# SDMA WITH A SUM FEEDBACK RATE CONSTRAINT

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***Abstract*— Space division multiple access (SDMA) is capable of achieving sum capacity that grows double logarithmically with the number of users. The sum rate for channel state information (CSI) feedback, however, increases linearly with the number of users, reducing the effective uplink capacity. To address this problem, a novel SDMA design is proposed, where the sum feedback rate is upper-bounded by a constant. This design consists of algorithms for CSI quantization, threshold based CSI feedback, and joint beamforming and scheduling. The key feature of the proposed approach is the use of feedback thresholds to select feedback users with large channel gains and small CSI quantization errors such that the sum feedback rate constraint is satisfied. Despite this constraint, the proposed SDMA design is shown to achieve a sum capacity growth rate close to the optimal one. Numerical results show that the proposed SDMA design is capable of attaining higher sum capacities than existing ones, even though the sum feedback rate is bounded.**

***Index Terms*—** Broadcast Channels, Space Division Multiplexing, Feedback Communication, Multiuser Channels

## I. Introduction

Given multiple transmit antennas, *space division multiple access* (SDMA) allows simultaneous transmission through the spatial separation of scheduled users. Compared with the optimal SDMA strategy that uses *dirty paper coding* [1], SDMA with transmit beamforming has suboptimal performance but a low-complexity transmitter. Various algorithms for SDMA with a beamforming constraint have been proposed recently (e.g. [2]–[5]). The existing SDMA algorithms requires users to send back their channel state information (CSI), which is required for beamforming and scheduling at a base station. Consequently, the sum feedback rate can potentially cause a uplink bottleneck for a SDMA system with a large number of users. That motivates us to address the following question: *How to design a SDMA downlink with a bounded sum feedback rate?*

As proposed in [6]–[8], the sum feedback rate of a downlink system can be reduced by applying a feedback threshold on signal-to-noise-ratio (SNR) or signal-to-noise-interference-ratio (SINR), where users below the threshold do not send back CSI. A common problem shared by these feedback algorithms is that the sum feedback rate increases linearly with the number of users. To constrain the sum feedback rate, an approach combining a feedback threshold and contention feedback is proposed in [9]. The drawbacks of this approach include the requirement of zero forcing equalization at receivers and the constraint of the number of simultaneous users by the number of receive antennas of each user. These drawbacks motivate us to consider a more practical downlink system.

In the literature of SDMA with transmit beamforming, a sum feedback rate constraint has not been considered as most work focuses on feedback reduction for individual users. For the *opportunistic SDMA* (OSDMA) algorithm proposed in [10], the feedback of each user is reduced to a few bits by constraining the choice of a beamforming vector to a set of orthogonal vectors. The sum capacity of OSDMA can be increased by selecting orthogonal beamforming vectors from multiple sets of orthogonal vectors, which motivates the *OSDMA with beam selection* (OSDMA-BS) [11] and the *OSDMA with limited feedback* (OSDMA-LF) [12] algorithms, where limited feedback refers to quantization and feedback of CSI [13]. These two algorithms assign beamforming vectors at mobiles and the base station, respectively. Existing SDMA algorithms share the drawback of having a sum feedback rate that increases linearly with the number of users. This motivates us to apply a sum feedback rate constraint on SDMA, thereby the sum feedback rate is upper bounded by a constant independent of the number of users.

The main contributions of this paper include an algorithm for a SDMA downlink with a sum feedback rate constraint, the design of feedback thresholds for realizing this constraint, and the analysis of sum capacity. The proposed algorithm is named *OSDMA with threshold feedback* (OSDMA-TF). First, the OSDMA-TF sub-algorithms for *CSI quantization at users*, *selection of feedback users using thresholds* and *joint beamforming and scheduling at a base station* are presented. Second, the feedback thresholds on users' channel power and channel quantization errors are designed such that the sum feedback rate constraint is satisfied. The design of feedback thresholds avoids a tedious numerical search for such thresholds and proves useful in analyzing the performance of the SDMA system under a sum feedback rate constraint. Third, it is shown that the

