EFFICIENT METRICS FOR SCHEDULING IN MIMO BROADCAST CHANNELS WITH LIMITED FEEDBACK

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ABSTRACT

We consider a downlink channel where a base station equipped with M transmit antennas communicates with $K \geq M$ single-antenna receivers and has partial channel knowledge obtained via a limited rate feedback channel. We propose scalar feedback metrics that provide an estimate of the received signal-to-noise plus interference ratio (SINR), which are combined with efficient user selection algorithms and zero-forcing beamforming. The asymptotic system sum rate for large K is analyzed and numerical results are provided, showing the performance of each metric in different scenarios.

***Index Terms*—** MIMO systems, Broadcast Channel, Limited Feedback, Multiuser Diversity, Scheduling.

1. INTRODUCTION

In multiple antenna broadcast channels, the capacity can be boosted by exploiting the spatial multiplexing capability of transmit antennas and transmit to multiple users simultaneously, by means of space division multiple access (SDMA). As the capacity-achieving dirty paper coding (DPC) is difficult to implement, downlink linear beamforming, although suboptimal, is of primary interest, as it has been shown to achieve a large portion of DPC capacity, exhibiting reduced complexity [1], [2]. Nevertheless, the capacity gain of multiuser multiple-input multiple-output (MIMO) systems seems to remain highly sensitive and dependent on the channel state information available at the transmitter (CSIT). Differently from the single-user case, the quality of CSIT affects the multiplexing gain of MIMO broadcast channels.

A finite rate feedback model for multiuser MIMO downlink channels for the case when the total number of users K equals the number of transmit antennas M was proposed in [3]. It was shown that the number of feedback bits per user must increase approximately linearly with the number of transmit antennas and the signal-to-noise ratio (SNR) (in dB) in order to achieve the full multiplexing gain. In this context, each user feeds back finite precision CSIT on its channel direction by quantizing its channel to the closest vector contained in a predetermined codebook. The system uses zero-forcing beamforming in conjunction with channel direction information (CDI). An alternative, very low-rate (scalar) feedback technique, coined as random opportunistic beamforming was proposed in [4], where M random orthonormal beamforming vectors are generated and the best user on each beam is scheduled. By exploiting multiuser diversity [5], this scheme is shown to yield the optimal capacity scaling of $M \log\log K$ for large number of users. However, the sum rate performance of this scheme is quickly degrading as the number of users decreases.

In this paper, we consider the finite rate feedback model of [3] for the case when $K \geq M$. In this scenario, information on the channel direction is not sufficient in order to benefit from multiuser diversity, and should be complemented with additional instantaneous channel quality information (CQI), as a means to intelligently select M spatially separable users with large channel gains. We suggest several scalar feedback metrics in the form of effective channel, which encapsulate information on the channel gain, as well as on the multiuser interference. These metrics can be interpreted as an estimate of the received signal-to-interference plus noise ratio (SINR). We combine them with greedy user selection algorithms and a system employing zero-forcing beamforming on the channel quantizations. The sum rate performance of the resulting schemes is analyzed, showing the utility and the gains of each metric in different SNR regions.

2. SYSTEM MODEL

We consider a multiple antenna broadcast channel with M transmit antennas and $K \geq M$ single-antenna receivers. The received signal y_k of the k-th user is mathematically described as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \qquad k = 1, \ldots, K \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the transmitted signal, $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is the channel vector, assumed to be perfectly known by each user k, and n_k is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian noise with zero mean and unit variance. The transmitted signal is subject to an average transmit power constraint $\mathbb{E}\{\|\mathbf{x}\|^2\} = P$. We consider an i.i.d. block Rayleigh flat fading channel and an homogeneous network where all users have the same average SNR.

