# DATA-AIDED KALMAN TRACKING FOR CHANNEL ESTIMATION IN DOPPLER-AFFECTED OFDM SYSTEMS

Paolo Banelli, Rocco Claudio Cannizzaro, and Luca Rugini

University of Perugia, D.I.E.I., 06125 Perugia, Italy

#### **ABSTRACT**

This paper considers a pilot-aided/data-aided Kalman channel estimator for OFDM in fast time-varying (TV) channels. Capitalizing on a basis expansion model (BEM) and on a frequency-domain estimation philosophy, the OFDM system is designed to periodically switch between a pilot-aided and a data-aided mode in order to reduce the rate penalty introduced by the training pilots. The suggested philosophy is effective in tracking the channel changes in the data-aided mode with a negligible channel mean squared error (MSE) penalty, if the Kalman filter prediction capability is coupled with an iterative data-aided estimation that is equipped by an opportune selection of the detected data. Appropriate data selection metrics for LMMSE and DFE equalizers are also provided, and the impact of the proposed channel estimation on the ultimate BER performance of each equalizer is investigated by simulations.

Index Terms - OFDM, doubly-selective channels, channel estimation, channel tracking, Kalman filter

# 1. INTRODUCTION

Wireless scenarios characterized by high-mobility destroy the subcarriers' orthogonality of OFDM systems, thus introducing intercarrier interference (ICI) that significantly degrades the performance of single-tap equalizers, which are classically employed in OFDM systems [1][2]. Thus, more complex equalizers are required for time-varying (TV) channels, such as those recently proposed in [3], [4], [5] and references therein. Obviously, TV channel estimation plays a crucial role for the performance of these equalizers, which require the knowledge of the channel variation within each OFDM block.

Recently, in [6] the authors proposed a Kalman-based channel estimator that capitalizes on a basis expansion model (BEM) [7] for the channel, and on a frequency-domain pilot-aided training. Specifically, the BEM basis functions capture the channel variation within each OFDM block, and the Kalman adaptively estimates the BEM coefficients from one OFDM block to another, thus significantly reducing the Kalman filter complexity. However the frequency-domain training, which reserves to pilots some subcarriers of each OFDM block, introduces a rate penalty that increases with the number of unknowns in the estimation problem (i.e. with the number of channel paths and the maximum Doppler spread [8]). In order to reduce the rate penalty induced by training, we propose an OFDM system that switches periodically between a frequency-domain pilot symbol assisted modulation (PSAM) mode and a virtual PSAM (V-PSAM). In the V-PSAM mode, all the OFDM subcarriers convey only data, which, after a first coarse equalization and detection, are used to refine the Kalman-filter channel estimation in an iterative fashion, thereby improving the overall BER. In order to make the channel estimation refinements effective in the V-PSAM mode, it is highly desirable that the virtual pilots correspond to data detected without any error. Thus, similarly

to layer sorting techniques for nulling and cancelling (NC)-MIMO detection [9], we propose virtual pilot selection strategies, by exploiting metrics that are aimed to maximize the a posteriori probability of correct data detection. We will show the great benefit provided by opportune virtual pilot selection strategies, and that the convenient selection metric depends on the TV equalization scheme. We will investigate by simulations the channel estimation MSE and the ultimate impact of the proposed data-aided estimation on the BER of TV OFDM systems that are equipped with block LMMSE and block DFE (BDFE) equalizers [5].

### 2. CHANNEL BASIS EXPANSION MODEL

A TV discrete-time channel induced by user mobility can be characterized by an (L+1)-tap TV FIR filter. In order to parsimoniously model the channel and reduce the channel estimation complexity, a useful approach is to approximate the channel time variation by a basis expansion model (BEM) [7] exploiting the superposition of 2Q+1 basis functions  $\lambda_q[n]$ , q=-Q,...,Q, as

$$h[n;l] \approx \sum_{q=-Q}^{Q} h_{q,l} \lambda_q[n], \quad n = 0, \dots, N-1$$

 $h[n;l] \approx \sum_{q=-Q}^{Q} h_{q,l} \lambda_q[n], \quad n=0,\cdots,N-1,$  where h[n,l] is the  $l^{th}$  channel tap at the  $n^{th}$  time interval. Equivalently, in matrix form

$$\mathbf{h}_{ll} = \mathbf{B}\mathbf{h}_{cl},\tag{1}$$

 $\mathbf{n}_{l} = \mathbf{B}\mathbf{n}_{cl}, \tag{1}$  where  $\mathbf{h}_{l} = [h[0;l], \cdots, h[N-1;l]]^{T}, \quad \mathbf{h}_{cl} = [h_{-Q,l}, \cdots, h_{Q,l}]^{T},$   $\mathbf{B} = [\lambda_{-Q}, \cdots, \lambda_{Q}], \text{ and } \lambda_{q} = [\lambda_{q}[0], \cdots, \lambda_{q}[N-1]]^{T}.$ 

