

AN ALGORITHM FOR 2-USER DOWNLINK SDMA BEAMFORMING WITH LIMITED FEEDBACK FOR MIMO-OFDM SYSTEMS

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ABSTRACT

Recent information theoretic results on MIMO broadcast channels have promoted research attention on devising practical transmit-receive strategies that achieve a reasonable fraction of the broadcast capacity. This paper proposes an algorithm for SDMA weight construction for two users. The weights are chosen to represent a subspace that lies between the dominant signal subspace of one user and the nullspace of the other. This proposal is unique due to its practical considerations - a broadband MIMO broadcast channel employing OFDM, imperfect CSI at the transmitter in the form of codebook based limited feedback, coded BER as a performance metric, and the ability to choose the users for SDMA arbitrarily. Simulation results in an IEEE 802.16e-style setup show that a coded BER within 1-2 dB of the case of perfect CSI can be achieved with a feedback of less than 1 bit per subcarrier. Additionally, maximum ratio combining receivers suffice and they may be oblivious to the multi-user nature of the SDMA transmission.

Index Terms— space division multiplexing, MIMO systems.

1. INTRODUCTION

The extra spatial dimension offered by a multiple-antenna array at the transmitter offers the possibility of transmitting simultaneously to multiple users. By modeling this scenario with a vector Gaussian broadcast channel model, the information theoretic capacity region has been derived [1]-[3], and dirty-paper coding (DPC) is an applicable transmission strategy having certain optimality criteria [1]. Since a potential for a multiplicative gain in the sum data-rate compared to a single-user transmission policy has been identified, significant research interest has been focused on devising a more practicable transmit-receive strategy than DPC. In this paper, we propose a strategy for broadcasting to two users over a broadband multiple-input multiple-output (MIMO) channel with limited feedback and evaluate it using an IEEE 802.16e-style simulation setup [4].

Space-division-multiple-access (SDMA) with linear transmit filters has been studied as a low-complexity, albeit sub-optimal method for multi-user transmission. Methods for constructing SDMA filters include zero-forcing (ZF) [5], minimum mean-squared error (MMSE) [6], solutions based on the generalized eigenvalue problem [7] or iterative algorithms [8], and these methods have been demonstrated to perform well with several criteria - sum-rate, SINR or BER. They, however, require perfect channel state information (CSI) for all users at the transmitter. Alternatively, methods to achieve a large fraction of the sum-rate by leveraging multi-user diversity have been proposed in [9],[10] and claim to reduce the per-user information required at the transmitter while requiring a large user population to guarantee

performance. It is evident that the quality of CSI at the transmitter is a bottleneck for implementing SDMA in a practical system.

Recent work in [11][12] takes a step further and actually quantizes the CSI at the mobiles before feeding it back to the transmitter using a limited capacity control channel. The results imply that maintaining a reasonable sum-rate performance leads to an explosion in the total feedback rate necessary for all the users. Additionally, a sum-rate criterion is sometimes restrictive due to the practicalities of adaptive modulation and coding (AMC) and channel considerations. This motivates us to consider a coded BER criterion (averaged across users) as our metric of performance. It may be noted that CSI can also be acquired for TDD systems using channel reciprocity but we restrict ourselves to a limited-feedback framework in this paper.

The main contribution of this paper is to demonstrate an algorithm for SDMA that is characterized by the following.

- We observe that a conventional SDMA weight construction algorithm (like ZF etc.) does not provide acceptable BER performance using coarse channel estimates derived from limited feedback. We propose a weight construction that is appropriate for limited feedback. The computed weights represent a mean-subspace between the dominant signal subspace of one user and the nullspace of the other.
- The mobile users can be oblivious to the SDMA nature of the transmission. Thus the receive processing and feedback is exactly the same as that for beamforming (MRT-MRC) in a limited feedback scenario [13]. This also means that the feedback overhead is the same as for beamforming.
- The algorithm selects the users arbitrarily and does not depend on multi-user diversity by selecting users from a user pool. Thus user scheduling and delay optimization is avoided. The coded BER performance can be improved by applying SDMA only to certain subcarriers or by using multiple receive antennas to suppress interference passively.
- The algorithm is evaluated using a COST-259-style channel model for MIMO-OFDM. QPSK symbols are coded (convolutional) over frequency.
- We assume two users chosen arbitrarily, single data-stream transmission to each user, perfect channel estimation at the mobiles and a zero-delay but highly quantized feedback link (imperfect CSI).