2.1. CDI Finite Rate Feedback Model

Consider a quantization codebook $\mathcal{V}_k = \{\mathbf{v}_{k1}, \mathbf{v}_{k2}, \ldots, \mathbf{v}_{kN}\}$ containing $N = 2^B$ unit norm vectors $\mathbf{v}_{ki} \in \mathbb{C}^M$, known to both the k-th receiver and the transmitter. Each receiver quantizes its channel to the vector that maximizes the following inner product [6], [7], [8]

$$\hat{\mathbf{h}}_k = \mathbf{v}_{kn} = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} |\bar{\mathbf{h}}_k^H \mathbf{v}_{ki}|^2 = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} \cos^2(\angle(\bar{\mathbf{h}}_k, \mathbf{v}_{ki}))$$

where $\bar{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$ corresponds to the channel direction. The corresponding quantization index n is sent back to the transmitter through an error-free, and zero-delay feedback channel using $B = \lceil \log_2 N \rceil$ bits. Clearly the performance of a system relying on quantized CSI depends on the codebook choice, but the problem

The research of the authors at Eurecom was supported in part by the Eurecom Institute, and by the national RNRT project Lao Tseu.

of optimum codebook design is not yet fully solved and it is beyond the scope of the paper. Note also that the complexity of using a different codebook for each user can be reduced by generating a common, general codebook $\mathcal{V}_g$ known at both ends of the link, and afterwards each user obtains its specific codebook through random unitary rotation of $\mathcal{V}_g$.

2.2. Zero Forcing Beamforming

Let $\mathbf{w}_k$ and s_k be the (normalized) beamforming vector and data symbol of the k-th user, respectively. Define $\mathbf{H} \in \mathbb{C}^{K\times M}$ as the concatenation of all user channels, $\mathbf{H} = [\mathbf{h}_1\, \mathbf{h}_2 \ldots \mathbf{h}_K]^H$. Let $\mathcal{G} = \{1, \cdots, K\}$ be the set of indices of all K users. Let $\mathcal{S} \in \mathcal{G}$, be one such group of $|\mathcal{S}| = \mathcal{M} \leq M$ users selected for transmission at a given time slot. Then $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, normalized beamforming vectors, uncorrelated data symbols and received signals respectively for the set of scheduled users. The signal model is given by

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathcal{P}\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{2}$$

where $\mathcal{P}$ is a diagonal power allocation matrix. We propose to use zero-forcing beamforming on the quantized channel directions available at the transmitter as a multiuser transmission strategy. The beamforming matrix is then given by

$$\mathbf{W}(\mathcal{S}) = \hat{\mathbf{H}}(\mathcal{S})^{\dagger} = \hat{\mathbf{H}}(\mathcal{S})\left(\hat{\mathbf{H}}(\mathcal{S})^H\hat{\mathbf{H}}(\mathcal{S})\right)^{-1}\mathbf{\Lambda} \tag{3}$$

where $\hat{\mathbf{H}}(\mathcal{S})$ is a matrix whose columns are the quantized channels $\hat{\mathbf{h}}_k$ (codevectors) of the users selected for transmission and $\mathbf{\Lambda}$ is a diagonal matrix that normalizes the columns of $\mathbf{W}(\mathcal{S})$. The SINR at the k-th receiver is

$$SINR_k = \frac{P_k|\mathbf{h}_k^H\mathbf{w}_k|^2}{\sum\limits_{j\in\mathcal{S}-\{k\}} P_j|\mathbf{h}_k^H\mathbf{w}_j|^2 + 1} \tag{4}$$

where $\sum_{i\in\mathcal{S}} P_i = P$ in order to satisfy the power constraint on the transmitted signal. We focus on the ergodic data rate which, assuming Gaussian inputs, is equal to

$$\mathcal{R}_k = \mathbb{E}\left\{\sum_{k\in\mathcal{S}}\log\left(1 + SINR_k\right)\right\} \tag{5}$$

3. EFFICIENT FEEDBACK METRICS EXPLOITING MULTIUSER DIVERSITY

In this section, we consider the problem of efficient feedback design under a feedback rate constraint and seek a scalar feedback metric ξ that allows us to exploit the multiuser diversity and achieve near-optimal capacity performance. Intuitively, the metric has to incorporate information on the channel gain and the quantized channel direction, as well as on the channel quantization error.