Several choices are possible for the basis functions (see [8] and references therein for further details). In our simulations, we will consider the so-called generalized complex exponentials (GCE). Whichever is the basis, the vectors  $\{\lambda_a\}$  are fixed, and therefore, in order to estimate the channel behaviour, we only need to estimate the  $N_h = (L+1)(2Q+1)$  expansion coefficients  $\{h_{q,l}\}$ . The number 2Q+1 of basis functions increases with the normalized maximum Doppler spread, defined as  $v_D = f_D / \Delta_f$ , where  $f_D$  is the maximum Doppler spread and  $\Delta_f$  is the OFDM subcarrier spacing.

## 3. OFDM SYSTEM MODEL

OFDM systems modulate the data  $\mathbf{s} = [s[0], \dots, s[N-1]]^T$  on N orthogonal subcarriers by IFFT processing. A cyclic prefix (CP), with length greater than the channel order L, is concatenated to the transmitted signal in order to convert the effect of the channel into a circular convolution. After CP removal, the received data can be reshaped by a time-domain windowing  $\mathbf{w} = [w_0, ..., w_{N-1}]^T$  in order to reduce Doppler effects [4], and are successively demodulated by an FFT processing. Summarising, the received data vector  $\mathbf{y} = [y[0], \dots, y[N-1]]^T$  can be expressed by

$$\mathbf{y} = \sum_{q=-O}^{O} \mathbf{C}_{q} \Delta_{q} \mathbf{s} + \mathbf{n}_{F} = \mathbf{H}_{F} \mathbf{s} + \mathbf{n}_{F},$$
 (2)

where, by applying the BEM model (1) to the windowed channel taps  $diag(\mathbf{w})\mathbf{h}_{d}$  [8],  $\mathbf{C}_{q} = \mathbf{F}diag(\lambda_{q})\mathbf{F}^{H}$  turns out to be circulant with  $[\mathbf{C}_q]_{k,m} = N^{-1} \sum_{l=1}^{N-1} \lambda_q [n] \mathrm{e}^{-j2\pi(k-m)n/N}$ , and  $\Delta_q = diag(\mathbf{F}_L \mathbf{h}^{(q)})$ , where  $\mathbf{F}$  is the unitary DFT matrix,  $\mathbf{F}_L$  contains the first L+1 columns of  $\sqrt{N}\mathbf{F}$ ,  $\mathbf{h}^{(q)} = [h_{q,0},\cdots,h_{q,L}]^T$ ,  $\mathbf{n}_F = \mathbf{F} diag(\mathbf{w})\mathbf{n}_T$  and  $\mathbf{n}_T = [n[0],\cdots,n[N-1]]^T$  is an additive white Gaussian noise. We remark that for TI channels Q=0 in (2), and  $\mathbf{H}_F=\mathbf{C}_0\Delta_0$  is a diagonal channel matrix. The more the channel is TV (i.e., the greater is  $v_D$ ), the more the channel matrix  $\mathbf{H}_F$  in (2) departs from a diagonal one, which means that more ICI is introduced.

### 4. CHANNEL ESTIMATION BY PSAM

A frequency-domain PSAM approach to estimate the channel was analyzed in [6][8], where, as suggested in [10][11], known pilots are grouped in P clusters, each of length  $L_P$ , and interleaved with the data to form the overall transmitted vector  $\mathbf{s} = [\mathbf{d}^{(1)^T}, \mathbf{p}^{(1)^T}, \cdots, \mathbf{d}^{(P)^T}, \mathbf{p}^{(P)^T}, \mathbf{d}^{(P+1)^T}]^T$ , as shown in Fig 1.

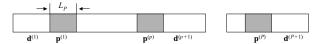


Fig. 1. Pilot placement.

