JOINT COMPENSATION OF OFDM FREQUENCY SELECTIVE TRANSMITTER AND RECEIVER IQ IMBALANCE

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ABSTRACT

Direct conversion architectures are currently receiving a lot of interest in OFDM based wireless transmission systems. However such systems are very sensitive to In-phase/Quadraturephase (IQ) imbalances in the front-end analog processing. In this paper the joint effect of frequency selective IQ imbalances at both transmitter and receiver is studied. When the cyclic prefix is long enough to accommodate the combined channel, transmitter and receiver filter impulse, we propose a low complexity two tap equalizer with LMS based adaptation. When the cyclic prefix is not sufficiently long, this results in Inter-Block-Interference (IBI) between the OFDM symbols. In this case we propose a frequency domain per-tone equalizer (PTEQ) initialized by RLS based adaptation. Both algorithms provide a very efficient post-FFT adaptive equalization and their performance is close to the ideal case.

Index Terms— Orthogonal frequency division multiplexing (OFDM), direct-conversion, IQ imbalance, compensation algorithms, analog impairments.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) [1] is a popular modulation technique for broadband wireless systems: it is used for Wireless LAN, Fixed Broadband Wireless Access, Digital Video & Audio Broadcasting, etc. Hence, a lot of effort is spent in developing integrated, cost and power efficient OFDM systems. The so-called zero-IF architecture (or direct-conversion architecture) is an attractive candidate as it converts the RF signal directly to baseband or vice-versa without any Intermediate Frequencies (IF). However the zero-IF architecture has an inherent two-path (In-phase/Quadraturephase, IQ) analog processing which results in the system being extremely sensitive to I and Q branch mismatches. The IQ imbalance can severely limit the achievable data rate and hence the performance of the system. Rather than trying to decrease the IQ imbalance by increasing the design time and the component cost of the analog processing, IQ imbalance can also be tolerated and then compensated digitally.

Several compensation algorithms considering either only receiver IQ imbalance or transmitter IQ imbalance have been developed in [2], [3], etc. Recently, joint compensation algorithms for frequency independent (constant over frequency) transmitter and receiver IQ imbalance have been proposed in [4]. In [5], a compensation scheme for frequency selective transmitter and receiver IQ imbalance is developed but the scheme is very complex due to the large number of equalizers and taps per equalizer needed, which also results in a slow convergence.

In this paper the joint effect of frequency selective IQ imbalances at both transmitter and receiver is studied. When the cyclic prefix is long enough to accommodate the combined channel, transmitter and receiver filter impulse, we propose a low complexity two tap equalizer with LMS based adaptation. Due to the small number of taps needed, the algorithm converges faster and provides a better performance. When the cyclic prefix is not long enough, there will be Inter-Block-Interference (IBI) and for this case we propose a frequency domain per-tone equalizer (PTEQ) [6] which shortens the combined impulse response to fit within the cyclic prefix and also compensates for the imperfection of the analog frontend. The present research is an extension of our previous work [7] and [8] where various compensation techniques for joint transmitter and receiver frequency independent IQ imbalance under carrier frequency offset have been developed.

This paper is organized as follows. Section 2 describes the joint model for transmitter and receiver frequency selective IQ imbalance. Section 3 explains the compensation schemes and adaptive algorithms used. Simulation results are shown in section 4 and finally conclusions are given in section 5.

Notation: Vectors are indicated in bold and scalar parameters in normal font. Superscripts *, ^T, ^H represent conjugate, transpose and hermitian respectively. **F** and **F**⁻¹ represent the $N \times N$ discrete Fourier transform and its inverse. **I**_N is the $N \times N$ identity matrix and $0_{M \times N}$ is the $M \times N$ all zero matrix. Operators \otimes , * and . denote Kronecker product, convolution and component-wise vector multiplication respectively.