3.1. Metric I: Upper Bound on SINR

Let $\phi_k = \angle(\hat{\mathbf{h}}_k, \bar{\mathbf{h}}_k)$ be the angle between the normalized channel vector and the quantized channel direction. We consider that each user provides information on its effective channel (SINR) by feeding back the following scalar metric

$$\xi_k^{UB} = \frac{P\left\|\mathbf{h}_k\right\|^2\cos^2\phi_k}{P\left\|\mathbf{h}_k\right\|^2\sin^2\phi_k + M} \tag{6}$$

proposed in parallel in [9], [10], [11]. This type of CQI encapsulates information on the channel gain as well as the CDI quantization error, defined as $\sin^2\phi_k = 1 - \left|\hat{\mathbf{h}}_k^H\bar{\mathbf{h}}_k\right|^2$, and it can be interpreted as an upper bound on each user's received SINR in a system where equal power is allocated over M beamforming vectors. Thus, it offers a good estimate of the multiuser interference at the mobile side without any user cooperation. If M orthogonal users can be found and the beamforming vectors at the transmitter are perfectly orthogonal, the upper bound (6) becomes tight and corresponds to the actual SINR.

3.2. Metric II: Lower Bound on SINR

In practice, the SINR values predicted by (6) cannot be achieved, since in general the beamforming vectors are not perfectly orthogonal, especially in networks with low to moderate number of users. Hence, another approach is to feed back a lower bound of the k-th user SINR, and for that, an upper bound of the multiuser interference is required. Let $\cos\theta_k = \left|\bar{\mathbf{h}}_k^H\mathbf{w}_k\right|$, and define the matrix $\mathbf{\Psi}_k(\mathcal{S}) = \sum_{j\in\mathcal{S},j\neq k}\mathbf{w}_j\mathbf{w}_j^H$, the operator $\lambda_{max}\{\cdot\}$, which returns the largest eigenvalue, and $\mathbf{U}_k \in \mathbb{C}^{M\times(M-1)}$ an orthonormal basis spanning the null space of $\mathbf{w}_k$.

Theorem 1: *Given an arbitrary set $\mathcal{S}$ of unit-norm beamforming vectors, an upper bound on the interference over the normalized channel $\bar{I}_k(\mathcal{S}) = \sum_{j\in\mathcal{S},j\neq k}\left|\bar{\mathbf{h}}_k^H\mathbf{w}_j\right|^2$ experienced by the k-th user is given by*

$$\bar{I}_k(\mathcal{S}) \leq \cos^2\theta_k\alpha_k(\mathcal{S}) + \sin^2\theta_k\beta_k(\mathcal{S}) + 2\sin\theta_k\cos\theta_k\gamma_k(\mathcal{S}) \tag{7}$$

where

$$\begin{cases} \alpha_k(\mathcal{S}) = \mathbf{w}_k^H\mathbf{\Psi}_k(\mathcal{S})\mathbf{w}_k \\ \beta_k(\mathcal{S}) = \lambda_{max}\{\mathbf{U}_k^H\mathbf{\Psi}_k(\mathcal{S})\mathbf{U}_k\} \\ \gamma_k(\mathcal{S}) = \left\|\mathbf{U}_k^H\mathbf{\Psi}_k(\mathcal{S})\mathbf{w}_k\right\| \end{cases} \tag{8}$$

The theorem can be proven by expressing $\bar{\mathbf{I}}_k = \bar{\mathbf{h}}_k^H\mathbf{\Psi}_k(\mathcal{S})\bar{\mathbf{h}}_k$ and decomposing $\bar{\mathbf{h}}_k$ into two parts, one that lies onto $\mathbf{w}_k$ and one onto $\mathbf{U}_k$. Each of the resulting terms is bounded separately by using matrix analysis tools. Consider now that we impose an ϵ-orthogonality constraint between the quantized channels as in [11], as well as worst-case orthogonality under zero-forcing beamforming as $\epsilon_{ZF} = \max_{i,j\in\mathcal{S}}\left|\mathbf{w}_i^H\mathbf{w}_j\right|$. The dependence on $\mathcal{S}$ can be dropped, expressing the worst interference received by the k-th user in terms of $\cos\theta_k$ and ϵ_{ZF}.