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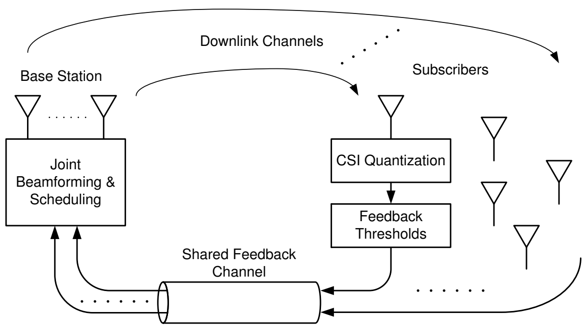


Fig. 1. SDMA Downlink system with feedback thresholds

growth rate of the sum capacity with the number of users can be made arbitrarily close to the optimal one by having a sufficiently large sum feedback rate. Last, OSDMA-TF is compared with several existing SDMA algorithms and is found to be capable of achieving higher sum capacities despite the sum feedback rate constraint.

## II. System Model

For the downlink system illustrated in Fig. 1, a base station with $N_t$ antennas transmits data simultaneously to $N_t$ scheduled users chosen from a total of $U$ users, each with one receive antenna. The base station separates the multi-user data streams by using beamforming vectors $\{\mathbf{w}_n\}_{n=1}^{N_t}$ that are selected as described in Section III-C. The received signal of the $n$th scheduled user is

$$y_n = \sqrt{P}\sum_{i=1}^{N_t} \mathbf{h}_n^{\dagger}\mathbf{w}_i x_i + \nu_n, \quad n = 1, \cdots, N_t. \quad (1)$$

$N_t$ is the number of transmit antennas and also the number of scheduled users; $\mathbf{h}_n$ is the downlink channel vector ($N_t \times 1$); $P$ is the transmit power; $\mathbf{w}_n$ is the beamforming vector ($N_t \times 1$) with $\|\mathbf{w}_n\|^2 = 1$; $x_n$ is the transmitted symbol with $|x_n| = 1$; $y_n$ is the received symbol; and $\nu_n$ is the AWGN sample with $\nu_n \in \mathcal{CN}(0, 1)$.

We assume that each user quantizes his/her CSI and sends it back following a feedback algorithm to be discussed in Section III-B. Furthermore, all users share a common feedback channel. Therefore, it is necessary to constrain the average sum feedback rate. Let $B$ denote the number of bits sent back by each feedback user and $K$ the number of feedback users. Since $B$ is a constant and $K$ a random variable, the constraint of the average sum feedback rate can be written as

$$\text{(Sum Feedback Rate Constraint)} \quad BE[K] \leq R, \quad (2)$$

where $R$ is the sum feedback rate constraint.

To simplify our analysis, we make the following assumption about the multi-user channels:

**AS 1:** *The downlink channel $\mathbf{h}_u$ is an i.i.d. vector whose coefficients are $\mathcal{CN}(0, 1)$.*

Given this assumption commonly made in the SDMA literature [5], [10], [11], the channel direction vector of each user follows a uniform distribution, which greatly simplifies the design of feedback thresholds in Section IV and capacity analysis in Section IV.

## III. Algorithms

In this section, we discuss OSDMA-TF algorithms for (i) CSI quantization at the subscribers, (ii) selection of feedback users using feedback thresholds, and (iii) joint beamforming and scheduling at the base station.

### A. CSI Quantization

Without loss of generality, the discussion in this section is focused on the $u$th user and the same algorithm for CSI quantization is used by other users. For simplicity, we assume:

**AS 2:** *The $u$th user has perfect receive CSI $\mathbf{h}_u$.*

This assumption allows us to neglect channel estimation error at the $u$th mobile. For convenience, the CSI, $\mathbf{h}_u$, is decomposed into two components: the *gain* and the *shape*, which are quantized separately. Hence, $\mathbf{h}_u = g_u \mathbf{s}_u$ where $g_u = \|\mathbf{h}_u\|$ is the gain and $\mathbf{s}_u = \mathbf{h}_u/\|\mathbf{h}_u\|$ is the shape. The channel shape $\mathbf{s}_u$ is quantized and sent back to the base station for choosing beamforming vectors. The channel gain $g_u$ is used for computing SINR, which is also quantized and sent back as a channel quality indicator. Due to the ease of quantizing SINR that is a scalar, we make the following assumption:

**AS 3:** *The SINR is perfectly known to the base station through feedback.*

The same assumption is made in [10], [11]. This assumption allows us to focus on quantization of the channel shape $\mathbf{s}_u$.