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2. IQ IMBALANCE MODEL

Let $S^{(i)}$ be the frequency domain OFDM symbol of size ($N \times 1$) where *i* is the time index of the symbol. We consider two successive OFDM symbols transmitted at time i - 1 and *i* respectively. The *i*th symbol is the symbol of interest, the previous symbol is included to model IBI. These symbols are transformed to the time domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length ν is then added to the head of each symbol. In the case of no IQ imbalance at the front end of the transmitter, the resulting time domain baseband signal is given as:

$$\mathbf{s} = (\mathbf{I}_2 \otimes \mathbf{P})(\mathbf{I}_2 \otimes \mathbf{F}^{-1}) \begin{bmatrix} \mathbf{S}^{(i-1)^T} & \mathbf{S}^{(i)^T} \end{bmatrix}^T$$
(1)

where **P** is the cyclic prefix insertion matrix given by:

$$\mathbf{P} = \frac{\begin{bmatrix} \mathbf{0}_{(\nu \times N - \nu)} & \mathbf{I}_{\nu} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}_{N} & \end{bmatrix}}$$

In the transmitter front-end, the amplifiers, filters and mixers generally result in frequency dependent IQ imbalance which can be approximated by two mismatched filters with frequency responses \mathbf{H}_{ti} and \mathbf{H}_{tq} . The IQ imbalance caused by the local oscillator can be categorized as frequency independent with a transmitter amplitude and phase mismatch g_t and ϕ_t . Following the derivation in [3], the baseband equivalent of the distorted and up-converted signal \mathbf{p} can be written as:

$$\mathbf{p} = \mathbf{g}_{t1} \star \mathbf{s} + \mathbf{g}_{t2} \star \mathbf{s}^* \tag{2}$$

where

$$\mathbf{g}_{t1} = \mathbf{F}^{-1} \{ \mathbf{G}_{t1} \} = \mathbf{F}^{-1} \left\{ \frac{\left[\mathbf{H}_{ti} + g_t e^{-j\phi_t} \mathbf{H}_{tq} \right]}{2} \right\}$$

$$\mathbf{g}_{t2} = \mathbf{F}^{-1} \{ \mathbf{G}_{t2} \} = \mathbf{F}^{-1} \left\{ \frac{\left[\mathbf{H}_{ti} - g_t e^{j\phi_t} \mathbf{H}_{tq} \right]}{2} \right\}$$

Here \mathbf{g}_{t1} and \mathbf{g}_{t2} are filters of length L_t padded with $N - L_t$ zero elements. They represent the combined frequency independent and dependent transmitter IQ imbalance.

When the distorted signal \mathbf{p} is transmitted through a multipath channel \mathbf{c} of length L, then equation (2) is modified as:

$$\mathbf{r} = \mathbf{c} \star \mathbf{g}_{t1} \star \mathbf{s} + \mathbf{c} \star \mathbf{g}_{t2} \star \mathbf{s}^* + \mathbf{v}$$

= $\mathbf{c}_1 \star \mathbf{s} + \mathbf{c}_2 \star \mathbf{s}^* + \mathbf{v}$ (3)

where \mathbf{c}_1 and \mathbf{c}_2 are the combined transmitter IQ and channel impulse responses of length $L + L_t - 1$ and \mathbf{v} is the additive white gaussian noise (AWGN). Substituting equation (1) in equation (3) we obtain:

$$\mathbf{r} = \mathbf{T}_{c1}(\mathbf{I}_{2} \otimes \mathbf{P})(\mathbf{I}_{2} \otimes \mathbf{F}^{-1}) \begin{bmatrix} \mathbf{S}^{(i-1)^{T}} & \mathbf{S}^{(i)^{T}} \end{bmatrix}^{T} + \mathbf{T}_{c2}(\mathbf{I}_{2} \otimes \mathbf{P})(\mathbf{I}_{2} \otimes \mathbf{F}^{-1}) \begin{bmatrix} \mathbf{S}_{m}^{*(i-1)^{T}} & \mathbf{S}_{m}^{*(i)^{T}} \end{bmatrix}^{T} + \mathbf{v}$$
(4)