Due to the fact that  $\mathbf{H}_F$  in (2) is almost banded, and neglecting its residual values out of the bandwidth B+1, we set,  $L_P=2B+1=4Q+1$ , P=L+1, where  $N_P=P(2B+1)$  and  $N_d=N-N_P$  are the total number of pilot and data symbols, denoted by  $\mathbf{p}=[\mathbf{p}^{(1)T},\cdots,\mathbf{p}^{(P)T}]^T$  and  $\mathbf{d}=[\mathbf{d}^{(1)T},\cdots,\mathbf{d}^{(P+1)T}]^T$ , respectively. The cluster  $\mathbf{d}^{(P+1)}$ , which is present only when  $N_d$  is not a multiple of P, has dimension  $N_d-P\lfloor N_d/P\rfloor$ . The B+1=2Q+1 samples of the received vector  $\mathbf{y}$  that correspond to the  $m^{th}$  pilot cluster can be approximated by

$$\tilde{\mathbf{y}}^{(m)} \approx \sum_{q=-Q}^{Q} \mathbf{C}_{q,m}^{(p)} \Delta_{q}^{(p)} \mathbf{p} + \mathbf{n}_{F}^{(m)}, \qquad (3)$$

if the data-induced interference is negligible, as in the optimal FDKD training [11], where each cluster  $\mathbf{p}^{(p)}$  contains a single pilot surrounded by B zeros at each edge. In (3)  $\{\mathbf{C}_{q,m}^{(p)}\}$  are the  $(B+1)\times N_p$  matrices representing the submatrices of  $\mathbf{C}_q$  that operate on the pilots, and  $\{\Delta_q^{(p)}\}$  are  $N_p\times N_p$  diagonal matrices carved out from  $\Delta_q$ , and correspond to the pilot positions (see [6] for further details). Moreover,  $\mathbf{n}_p^{(m)}$  is the corresponding part of  $\mathbf{n}_F$ . In order to make explicit the dependence on  $\mathbf{h} = [\mathbf{h}^{(-Q)T}, \cdots, \mathbf{h}^{(Q)T}]^T$ , equation (11) can be opportunely rearranged as

$$\tilde{\mathbf{y}}^{(m)} = \mathbf{C}^{(m)} \mathbf{V}_{n} \mathbf{h} + \mathbf{n}_{F}^{(m)}, \tag{4}$$

where  $\mathbf{C}^{(m)} = [\mathbf{C}_{-\mathcal{Q},m}^{(p)},\cdots,\mathbf{C}_{\mathcal{Q},m}^{(p)}]$ ,  $\mathbf{V}_p = \mathbf{I}_{2\mathcal{Q}+1} \otimes diag(\mathbf{p}) \, \mathbf{F}_l^{(p)}$ , and  $\mathbf{F}_l^{(p)}$  collects the rows of  $\mathbf{F}_L$  corresponding to the pilots positions. Stacking all the data in the column vector  $\tilde{\mathbf{y}} = [\tilde{\mathbf{y}}^{(1)^T} \cdots \tilde{\mathbf{y}}^{(P)^T}]^T$ , we obtain the estimation problem

$$\tilde{\mathbf{y}} = \begin{bmatrix} \mathbf{C}^{(1)} \mathbf{V}_p \\ \vdots \\ \mathbf{C}^{(P)} \mathbf{V}_p \end{bmatrix} \mathbf{h} + \begin{bmatrix} \mathbf{n}_F^{(1)} \\ \vdots \\ \mathbf{n}_F^{(P)} \end{bmatrix} = \mathbf{P} \mathbf{h} + \tilde{\mathbf{n}}_F , \qquad (5)$$

where **P** is known and depends on pilot values and positions [8].

#### 5. KALMAN FILTER CHANNEL ESTIMATOR

As shown in [6], a 1<sup>st</sup> order Gauss-Markov process is accurate enough to model the variation of the BEM coefficients from an OFDM block to the next. Thus, it is possible to assume

$$\mathbf{h}_k = \mathbf{A}\mathbf{h}_{k-1} + \mathbf{v}_k,\tag{6}$$

where  $\mathbf{h}_k$  are the BEM coefficients that represent the channel during the  $k^{th}$  OFDM block,  $\mathbf{A}$  drives the model evolution, and  $\mathbf{v}_k$  is

the channel innovation, characterized by

$$E(\mathbf{v}_k) = \mathbf{0}_{N_k \times 1}; \ E(\mathbf{h}_k \mathbf{v}_{k-m}^H) = \mathbf{0}_{N_k}; \ E(\mathbf{v}_k \mathbf{v}_{k-m}^H) = \mathbf{Q} \delta[m] \ . \tag{7}$$

Thus, for the  $k^{th}$  OFDM block, (5) can be written as

$$\tilde{\mathbf{y}}_{k} = \mathbf{P}\mathbf{h}_{k} + \tilde{\mathbf{n}}_{k} , \qquad (8)$$

where we omitted the subscript  $_F$  for simplicity.