It may be mentioned that the algorithm, in theory, naturally generalizes to multiple data-stream transmission, though it is evaluated and optimized for practical implementation only for single-stream SDMA.

2. SYSTEM MODEL

A MIMO-OFDM system is considered with M_t transmit and M_r receive antennas and N data-carrying subcarriers for both users.

We assume that the transmit power is uniformly assigned to all the subcarriers and users. We consider the cyclic prefix to be longer than sampled impulse response of the channel. Then the channel for the k^{th} subcarrier after the discrete Fourier transform may be described by $M_r \times M_t$ matrices $\mathbf{H}_1(k)$ and $\mathbf{H}_2(k)$ for the two users. The received signals for the k^{th} subcarrier at the two users (denoted by subscripts 1,2) after combining may be expressed as

$$\begin{aligned} x_1(k) &= \mathbf{G}_1^H(k) \{ \mathbf{H}_1(k) \mathbf{W}_1(k) s_1(k) + \mathbf{H}_1(k) \mathbf{W}_2(k) s_2(k) + \mathbf{N}_1(k) \} \\ x_2(k) &= \mathbf{G}_2^H(k) \{ \mathbf{H}_2(k) \mathbf{W}_2(k) s_2(k) + \mathbf{H}_2(k) \mathbf{W}_1(k) s_1(k) + \mathbf{N}_2(k) \} \end{aligned} \quad (1)$$

where $x_1(k)$, $x_2(k)$, $s_1(k)$, $s_2(k)$ are complex scalars, $\mathbf{W}_1(k)$, $\mathbf{W}_2(k)$ are the SDMA weights ($M_t \times 1$), $\mathbf{N}_1(k)$, $\mathbf{N}_2(k)$ are AWGN noise vectors with variances σ_1^2 and σ_2^2 , $\mathbf{G}_1(k)$, $\mathbf{G}_2(k)$ are MRC receivers (expressed by $\mathbf{G}_1(k) = \mathbf{H}_1(k) \mathbf{W}_1(k) / \|\mathbf{H}_1(k) \mathbf{W}_1(k)\|_2$ and similarly for $\mathbf{G}_2(k)$). For the sake of clarity we use the following terminology - downlink transmission for broadcast, base-station for the transmitter and Mobile-1, Mobile-2 for the two receivers.

2.1 System Model with Perfect CSI

Mobile-1 and Mobile-2 are arbitrarily chosen and have perfect knowledge of $\mathbf{H}_1(k)$ and $\mathbf{H}_2(k)$ respectively. The base has perfect knowledge of both $\mathbf{H}_1(k)$ and $\mathbf{H}_2(k)$ and computes the weights $\mathbf{W}_1(k)$ and $\mathbf{W}_2(k)$. The mobiles construct the receivers $\mathbf{G}_1(k)$ and $\mathbf{G}_2(k)$ using independent training, which is again perfect.

2.2 System Model with Limited Feedback

Mobile-1 and Mobile-2 have perfect knowledge of $\mathbf{H}_1(k)$ and $\mathbf{H}_2(k)$ respectively. In addition, the mobiles quantize the CSI using a vector quantization (VQ) approach where a fixed predetermined codebook is known to the transmitter and to both users. Based on CSI, the users select a set of indices from the codebook that are sent back to the transmitter using a feedback channel. The exact information that is quantized and the codebooks will be discussed later. The transmitter uses this index information from both users to derive the weights $\mathbf{W}_1(k)$ and $\mathbf{W}_2(k)$. The channel is assumed to be constant within a block spanning the estimation and feedback of CSI and the subsequent transmission. $\mathbf{G}_1(k)$ and $\mathbf{G}_2(k)$ are known perfectly at the mobiles.