Lemma 1: The orthogonality of the set of M zero-forcing normalized beamforming vectors (ϵ_{ZF}) and alignment with the normalized channel ($\cos\theta_k$) are bounded as a function of $\cos\phi_k$ and ϵ as follows:

$$\epsilon_{ZF} \leq \vartheta \text{ and } \cos\theta_k \geq \frac{\left|\cos\phi_k - \sqrt{\vartheta}\right|}{1+\vartheta} \text{ with } \vartheta = \frac{\epsilon}{1-(M-1)\epsilon}$$

Assuming equal power allocation, i.e. $P_k = P/M$, we obtain the following result:

Theorem 2 *Given a user set $\mathcal{S}$, with $|\mathcal{S}| = M$, constrained to be ϵ-orthogonal, a system that performs zero-forcing beamforming can guarantee the following SINR for the k-th user*

$$SINR_k^{ZF} \geq \frac{P\left\|\mathbf{h}_k\right\|^2\cos^2\theta_k}{P\left\|\mathbf{h}_k\right\|^2\bar{I}_{UB_k} + M} \tag{9}$$

where $\bar{I}_{UB_k} = \cos^2\theta_k\,(M-1)\epsilon_{ZF}^2 + \sin^2\theta_k\,[1+(M-2)\epsilon_{ZF}] + 2\sin\theta_k\cos\theta_k\,(M-1)\epsilon_{ZF}$

and $\cos\theta_k$, ϵ_{ZF} are as described in Lemma 1 substituting the inequalities by equalities.

The bound is found by expressing the worst-case interference received by the k-th user in terms of ϵ_{ZF} and $\cos\theta_k$ and substituting these values in (7). Motivated by the above results, we propose that each user provides information on its SINR lower bound and reports the following scalar metric

$$\xi_k^{LB} = \frac{\frac{P}{(1+\vartheta)^2}\|\mathbf{h}_k\|^2(\cos\phi_k - \sqrt{\vartheta})^2}{P\|\mathbf{h}_k\|^2\,\overline{I}_{UB_k} + M} \tag{10}$$

3.3. Metric III: Decoupling the feedback into two scalars

The main drawback of the above metrics is that they both estimate the SINR by assuming $\mathcal{M} = M$. However, in the high SNR regime, or for low number of users, it is better to transmit to $\mathcal{M} < M$ users. For that reason, we propose to decouple the CSIT by letting each user feed back the following two scalar values: 1) the alignment $\cos^2\phi_k$, and 2) the channel norm, $\|\mathbf{h}_k\|^2$. Under this feedback strategy, the base station is able to calculate a more accurate SINR estimate for any set of scheduled users with cardinality $\mathcal{M} \leq M$. Obviously, under fixed number of feedback bits, each scalar value is quantized with reduced accuracy compared to the case of only one scalar feedback metric. Based on Theorem 2, we suggest that the transmitter selects the user based on the following lower bound on the received SINR

$$\xi_k^{LBd} = \frac{P\|\mathbf{h}_k\|^2\rho_k^2}{P\|\mathbf{h}_k\|^2\,\overline{I}_{UBd_k} + \mathcal{M}} \tag{11}$$

where $\overline{I}_{UBd_k} = \rho_k^2\alpha_k(\mathcal{S}) + \left(1-\rho_k^2\right)\beta_k(\mathcal{S}) + 2\rho_k\sqrt{1-\rho_k^2}\gamma_k(\mathcal{S})$ can be explicitly calculated at the transmitter using (8), and $\rho_k^2 = \cos^2(\phi_k + \angle(\hat{\mathbf{h}}_k, \mathbf{w}_k))$. Note that as $\epsilon \to 0$, $\overline{\mathrm{I}}_{UBd_k} \to \sin^2\phi_k$, and a more refined metric can be used

$$\xi_k^{UBd} = \frac{P\|\mathbf{h}_k\|^2\rho_k^2}{P\|\mathbf{h}_k\|^2\sin^2\phi_k + \mathcal{M}} \tag{12}$$

4. JOINT SCHEDULING AND BEAMFORMING SCHEMES

The above metrics are combined with two user selection algorithms in a system that uses zero-forcing beamforming. In order to avoid the prohibitively high complexity of exhaustive search for K increasing, we focus on low-complexity, greedy user selection approaches.