The Kalman filtering algorithm for the model described by (6) and (8) is classically expressed as follows [12]

$$\mathbf{M}_{k}^{F} = \mathbf{A}\mathbf{M}_{k-1}^{A}\mathbf{A}^{H} + \mathbf{Q} \qquad Forward \ Error \ Cov.$$

$$\mathbf{K}_{k} = \mathbf{M}_{k}^{F}\mathbf{P}^{H}(\mathbf{R}_{\tilde{n}} + \mathbf{P}\mathbf{M}_{k}^{F}\mathbf{P}^{H})^{-1} \qquad Kalman \ Gain$$

$$\hat{\mathbf{h}}_{k} = \mathbf{A}\hat{\mathbf{h}}_{k-1} + \mathbf{K}_{k}(\tilde{\mathbf{y}}_{k} - \mathbf{P}\mathbf{A}\hat{\mathbf{h}}_{k-1}) \qquad Posteriori \ Estimation$$

$$\mathbf{M}_{k}^{A} = (\mathbf{I} - \mathbf{K}_{k}\mathbf{P})\mathbf{M}_{k}^{F} \qquad Posteriori \ Error \ Cov.$$

$$(9)$$

where, for wide sense stationary (WSS) channel statistics, **A** and **Q** can be easily derived [6] by Yule-Walker equations [12]. However, **A** and **Q** can be adaptively estimated also in non-WSS environments (e.g., when the Doppler spread  $\nu_D$  changes due to changes of the mobile speed). We refer to [6] for the Kalman filter initialization, recursion and model update (i.e.,  $\mathbf{A}_k, \mathbf{Q}_k$ ).

### 6. DATA-AIDED VIRTUAL PSAM

In order to reduce the rate penalty  $N_p/N$  induced by the PSAM approach, we propose to periodically alternate PSAM OFDM blocks of Fig.1 with V-PSAM OFDM blocks, where the transmitted vector  $\mathbf{s}$  contains only data. V-PSAM channel estimation/tracking is pursued by the following steps (S).

S-1) The channel values for the  $k^{th}$  block are predicted from the values estimated in the previous block. The optimum predictor for the Kalman filter, is well known to be [12]

$$\hat{\mathbf{h}}_{k}^{(p)} = \mathbf{A}\hat{\mathbf{h}}_{k-1}. \tag{10}$$

S-2) The channel prediction  $\hat{\mathbf{h}}_k^{(p)}$  is used to recover the data transmitted during the  $k^{th}$  block from the received vector  $\mathbf{y}_k$  of (2) by

$$\hat{\mathbf{s}}_{k}^{(1)} = f_{dec}(\check{\mathbf{y}}_{k}^{(1)}), \qquad \check{\mathbf{y}}_{k}^{(1)} = g_{eq}(\mathbf{y}_{k}; \hat{\mathbf{h}}_{k}^{(p)}), \qquad (11)$$

where  $g_{eq}(\cdot;\mathbf{h})$  represents a suitable equalization strategy (linear or non linear) for a given channel  $\mathbf{h}$ , and  $f_{dec}(\cdot)$  the hard decision. S-3) A set  $\overline{\Omega}^{(1)}$  of  $N_p$  indexes has to be selected from the total ordered set of active subcarriers  $\Omega_N = <1,2,\cdots,N>$ , according to an opportune metric, in order to identify as virtual pilots (VP) those subcarriers where data have been detected without any error, with the highest probability. Thus, the VP vector  $\mathbf{p}_{\underline{u}}^{(1)}$  obtained selecting the elements of  $\hat{\mathbf{s}}_k^{(1)}$  that correspond to the set  $\Omega^{(1)}$ , is expressed by

$$\mathbf{p}_{v}^{(1)} = \hat{\mathbf{s}}_{k}^{(1)}(\overline{\Omega}^{(1)}), \qquad (12)$$

where, taking advantage of the soft information contained in the equalized vector  $\tilde{\mathbf{y}}_k^{(1)}$ , the virtual pilot set is selected according to

$$\overline{\Omega}^{(1)} = \underset{\Omega \subset \Omega_{N}}{\arg\min} \{ P\{ \hat{\mathbf{s}}_{k}^{(1)}(\Omega) \neq \mathbf{s}_{k}(\Omega) \, | \, \overline{\mathbf{y}}_{k}^{(1)}(\Omega) \} \} . \tag{13}$$