3. SDMA WITH PERFECT CSI

Given complete knowledge of $\mathbf{H}_1(k)$ and $\mathbf{H}_2(k)$, several algorithms may be used for deriving $\mathbf{W}_1(k)$ and $\mathbf{W}_2(k)$, including ZF, MMSE, generalized eigenvalue method or iterative solutions. All of these perform similarly (in terms of BER) when the number of users is small compared to the number of transmit antennas. We have chosen the generalized eigenvalue method (easily derived from the uplink array processing algorithms in [7]) to indicate our baseline performance for SDMA with perfect CSI since it provides a good balance between complexity and performance. The SDMA weight for Mobile-1 for the k^{th} subcarrier $\mathbf{W}_1(k)$ is defined as the dominant normalized eigenvector of \mathbf{L}_1

$$\mathbf{L}_1(k) = [\sigma_1^2 \mathbf{I} + \mathbf{H}_2^H(k) \mathbf{H}_2(k)]^{-1} \mathbf{H}_1^H(k) \mathbf{H}_1(k) \quad (2)$$

and $\mathbf{W}_2(k)$ is defined analogously for Mobile-2.

4. SDMA WITH LIMITED FEEDBACK

The conventional algorithms for arriving at SDMA weights (with perfect CSI) involve a matrix inverse in some form. At practical

quantization resolutions, the quantization noise added to the CSI due to limited feedback is magnified due to the inversion process and severely degrades the solution. Compared to a sum-rate criterion the coded BER criterion is more sensitive to the error in the SDMA weights due to imperfect CSI. Coding across frequency helps to mitigate some of the errors due to the mismatch in SDMA weights and provides robustness to quantization noise.

4.1 Intuitive Description of the Algorithm

In the following we describe the proposed subspace-based SDMA algorithm which is motivated from empirical observations discussed above. Let us define a channel subspace $\mathbf{V}_1(k)$ and a null subspace $\mathbf{N}_1(k)$ corresponding to the channel $\mathbf{H}_1(k)$ as follows. $\mathbf{V}_1(k)$ is defined as the subspace spanned by the dominant right singular vector of $\mathbf{H}_1(k)$, and $\mathbf{N}_1(k)$ is defined as the subspace (of dimension $M_t - M_r$) spanned by the singular vectors of $\mathbf{H}_1(k)$ corresponding to its zero singular values. Similarly $\mathbf{V}_2(k)$ and $\mathbf{N}_2(k)$ can be defined for the channel $\mathbf{H}_2(k)$.

If $\mathbf{V}_1(k)$ and $\mathbf{V}_2(k)$ are orthogonal, we can simply set $\mathbf{W}_1(k) = \mathbf{V}_1(k)$ and $\mathbf{W}_2(k) = \mathbf{V}_2(k)$ resulting in zero interference between the mobiles as well as aligning the SDMA weights perfectly to the channel subspaces (and maximizing the delivered power). Note that in this case only the channel subspace information is required for obtaining $\mathbf{W}_1(k)$ and $\mathbf{W}_2(k)$ (and not the entire matrices $\mathbf{H}_1(k)$, $\mathbf{H}_2(k)$). In general, it may be observed that if $\mathbf{W}_1(k)$ is contained in the nullspace of $\mathbf{H}_2(k)$ then $\mathbf{W}_1(k)$ provides no interference to Mobile-2. The same may be said for $\mathbf{W}_2(k)$. Thus intuitively $\mathbf{W}_1(k)$ should lie somewhere between $\mathbf{V}_1(k)$ and $\mathbf{N}_2(k)$ to provide a balance between the amount of power delivered to Mobile-1 and the interference caused to Mobile-2. Similarly $\mathbf{W}_2(k)$ should be in between $\mathbf{V}_2(k)$ and $\mathbf{N}_1(k)$. This forms the central idea of our algorithm. In simple terms we propose $\mathbf{W}_1(k)$ to be a subspace lying midway between $\mathbf{V}_1(k)$ and $\mathbf{N}_2(k)$ while $\mathbf{W}_2(k)$ to be a subspace lying midway between $\mathbf{V}_2(k)$ and $\mathbf{N}_1(k)$. It immediately follows that the computation of $\mathbf{W}_1(k)$, $\mathbf{W}_2(k)$ only requires the subspace and nullspace information for each channel (and not the channel gains or the singular values). The technicalities regarding the subspace averaging are provided below.