4.1. Greedy-SUS algorithm

We review a heuristic user selection algorithm based on semi orthogonal user selection (SUS) [11]. Using ξ_k^{UB} and $\hat{\mathbf{h}}_k$, $k = 1,\ldots,K$, the base station performs user selection to support up to M out of K users at each time slot. The first user is selected from the set $\mathcal{Q}^0 = \{1,\ldots,K\}$ as the one having the highest channel quality, i.e., $k_1 = \arg\max_{k\in\mathcal{Q}^0}\xi_k^{UB}$. For $i = 1,\ldots,M$, the $(i+1)$-th user is selected as $k_{i+1} = \arg\max_{k\in\mathcal{Q}^i}\xi_k$ among the user set $\mathcal{Q}^i = \left\{1 \leq k \leq K : |\hat{\mathbf{h}}_k^H\hat{\mathbf{h}}_{k_j}| \leq \epsilon, 1 \leq j \leq i\right\}$. The system parameter ϵ defines the maximum allowed non-orthogonality (maximum correlation) between quantized channels and it is a parameter set in advance. The above algorithm selects users with high channel qualities and mutually semi-orthogonal quantized channel directions $\hat{\mathbf{h}}_k$. Evidently, if ϵ is very large, the selected users may cause significant multiuser interference, reducing the system sum rate. Conversely, if ϵ is too small, the scheduler cannot find enough semi-orthogonal users to transmit to.

4.2. Greedy-US algorithm

We propose here a low-complexity greedy user selection scheme, taken from [1]. Each user feeds back its quantized channel direction based on a predetermined codebook $\mathcal{V}_k$ and scalar instantaneous feedback, which are used to perform joint scheduling and beamforming with quantized CSIT. The algorithm is summarized as follows

Table 1. Outline of Greedy-US Algorithm

Step 0 Set $\mathcal{S}_0 = \emptyset$, $\mathcal{S} \subseteq \mathcal{G}$ and $\mathcal{R}(\mathcal{S}_0) = \emptyset$
For $i = 1, 2, \ldots, M$ repeat

Step 1 $k_i = \arg\max_{k\notin\mathcal{S}_{i-1}} \mathcal{R}(\mathcal{S}_{i-1}\cup\{k\})$

Step 2 if $\mathcal{R}(\mathcal{S}_{i-1}\cup\{k\}) < \mathcal{R}(\mathcal{S}_{i-1})$
set $\mathcal{S} = \mathcal{S}_{i-i}$ and break
else if $i = M$ then $\mathcal{S} = \mathcal{G}$ and break
else set $\mathcal{S}_i = \mathcal{S}_{i-1}\cup\{k\}$ and go to **Step 1**

where $\mathcal{R}(\mathcal{S}_i) = \sum_{k\in\mathcal{S}_i}\log_2(1+\xi_k)$, with ξ_k being either: ξ_k^{UB}, ξ_k^{LB}, ξ_k^{UBd} or ξ_k^{LBd}. As outlined in Table 1, the user with the highest rate (thus ξ_k metric) among K users is first selected, and at each iteration, a user is added only if the approximate sum rate (based on the estimated SINR) is increased. At each step, it is important to reprocess the set of previously selected users (thus, re-calculating the zero-forcing beamformers) once a user is added to the set $\mathcal{S}$. Note also that this algorithm does not necessarily need the use of a predetermined system parameter ϵ.

5. PERFORMANCE ANALYSIS

In this section, we analyze the sum rate performance $\mathcal{R}$ of a system using the Greedy-SUS algorithm in conjunction with (10) considering the asymptotic case of $K \to \infty$ and M fixed. A sum rate bound can be obtained as follows:

$$\mathcal{R} \geq M\left(1 - \Pr\{|\mathcal{S}| = 0\}\right)\log_2\left(1 + SINR_{min}^{LB}\right) \tag{13}$$

where $SINR_{min}^{LB} = \min_{k\in\mathcal{S}}\xi_k^{LB}$. Assuming that ϵ is set so that $\lim_{K\to\infty}\epsilon = 0$, then $SINR_{min}^{LB} \overset{K\to\infty}{=} \min_{k\in\mathcal{S}}\xi_k^{UB}$.

Using asymptotic results of [12], we have that, for sufficiently large K,

$$\xi_k^{UB} = \frac{P}{M}\log\left(\frac{K}{\delta}\right) + O(\log\log K), \qquad \delta = (P/M)^{M-1}2^{-B}$$

Since $\lim_{K\to\infty}\left(1 - \Pr\{|\mathcal{S}| = 0\}\right) = 1$, we can show that the expected system sum rate is asymptotically optimal as $K \to \infty$, i.e.

$$\lim_{K\to\infty}\left(\mathcal{R}_{ZF}^{opt} - \mathcal{R}\right) = \lim_{K\to\infty}\left[M\log_2\frac{1+\frac{P}{M}\log K}{1+\frac{P}{M}\log\left(\frac{K}{\delta}\right)}\right] = 0$$

where $\mathcal{R}_{ZF}^{opt}$ is the maximum sum rate achievable by zero-forcing with full CSIT, which at large K, $\mathcal{R}_{ZF}^{opt} \sim M\log_2\left(1+\frac{P}{M}\log K\right)$. Thus, the above result implies that, for large K, the sum rate of a system using an efficient user selection algorithm and beamforming based on these scalar feedback metrics converges to the optimum capacity scaling of the MIMO broadcast channel $M\log\log K$.