S-4) The first VP-aided estimation  $\hat{\mathbf{h}}_k^{(1)}$  of the channel is obtained by using the VP vector  $\mathbf{p}_v^{(1)}$  instead of  $\mathbf{p}$  to generate the matrix  $\mathbf{P}$  in (5) (see also (3)); then solve the estimation problem of (8) by updating the Kalman filter estimator in (9).

S-5) A more reliable data vector  $\hat{\mathbf{s}}_k^{(2)}$  is obtained by a second equalization/detection, plugging the refined estimate  $\hat{\mathbf{h}}_k^{(5)}$  in (11). S-6) Denoting with  $N_{eq}$  the total number of equalizations for each OFDM block, iterate  $(N_{eq}-2)$  times the steps S-3, S-4 and S-5, to finally obtain  $\hat{\mathbf{h}}_k^{(N_{eq}-1)}$  and  $\hat{\mathbf{s}}_k^{(N_{eq})}$ .

S-7) (Optional) In order to improve the channel prediction for the next OFDM block, perform S-3) and S-4) to obtain  $\hat{\mathbf{h}}_k^{(N_{eq})}$ .

S-8) Move to the next OFDM block and restart from Step S-1.

# **6.1 Virtual Pilot Selection**

In order to solve the VP selection problem (13), a reasonable simplifying assumption is the independence of correct decisions on each subcarrier. Thus, defining  $\Omega$  a pilot pattern that belongs to the set  $\Omega_{N_n} = \{ \langle \omega_1, \omega_2, \dots, \omega_{N_n} \rangle : \omega_i \in [1, N] \subset \mathbb{N}^+ \}$  of all the possible patterns, we can express the probability of erroneous detection on pilot pattern  $\,\Omega\,$  conditioned on the soft equalizer output  $\,\check{\mathbf{y}}_{\scriptscriptstyle k}(\Omega)\,$  as

$$P\{\hat{\mathbf{s}}_{k}(\Omega) \neq \mathbf{s}_{k}(\Omega) \mid \breve{\mathbf{y}}_{k}(\Omega)\} = 1 - P\{\hat{\mathbf{s}}_{k}(\Omega) = \mathbf{s}_{k}(\Omega) \mid \breve{\mathbf{y}}_{k}(\Omega)\}$$

$$= 1 - \prod_{i=1}^{N_{p}} P\{[\hat{\mathbf{s}}_{k}(\Omega)]_{i} = [\mathbf{s}_{k}(\Omega)]_{i} \mid [\breve{\mathbf{y}}_{k}(\Omega)]_{i}\},$$
(14)

which allows to rewrite (13) as

$$\overline{\Omega} = \underset{\Omega \in \Omega_{N_p}}{\operatorname{arg\,max}} \prod_{i=1}^{N_p} P\{ [\hat{\mathbf{s}}_k(\Omega)]_i = [\mathbf{s}_k(\Omega)]_i \mid [\check{\mathbf{y}}_k(\Omega)]_i \}. \tag{15}$$

This problem resembles a vector generalization of the problem to find the most reliable layer in a NC-MIMO detection scheme [9]. For a block LMMSE equalizer  $g_{eq}(\cdot; \mathbf{h})$  and QPSK, [9] proved that the most reliable single layer/subcarrier can be selected by

$$i_{opt} = \underset{i \in \{1,..,N\}}{\arg \max} P\{[\hat{\mathbf{s}}_k]_i = [\mathbf{s}_k]_i \mid [\breve{\mathbf{y}}_k]_i\} \approx \underset{i \in \{1,..,N\}}{\arg \max} \left\{ \frac{|[\breve{\mathbf{y}}_k]_i|}{MSE_i} \right\}, \quad (16)$$

where  $MSE_i = E\{|[\breve{\mathbf{y}}_k - \mathbf{s}_k]_i|^2\}$ . Expressions (15) and (16) would suggest to select the  $N_p$  entries of  $\overline{\Omega} = \langle \overline{\omega}_1, \overline{\omega}_2, \cdots, \overline{\omega}_{N_p} \rangle$  as those indexes among all the N subcarriers with the first  $N_p$  most reliable metrics (16). This is equivalent to maximize the product of the metrics, thus leading to

$$\overline{\Omega} = \underset{\Omega \in \Omega_{N_p}}{\arg \max} \prod_{i=1}^{N_p} \frac{|[\overline{\mathbf{y}}_k]_i|}{MSE_i}.$$
(17)