Quantization of the channel shape $\mathbf{s}_u$ is the process of matching it to a member of a set of pre-determined vectors, called *a codebook*. Different from [5], [12] where code vectors are randomly generated, we propose a structured codebook constructed as follows. The codebook, denoted as $\mathcal{F}$, is comprised of $M$ sub-codebooks: $\mathcal{F} = \cup_{m=1}^{M}\mathcal{F}_m$, each of which is comprised of $N_t$ orthogonal vectors. The sub-codebooks $\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_M$ are independently and randomly generated. Each sub-codebook provides a potential set of orthogonal beamforming vectors for downlink transmission. Given a codebook $\mathcal{F}$ thus generated, the quantized channel shape, denoted as $\hat{\mathbf{s}}_u$, is the member of $\mathcal{F}$ that forms the smallest angle with the channel shape $\mathbf{s}_u$ [14]

$$\hat{\mathbf{s}}_u = \mathcal{Q}(\mathbf{s}_u) = \arg\max_{\mathbf{f}\in\mathcal{F}} \left|\mathbf{f}^{\dagger}\mathbf{s}_u\right|, \quad (3)$$

where the function $\mathcal{Q}$ represents the CSI quantization process. We define the *quantization error* as

$$\text{(Quantization Error)} \quad \delta_u = \sin^2(\angle(\hat{\mathbf{s}}_u, \mathbf{s}_u)). \quad (4)$$

### B. Feedback Algorithm

To satisfy the sum feedback rate constraint (2), we propose a threshold-based feedback algorithm, which allows only users with high SINRs to send back CSI to the base station. The SINR of the $u$th user is a function of the channel power $\rho_u = \|\mathbf{h}_u\|^2$ and the quantization error $\delta_u$ in (4) [10]

$$\mathrm{SINR}_u = (1 + P\rho_u)/(1 + P\rho_u\delta_u) - 1. \quad (5)$$

Therefore, the feedback algorithm employs two feedback thresholds for feedback user selection: the channel power threshold ($\gamma$), and the quantization error threshold ($\epsilon$). It follows that the $u$th user meets the feedback criteria if $\rho_u \geq \gamma$ and $\delta_u \leq \epsilon$. When selected, the $u$th user sends back the quantized channel shape $\hat{\mathbf{s}}_u$ to the base station through a finite-rate feedback channel, which requires $\log_2 N$ feedback bits where $N = |\mathcal{F}|$ [13], [14].

### C. Joint Beamforming and Scheduling

Among feedback users, the base station schedules a subset of users for downlink transmission using the criterion of maximizing sum capacity and under the constraint of orthogonal beamforming. To facilitate the description of the procedure for joint beamforming and scheduling, we group feedback users according to their quantized channel shapes by defining the following index sets: $\forall\ 1 \leq m \leq M$ and $1 \leq n \leq N_t$

$$\mathcal{I}_{m,n} = \{1 \leq u \leq U \mid \rho_u \geq \gamma,\ \delta_u \leq \epsilon,\ \mathcal{Q}(\mathbf{s}_u) = \mathbf{f}_{m,n}\}, \quad (6)$$

where $\mathcal{Q}(\cdot)$ is the quantization function in (3) and $\mathbf{f}_{m,n} \in \mathcal{F}$ is the $n$th member in the $m$th sub-codebook $\mathcal{F}_m \subset \mathcal{F}$. The base-station adopts a two-step procedure for joint beamforming and scheduling. First, it selects the user with maximum SINR from each index set defined (6). Second, from these selected users, the base station schedules up to $N_t$ users for downlink transmission under the constraint of orthogonal beamforming. The resultant sum capacity is

$$\mathcal{C} = E\left[\max_{m=1,\cdots,M} \sum_{n=1}^{N_t} \log_2(1 + \max_{u \in \mathcal{I}_{m,n}} \mathrm{SINR}_u)\right], \quad (7)$$

where the two "max" operators correspond to the two steps in the procedure for joint beamforming and scheduling.