4.2 Subspace Averaging

In this section we describe averaging of two given M_s dimensional subspaces. Note that the rest of the paper only considers averaging 1-dimensional subspaces. Literature on this may be found in [14]. Let \mathbf{U}_1 and \mathbf{U}_2 be complex unitary matrices of dimension $M_s \times M_s$ and let us abuse notation and also denote their M_s - dimensional column spaces by \mathbf{U}_1 and \mathbf{U}_2 respectively. Let $\mathbf{U}_{\text{avg}} = \text{avg}(\mathbf{U}_1, \mathbf{U}_2)$ denote the average subspace lying midway between \mathbf{U}_1 and \mathbf{U}_2 expressed mathematically as

$$\mathbf{U}_{\text{avg}} = \arg \min_{\mathbf{U}} \{ d(\mathbf{U}, \mathbf{U}_1) + d(\mathbf{U}, \mathbf{U}_2) \} \quad (3)$$

where $d(\mathbf{U}_1, \mathbf{U}_2)$ is the chordal distance defined as

$$d(\mathbf{U}_1, \mathbf{U}_2) = \left\| \mathbf{U}_1 \mathbf{U}_1^H - \mathbf{U}_2 \mathbf{U}_2^H \right\|_F \quad (4)$$

and is a metric in the space of all such subspaces (technically a complex Grassmann manifold). The minimization in (3) is over all M_s dimensional subspaces. The M_s dominant eigenvectors of $(\mathbf{U}_1 \mathbf{U}_1^H + \mathbf{U}_2 \mathbf{U}_2^H)$ stacked as columns in a $M_t \times M_s$ matrix constitute \mathbf{U}_{avg} .

4.3 SDMA Algorithm

First, let us define a quantization function Q that represents the subspaces $\mathbf{V}_1(k)$, $\mathbf{V}_2(k)$ using a fixed number of bits. Consider a finite collection of 1-dim subspaces $C=\{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_B\}$, also called the codebook, to be known both at the mobiles and the base. The function Q is used at the mobiles to map a channel subspace into one of the elements of C . The bit-efficiency of quantization may be increased by grouping adjacent subcarriers into clusters [13]. The channel subspaces of all the subcarriers within a cluster map to the same element in C . Let us partition N subcarriers into N/K clusters of size K and let subcarrier k belong to an arbitrary cluster containing subcarriers numbered from 1 to K . The quantization map may be expressed as (say for Mobile-1)

$$Q(\mathbf{V}_1(k)) = \arg \max_{\mathbf{F} \in C} \sum_{j=1}^K \left\| \mathbf{H}_1(j) \mathbf{F} \right\|_{\mathbf{F}}^2 \quad \text{for } k = 1, \dots, K. \quad (5)$$

The design and construction of the codebook C is outside the present scope and may be found in [15]. In the particular case of a MISO channel, and if the channel subspace $\mathbf{V}_1(k)$ is perfectly known at the base, the nullspace $\mathbf{N}_1(k)$ can be derived as the orthogonal complement of $\mathbf{V}_1(k)$. In order to conserve feedback bits, we do not quantize and feedback information about $\mathbf{N}_1(k)$. Instead, the base obtains an approximate version of $\mathbf{N}_1(k)$, denoted by $\mathbf{N}_1^*(k)$ as the orthogonal complement of $Q(\mathbf{V}_1(k))$. In the case of multiple receive antennas, even though $\mathbf{N}_1(k)$ and $\mathbf{V}_1(k)$ are not orthogonal complements of each other, the base still constructs $\mathbf{N}_1^*(k)$ as the orthogonal complement of $Q(\mathbf{V}_1(k))$.

Since an element of C may be represented by $\log_2(B)$ bits, therefore the total feedback load for all the subcarriers for a single channel realization is $(N/K) \log_2(B)$ bits or $(1/K) \log_2(B)$ bits per tone for each user.

In the following, the steps for computing the SDMA weights $\mathbf{W}_1(k)$ and $\mathbf{W}_2(k)$ for a cluster consisting of subcarriers $k=1, \dots, K$ are described. This algorithm transmits with equal power to both users. Since the same SDMA weights are used for all the subcarriers in a cluster, we arbitrarily choose a subcarrier index k .