6. NUMERICAL RESULTS

We evaluate the sum rate performance of a system that performs zero-forcing beamforming using the presented scheduling algorithms and scalar feedback metrics. We consider $M = 2$ antennas, $\epsilon = 0.4$ and codebooks generated using random vector quantization (RVQ) [13] of size $B = 4$ bits. Random beamforming [4] and zero-forcing beamforming with full CSIT are also simulated as a performance reference. Fig. 1 shows similar performance for the scheme with Metric I and II, exhibiting the same bounded behavior at high SNR since M users are always scheduled. However, the scheme using Metric III in eq.(11) provides more flexibility by transmitting to $\mathcal{M} \leq M$ users, thus keeping a linear sum rate growth in the interference-limited region and converging to TDMA for $P \to \infty$, where $\mathcal{M} = 1$ is optimal. In Fig. 2 it can be seen that both scalar metrics can efficiently exploit the multiuser diversity gain. In a system with fixed orthogonality factor ϵ, the accuracy of the lower bound (metric II) does not improve as K increases. On the other hand, the upper bound (metric I) becomes more realistic due to a higher probability of finding orthogonal quantized channels, hence yielding better user selection.

7. CONCLUSION

We studied a multiple antenna broadcast channel in which partial CSIT is conveyed via a limited rate feedback channel. We proposed scalar feedback metrics which, combined with efficient joint scheduling and zero-forcing beamforming, can achieve a large portion of the optimum capacity by exploiting multiuser diversity.

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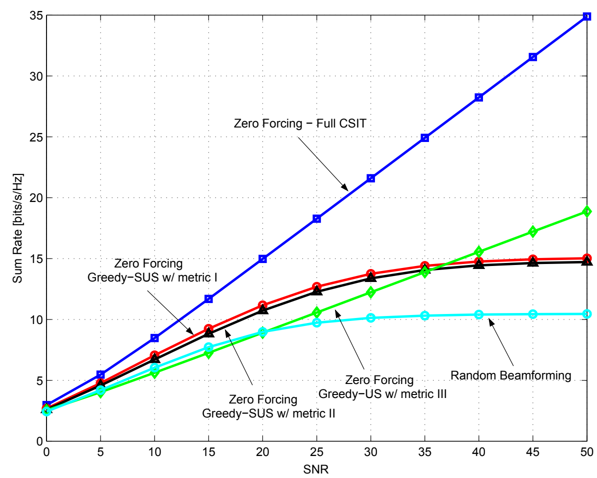

Fig. 1. Sum rate versus the average SNR for $B = 4$ bits, $M = 2$ transmit antennas and $K = 20$ users.

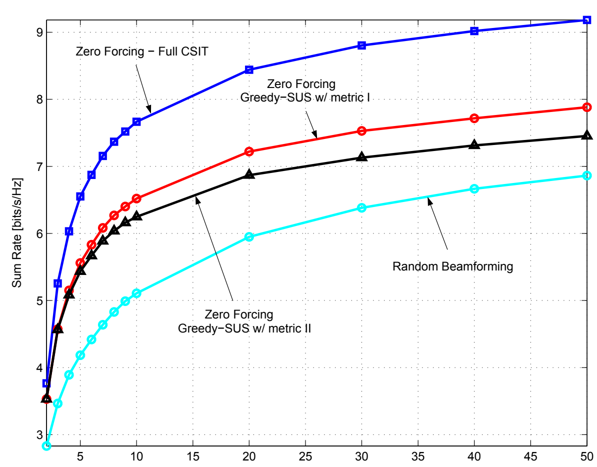

Fig. 2. Sum rate as a function of the number of users for $B = 4$ bits, $M = 2$ transmit antennas and $SNR = 10$ dB.