This solution of the VP selection problem is characterized by a quite high complexity due to the evaluation of (17) over all the possible  $N!/(N_p!(N-N_p)!)$  combinations of  $N_p$  subcarriers out of N. Moreover, the almost random VP pattern that emerges from the unconstrained optimization (13), although reliability-wise optimal (for LMMSE equalizers), could lead to an ill-conditioned matrix **P** in the estimation problem (8), thus potentially causing a significant estimation performance loss.

In order to both reduce the implementation complexity and avoid an ill-conditioned estimation, a smart idea is to constrain the VPs to a set  $\Omega_{N_p}^{(eq)} \subset \Omega_{N_p}$  of P equally-spaced  $L_p$ -long clusters defined by  $\Omega_{N_p}^{(eq)} = \{\Omega_{L_p}(i_1) \cup \cdots \cup \Omega_{L_p}(i_1 + (P-1)\Delta) : i_1 \in \mathbb{N}_{\text{mod}\Delta}\}, \quad (18)$ 

$$O^{(eq)} = \{O_{-}(i) \cup \cdots \cup O_{-}(i+(P-1)\Lambda): i \in \mathbb{N} \}$$
 (18)

where  $\Omega_{L_p}(i_1) = \langle i_1, i_1 + 1, \dots, i_1 + (L_p - 1) \rangle$ , and  $\Delta = \lfloor N/P \rfloor$  is the distance between two successive pilot clusters. This reduces to  $\Delta$ the searches for the most reliable VP set, where  $\Delta$  represents all the possible VP patterns obtainable as shifted versions of the equi-spaced PSAM training pilots of Fig.1. Noteworthy, in V-PSAM the VP cannot be the optimal FDKD training [11] that we exploit during the PSAM mode, because in V-PSAM mode there are not zeros, but just data. However, groups of equi-spaced clusters [14], although suboptimal with respect to the optimal FDKD, are reliable for channel estimation of TV OFDM systems [13], reducing both the ill-conditioning of eq. (5) (thanks to the almost "uniform" sounding of the channel) and the data-induced ICI.

However, care must be taken in exploiting (17) to solve (15) when the search is constrained over the subset of equi-spaced clusters  $\Omega_{N_{-}}^{(eq)}$  rather than over the total set  $\Omega_{N_{-}}$ . For instance, in the constrained case, a VP pattern with all the subcarriers characterized by an average reliability (16), but not maximum product (17), could be preferable to a second pattern with maximum product (17) and with a very high reliability (16) for all-but-one subcarriers. Indeed, if this worst subcarrier is not reliable, it could be in error, differently from all the subcarriers in the first pattern. This intuition can be formalized by noting that, at reasonably low BER, the probability of erroneous detection in (13) can be approximated by the probability to have just one error, which dominates the error events, as

$$P\{\hat{\mathbf{s}}_{k}^{(1)}(\Omega) \neq \mathbf{s}_{k}(\Omega)\} = P\{1 \text{ err}\} + \sum_{n=2}^{N_{p}} P\{n \text{ err}\} \approx P\{1 \text{ err}\}.$$
 (19)

In this view, a global reliability function that minimizes the probability to have a pilot pattern with a single subcarrier in error is the one that maximizes the reliability of the less reliable subcarrier in the pattern. Thus, we have to maximize the minimum rather than the product of the metrics of all the subcarriers that, exploiting (16) for LMMSE equalizers, leads to the following pilot selection rule

$$\overline{\Omega}^{(eq)} = \underset{\Omega \in \Omega_{N_p}^{(eq)}}{\operatorname{arg\,max\,min}} \left\{ \frac{|[\check{\mathbf{y}}_k]_{\omega_i}|}{MSE_{\omega_i}} \right\}. \tag{20}$$