1. $\mathbf{N}_1^*(k)$ is constructed as a $M_r \times (M_r - 1)$ unitary matrix by stacking the $(M_r - 1)$ lower singular vectors of $Q(\mathbf{V}_1(k))Q(\mathbf{V}_1(k))^H$. Similarly $\mathbf{N}_2^*(k)$ is constructed from $Q(\mathbf{V}_2(k))Q(\mathbf{V}_2(k))^H$.
2. Let us denote the orthogonal projection matrices corresponding to $\mathbf{N}_1^*(k)$ and $\mathbf{N}_2^*(k)$ by $\mathbf{P}_1(k)$ and $\mathbf{P}_2(k)$ respectively. Then compute $\mathbf{P}_1(k) = \mathbf{N}_1^*(k)\mathbf{N}_1^*(k)^H$ and $\mathbf{P}_2(k) = \mathbf{N}_2^*(k)\mathbf{N}_2^*(k)^H$.
3. Obtain $\mathbf{W}_1(k) = \text{avg}(Q(\mathbf{V}_1(k)), \mathbf{P}_2(k)^*Q(\mathbf{V}_1(k)))$ and $\mathbf{W}_2(k) = \text{avg}(Q(\mathbf{V}_2(k)), \mathbf{P}_1(k)^*Q(\mathbf{V}_2(k)))$.

4.4 Cluster Selection

It was observed that the BER performance of the proposed SDMA algorithm degraded in highly frequency selective channels considering reasonable feedback overhead (see next section). As a result, we also propose and simulate a strategy to improve the BER performance by choosing to refrain from SDMA for certain clusters. Intuitively when the quantized channel subspaces of the two users are very closely aligned, we refrain from applying SDMA. The criterion for selecting a cluster for SDMA is described here. A cluster containing subcarrier k is chosen if the following criterion is satisfied.

$$\left\| Q(\mathbf{V}_1(k))Q(\mathbf{V}_1(k))^H - \mathbf{P}_2(k)Q(\mathbf{V}_1(k))Q(\mathbf{V}_1(k))^H \mathbf{P}_2^H(k) \right\|_{\mathbf{F}} < \partial$$

$$\text{and } \left\| Q(\mathbf{V}_2(k))Q(\mathbf{V}_2(k))^H - \mathbf{P}_1(k)Q(\mathbf{V}_2(k))Q(\mathbf{V}_2(k))^H \mathbf{P}_1^H(k) \right\|_{\mathbf{F}} < \partial$$

where ∂ is an experimentally determined constant. In case a cluster is not selected for SDMA one of the users is chosen arbitrarily and single stream beamforming is applied using $Q(\mathbf{V}_1(k))$ or $Q(\mathbf{V}_2(k))$ as appropriate. The implications of cluster selection are twofold – a reduction in the sum data-rate and a feedforward overhead for identifying which clusters use SDMA transmission. It may be pointed out that the proposed cluster selection does not leverage multi-user diversity.

5. SIMULATION RESULTS

The simulations use OFDM parameters borrowed from IEEE 802.16e. We use a COST-259-style spatial channel model with a vehicle velocity set to 60 miles per hour, consisting of a single scattering zone having 100 discrete multipath rays, a 15° multipath angular spread with respect to the base antenna array, and a 360° multipath angular spread with respect to the mobile antenna array. The base has a uniform linear array of 4 antennas with a 5 wavelength spacing between antenna elements and each mobile has 1 or 2 element uniform linear array with 1 wavelength spacing. The OFDM system uses a 2048 point FFT with 11.2 kHz subcarrier spacing at 2.6 GHz carrier frequency. The number of subcarriers with data is $N=1728$ which span 19.35 MHz. The cyclic prefix length is 256 (11.16 μsec) and the total OFDM symbol duration is 100.45 μsec . A 1/2 rate convolutional encoder with QPSK is used with bit interleaving across all 1728 data carrying subcarriers.

Figure 1 shows the coded BER performance of subspace average SDMA with limited feedback compared to the generalized eigenvalue method with perfect CSI. The frequency selectivity of the channel is relatively small with the RMS delay spread set to 0.5 μsec and $M_r = 1$. Each cluster consists of 16 adjacent subcarriers ($K=16$), and a 7-bit codebook ($B=128$) is used resulting in a feedback load of 0.44 bits per tone. SDMA is applied for all the clusters all the time. Figure 2 shows the coded BER performance for a channel with high frequency selectivity, the RMS delay spread set to 2 μsec and $M_r = 1$. The cluster size K is reduced to 8 subcarriers and a 7-bit codebook is used, implying a feedback load of 0.87 bits per tone. The coded BER performance shows significant improvement with cluster selection. A value of $\delta=0.7$ is set, and consequently SDMA is not applied to 20% of the subcarriers on an average (implying a 10% hit on the sum data-rate). An additional feedforward overhead of 1 bit per cluster (or 0.125 bits per tone) is also necessary for identifying the SDMA clusters. Figure 3 shows the impact of adding one more receive antenna at the mobiles. The conditions of the experiment are the same as Figure 1 (delay spread of 0.5 μsec) as well as the feedback rate. Figure 3 confirms that the advantage of one extra receive antenna is preserved with limited feedback. The results show that in a realistic propagation environment with a feedback overhead of less than 1 bit per tone, SDMA could substantially increase the number of users supported in a cell by a single base.

