colorado, 911 Sep. 1998