## IV. Feedback Thresholds

In this section, the feedback thresholds for OSDMA-TF (cf. Section III-B) are designed as functions of the number of users $U$ under the sum feedback constraint in (2).

The sum feedback rate, denoted as $R$, can be expressed as $R = E[K]B$ where $E[K]$ denotes the average number of feedback users and $B$ the number of bits sent back by each of them. We derive and show in the following theorem a set of feedback thresholds causing $E[K]$ to be limited by an upper-bound, which is independent of $U$. By choosing a proper value for the upper-bound, we can thus satisfy any given constraint on the sum feedback rate $R$.

**Theorem 1:** *Consider the following channel power and quantization error thresholds*

$$\gamma = \log U - \lambda \log\log U, \quad \lambda > 0, \quad (8)$$

$$\epsilon = \left[U^{1-\varphi}(\log U)^{\varphi\lambda}\right]^{-1/(L-1)}, \quad (9)$$

*where* $\varphi = -\gamma \ln\left(\frac{1}{L!}\int_\gamma^\infty \rho^{L-1}e^{-\rho}d\rho\right)$ *. Given these thresholds, the average number of feedback users* $E[K]$ *is upper-bounded as* $E[K] \leq NN_t$*, where* $N$ *is the cardinality of the CSI quantization codebook* $\mathcal{F}$*.*

The proof is given in [15]. A few remarks are in order. First, given the feedback thresholds in Theorem 1, the sum feedback rate is bounded as $R \leq BNN_t$, where $B$ is the number of feedback bits per user. Second, the power and quantization thresholds in (8) and (9) are chosen jointly to ensure the sum capacity grows with the number of users $U$ at an optimal rate (cf. Section V). Third, the optimal value of $\lambda$ in (8) and (9) for maximizing sum capacity can be chosen numerically since analytical methods seem difficult.

Last, we provide the following proposition, which shows that the upper-bound on the average number of feedback users $E[K]$ is tight if the number of users $U$ is large. We define the minimum distance between any two members of the codebook $\mathcal{F}$ as $\Delta\delta_{\min} = \min_{1\leq a,b\leq N}[1 - |\mathbf{f}_a^H\mathbf{f}_b|^2]$, where $\mathbf{f}_a, \mathbf{f}_b \in \mathcal{F}$.

**Proposition 1:** *For any codebook* $\mathcal{F}$ *with* $\Delta\delta_{\min} > 0$*, there exists an integer* $U_0$ *such that* $\forall\ U \geq U_0$*, the average number of feedback users* $E[K]$ *is equal to* $NN_t$*.*

The proof is given in [15].

## V. Asymptotic Sum Capacity

In this section, for a large number of users $U$, we show in the following theorem that the sum capacity of OSDMA-TF can grow at a rate close to the optimal one, namely $N_t \log_2 \log_2 N$, if the sum feedback rate is sufficiently large.

**Theorem 2:** *For a large number of users (*$U \to \infty$*), the sum capacity of OSDMA-TF grows with the number of transmit antennas* $N_t$ *linearly and with the number of users double logarithmically*

$$1 \geq \lim_{U\to\infty} \frac{\mathcal{C}}{N_t \log_2\log_2 U} > 1 - P_\beta, \quad \lambda \geq N_t - 1, \quad (10)$$

*where* $P_\beta = (N_t e^{-N_t})^M$ *and* $\lambda$ *is the parameter of the power threshold in (8).*

The proof is given in [15]. The above theorem shows the effect of a sum feedback rate constraint is to decrease the growth rate of the sum capacity with respect to that for feedback from all users, namely $N_t \log_2 \log_2 U$ [10], [12]. Nevertheless, such difference in growth rate can be made arbitrarily small by increasing the sum feedback rate, or equivalently the number of feedback bits per feedback user, as stated in the corollary.