Actually, (16) is a combining of two other metrics [9]: the first is the classical average selection metric of the NC-MIMO receiver

$$i_{MSE} = \underset{i \in \{1,..,N\}}{\operatorname{arg max}} \left\{ 1 / MSE_i \right\}, \tag{21}$$

which depends on the channel  $MSE_i = E\{|[\breve{\mathbf{y}} - \mathbf{s}]_i|^2\}$  of the soft-output  $\breve{\mathbf{y}}$  of the MMSE equalizer  $g_{mmse}(\cdot;\mathbf{h})$ ; the second is the instantaneous reliability metric of the equalizer soft output  $[\breve{\mathbf{y}}]_i$  for the  $i^{th}$  layer/subcarrier, expressed by

$$i_{Abs} = \underset{i \in \{1, \dots, N\}}{\arg \max} \left\{ \left| \left[ \mathbf{\breve{y}}_{k} \right]_{i} \right| \right\}. \tag{22}$$

Expressions (21) and (22) are possible alternative reliability metrics, for those equalizers for which the metric (16) is not guaranteed to be optimal, such as the BDFE or the low-complexity banded equalizers of [5]. The performance of selection metrics (20), (21) and (22), as well as the performance of the minimum and product metric combiners (20) and (17) will be compared by simulations in the next section to assess the best strategy to weapon the Kalman filter tracking in V-PSAM mode.

# 7. SIMULATION RESULTS

We show simulation results for QPSK OFDM systems with  $N_A$ active carriers,  $N_V = N - N_A$  guard-band subcarriers, and a Jakes' channel with constant power delay profile. The MBAE-SOE design proposed in [5] is employed for the receiver window  $\mathbf{w}$ . We show results for N = 256,  $N_A = 244$ , Q = 2, L = 4, B = 2Q = 4 and due to space limitation only for a Doppler spread  $v_D = 10.24\%$ , which represents a quite high mobility scenario and a hard test for channel tracking/equalization of OFDM systems not equipped by a permanent PSAM training.

Simulations were organized by transmitting three PSAM OFDM blocks followed by five V-PSAM OFDM blocks. The transmission of these eight OFDM blocks was iterated 250 times in order to test the average tracking capability of the Kalman filter in the data-aided scenario. Performances are shown in terms of channel estimation normalized mean squared error (NMSE) [6] as well as in terms of the data BER, obtained by coupling the proposed channel estimation/tracking strategy with the low-complexity banded-LMMSE and banded-BDFE equalizers proposed in [5]. NMSE and BER are in a close liaison in this context due to the data-aided channel tracking in the V-PSAM mode. Indeed, a very low BER on the VP is necessary to obtain a good channel estimate (low NMSE), and vice versa. Thus we will use the BER on the VP to establish which selection metric among the proposed ones, most reliably identifies the detected data. Fig.2 shows the BER at the first V-PSAM block (the 4<sup>th</sup> in our test) for a banded-LMMSE equalizer: it is evident the capability of the "minimum" reliability metric (20) to select as VP those subcarriers with BERs significantly lower than the data BER after the first equalization step S-2. Fig.2 also shows that, as expected for full-LMMSE equalizers, the "Combined" metric (16) outperforms the separate "MSE" and "Abs" metrics ((21) and (22)). The slight advantage in terms of data BER obtained after the second equalization/detection of step S-5, has not to be disregarded in a tracking mode scenario. Indeed, due to the prediction step S-1, the three metrics could lead to significant different performance after few OFDM blocks (see also Fig.4 for BER time evolution). Fig. 3 shows similar results for a banded-BDFE equalizer [5], where the "Combined" metric is no longer optimal and the intuitive "Abs" metric shows the best performance. Fig. 4 shows, for a fixed SNR, the NMSE and BER evolution of a banded-BDFE for the eight OFDM blocks. We compared  $N_{eq} = 2$  with  $N_{eq} = 3$  equalizations, using the "Abs" metric. Note that all the BERs at a given OFDM block depend on the number of channel estimates performed at the previous blocks. It is evident that the two (S-4 steps) channel estimates performed in our procedure when  $N_{eq} = 3$  give excellent performance both in terms of NMSE and data BER, which during the V-PSAM mode (blocks 4-8) are almost equivalent to those in PSAM mode (blocks 1-3). However, the third equalization (S-5 at the second iteration), does not improve significantly the performance in terms of BER. Thus, the suggested choice is to perform two equalizations (S-2 and S-5) with  $N_{eq} = 2$  and enable option S-7 for the second channel estimation. Fig.4 also shows that the "minimum" combining strategy (20), under the constraint (18), outperforms, as expected, the "product" combining (17).