6. CONCLUSION

This paper proposes an algorithm for SDMA beamforming with limited feedback for 2 users (chosen arbitrarily) where the receivers do not need to know the multi-user nature of the trans-

mission. Simulation with an IEEE 802.16e-style system validate that the coded BER can be maintained within a couple of dBs of the case with perfect CSI with a feedback of less than 1 bit per tone.

Advanced link level simulations with channel estimation and feedback delay are of interest for future work. In principle the algorithm extends naturally to multi-stream SDMA. Future work on extending the algorithm for more than 2 users and optimizing for multi-stream SDMA would be promising.

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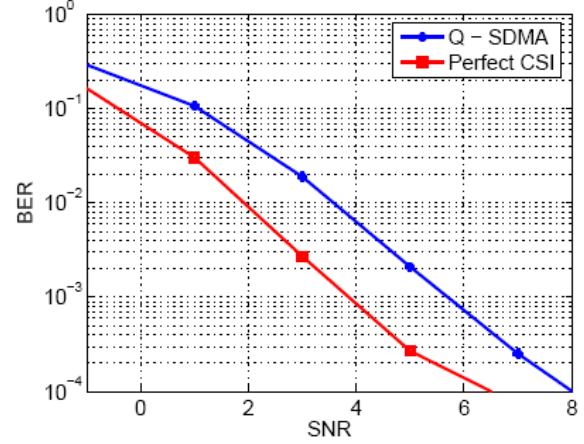


Figure 1. BER comparison of proposed limited feedback SDMA (Q-SDMA) with SDMA using generalized eigenvalue method using perfect CSI.

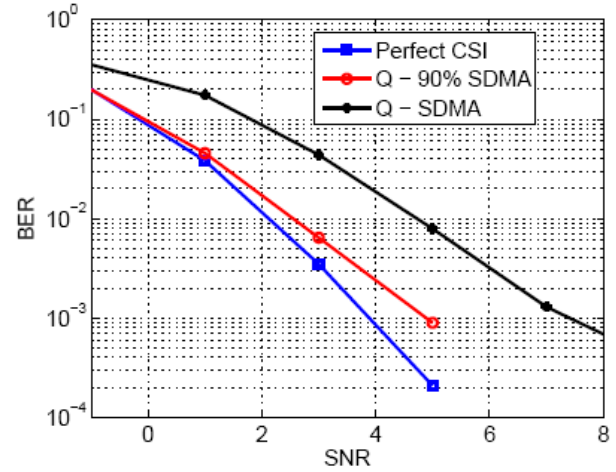


Figure 2. BER comparison in a channel with high frequency selectivity. The proposed algorithm (Q-SDMA) degrades due to limited feedback. Using cluster selection (Q-90% SDMA) improves the performance.

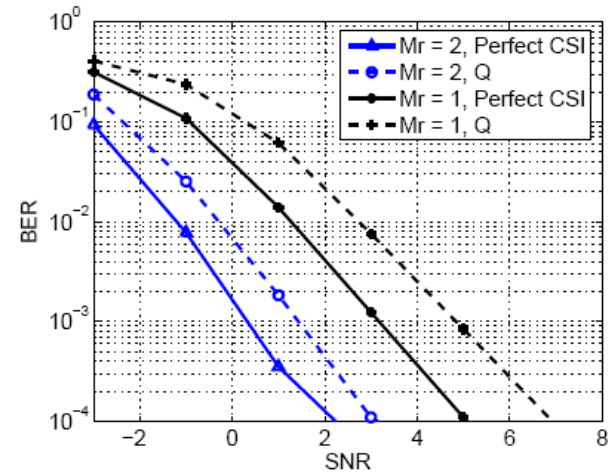


Figure 3. SDMA transmission to mobiles with $M_r=2$ and $M_r=1$ are compared. Both employ the same feedback rate.