**Corollary 1:** *By increasing the number of feedback bits per feedback user (*$\log_2 N$*), the sum capacity of OSDMA-TF*

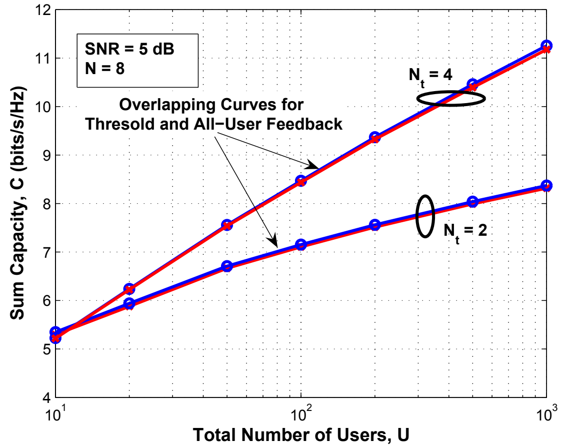


Fig. 2. Sum capacities of threshold feedback (OSDMA-TF) and all-user feedback

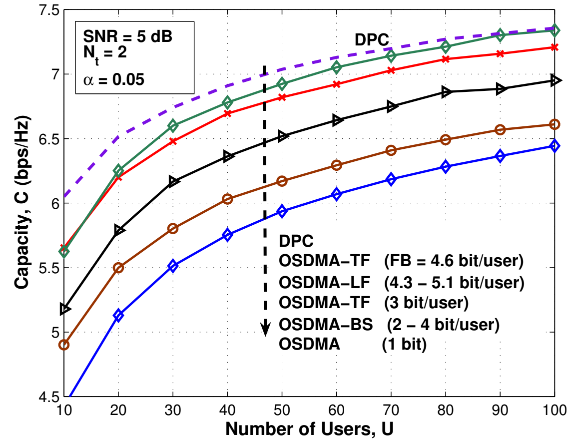


Fig. 3. Sum capacities and sum feedback rates of OSDMA-TF, OSDMA-LF, OSDMA-BS and OSDMA.

*can grow at the optimal rate:*

$$\lim_{N\to\infty}\lim_{U\to\infty}\frac{\mathcal{C}}{N_t\log_2\log_2 U}=1,\quad \textit{if}\ \lambda\geq N_t-1. \qquad (11)$$

**Proof:** The result follows from (10) and $N = MN_t$. □

## VI. Numerical Results

First, we compare the sum capacity and the sum feedback rate of OSDMA-TF with the case of all-user feedback for a varying number of users $U$ in Fig. 2. For this comparison, $N_t = \{2, 4\}$ transmit antennas and $\lambda = \{1, 1.5\}$ for the feedback thresholds in (8) and (9). Each user quantizes his/her channel shape using a codebook of a size $N = 8$ for both $N_t = 2$ and $N_t = 4$. It can be observed from Fig. 2 that the curves for threshold feedback (OSDMA-TF) and all-user feedback overlap, indicating the sum feedback rate constraint for SDMA-TF incurs no loss in sum capacity. The number of feedback users for OSDMA-TF is bounded at 16 for $N_t = 2$ or 32 for $N_t = 4$.

Second, in Fig. 3, the sum capacity of OSDMA-TF is compared with that of OSDMA-LF, OSDMA-BS and OSDMA for different numbers of users $U$, with $N_t = 2$ and an SNR of 5 dB. The number of feedback bits per feedback user differs for the algorithms in comparison since they use different sizes for quantization codebooks or different feedback algorithms. For OSDMA-TF, two codebook sizes $N = 8$ and $N = 24$ are considered, corresponding to 3 and 4.6 feedback bits for each feedback user, respectively. Feedback bits for other algorithms are indicated in Fig. 3. For fair comparison, we also apply a feedback penalty factor ($\alpha$), used in [11] to all algorithms. From Fig. 3, we can observe that OSDMA-TF ($N = 24$) yields the highest sum capacity and OSDMA-TF ($N = 8$) is outperformed only by OSDMA-LF. Moreover, the sum capacity of OSDMA-TF converges to DPC rapidly. Due to the sum feedback constraint, OSDMA-TF requires a smaller sum feedback rate than other algorithms [15].

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