### 8. CONCLUSIONS

A data-aided Kalman channel tracking for OFDM system in fast time-varying channels has been considered. The suggested technique effectively tracks the channel changes in the data-aided mode. It has been proved by simulations that for low-complexity banded-LMMSE and BDFE equalizers, the channel estimation NMSE penalty, as well as the BER degradation, could be taken under control by an opportune selection of the most reliably detected data. A deeper analytical insight on the performance limit of the proposed technique could be the subject of future work.

# 9. REFERENCES

- [1] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications: where Fourier meets Shannon," *IEEE Sig. Proc. Magazine*, vol. 17, pp. 29–48, May 2000.
- [2] M. Russell and G. L. Stuber, "Interchannel interference analysis of OFDM in a mobile environment," *IEEE VTC'95*, pp. 820–824, Jul 1995.
- [3] A. Gorokhov and J.-P. Linnartz, "Robust OFDM receivers for dispersive time-varying channels: Equalization and channel acquisition," *IEEE Trans. Commun.*, vol. 52, pp. 572-583, Apr. 2004.
- [4] P. Schniter, "Low-complexity equalization of OFDM in doubly selective channels," *IEEE Trans. Sig. Proc.*, vol. 52, pp. 1002–1011, Apr 2004.
- [5] L. Rugini, P. Banelli, G. Leus, "Low-complexity banded equalizers for OFDM systems in Doppler spread channels," *EURASIP Journal on Applied Signal Processing*, vol. 2006, Article ID 67404, 2006.
- [6] C.R. Cannizzaro, P. Banelli, G. Leus, "Adaptive channel estimation for OFDM systems with Doppler spreads," *IEEE SPAWC'06*, Cannes, France, July 2006.
- [7] M. K. Tsatsanis and G. B. Giannakis, "Modeling and equalization of rapidly fading channels," *Int. Jour. of Adapt. Control and Sig. Proc.*, vol. 10, pp. 159–176, Mar 1996.
- [8] Z. Tang, R. C. Cannizzaro, G. Leus, P. Banelli "Pilot-assisted time-varying channel estimation for OFDM systems," accepted on *IEEE Trans. Sig. Proc.*, July 2006. [online] <a href="http://cas.et.tudelft.nl/~leus/publications.html">http://cas.et.tudelft.nl/~leus/publications.html</a>

- [9] D. Seethaler, H. Artés and F. Hlawatsch, "Dynamic nulling and cancelling with near-ML performance for MIMO communication systems," *IEEE ICASSP* '04, vol. 4, pp.777–790, May 2004.
- [10] X. Ma, G. B. Giannakis, and S. Ohno, "Optimal training for block transmissions over doubly-selective wireless fading channels," IEEE Trans. Sig. Proc., vol. 51, pp. 1351–1366, May 2003.
- Sig. Proc., vol. 51, pp. 1351–1366, May 2003.
  [11] A. P. Kannu and P. Schniter, "MSE-optimal training for linear time-variyng channels," *IEEE ICASSP'05*, vol. 3, pp. 789–792, Mar. 2005.
- [12] S. Haykin, Adaptive Filter Theory, 3rd ed., Prentice-Hall, 1996.
- [13] A. Stamoulis, S. N. Diggavi, N. Al-Dhahir, "Intercarrier interference in MIMO OFDM," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 2451-2464, Oct. 2002. [14] M.Dong; L. Tong, "Optimal design and placement of pilot symbols for channel estimation," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 3055-3069, Dec. 2002.

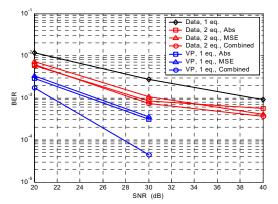


Fig.2. Banded-LMMSE with "minimum" metric combiner.

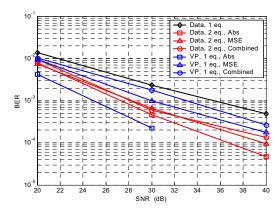


Fig.3. Banded-BDFE with "minimum" metric combiner.

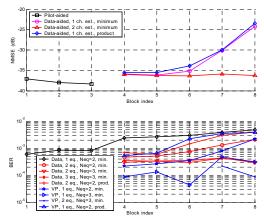


Fig.4. BER and NMSE evolution for Banded-BDFE (SNR=30 dB).