where **r** is of dimension $(2N + 2\nu - L - L_t + 2 \times 1)$. **T**_{cd} (for d = 1, 2) is an $(2N + 2\nu - L - L_t + 2 \times 2N + 2\nu)$ Toeplitz matrix with first column $[\mathbf{c}_{d(L+L_t-2)}, \mathbf{0}_{(1\times 2N+2\nu-L-L_t+1)}]^T$ and first row $[\mathbf{c}_{d(L+L_t-2)}, \dots, \mathbf{c}_{d(0)}, \mathbf{0}_{(1\times 2N+2\nu-L-L_t+1)}]$.

Here $()_m$ denotes the mirroring operation in which the vector indices are reversed, such that $\mathbf{S}_m[l] = \mathbf{S}[l_m]$ where $l_m = 2 + N - l$ for $l = 2 \dots N$ and $l_m = l$ for l = 1.

An expression similar to equation (2) can be used to model IQ imbalance at the receiver. Let z be the baseband equivalent of the distorted and down-converted signal given as:

$$\mathbf{z} = \mathbf{g}_{r1} \star \mathbf{r} + \mathbf{g}_{r2} \star \mathbf{r}^*$$

= [**O**₁|**T**_{r1}]**r** + [**O**₁|**T**_{r2}]**r**^{*} (5)

where \mathbf{g}_{r1} and \mathbf{g}_{r2} are filters of length L_r representing the combined frequency independent and dependent receiver IQ imbalance. \mathbf{z} is of size $(N \times 1)$, $\mathbf{O}_1 = \mathbf{0}_{(N \times N + 2\nu - L - L_T - L_r + 3)}$. \mathbf{T}_{rd} (for d = 1, 2) is an $(N \times N + L_r - 1)$ Toeplitz matrix with first column $[\mathbf{g}_{rd(L_r-1)}, \mathbf{0}_{(1 \times N-1)}]^T$ and first row $[\mathbf{g}_{rd(L_r-1)}, \dots, \mathbf{g}_{rd(0)}, \mathbf{0}_{(1 \times N-1)}]$. When the impact of both transmitter and receiver IQ imbalance is considered, then equation (5) is modified as:

$$\mathbf{z} = (\mathbf{g}_{r1} \star \mathbf{c}_1 + \mathbf{g}_{r2} \star \mathbf{c}_2^*) \star \mathbf{s} + (\mathbf{g}_{r1} \star \mathbf{c}_2 + \mathbf{g}_{r2} \star \mathbf{c}_1^*) \star \mathbf{s}^* + \mathbf{v} \quad (6)$$

here $\tilde{\mathbf{v}}$ is additive noise which may also be modified by the receiver imbalances. In the frequency domain, if the cyclic prefix is long enough ($\nu \ge L_t + L + L_r - 2$), then equation (6) can be given as:

$$\mathbf{Z} = (\mathbf{G}_{r1}.\mathbf{G}_{t1}.\mathbf{C} + \mathbf{G}_{r2}.\mathbf{G}_{t2m}^*.\mathbf{C}_m^*).\mathbf{S}^{(i)} + (\mathbf{G}_{r1}.\mathbf{G}_{t2}.\mathbf{C} + \mathbf{G}_{r2}.\mathbf{G}_{t1m}^*.\mathbf{C}_m^*).\mathbf{S}_m^{*(i)} + \overset{\sim}{\mathbf{V}}$$
(7)

where \mathbf{Z}, \mathbf{C} and $\widetilde{\mathbf{V}}$ are frequency domain representations of \mathbf{z}, \mathbf{c} and $\widetilde{\mathbf{v}}$. Equation (7) shows that due to the IQ imbalance, power leaks from the signal on the mirror carrier $(\mathbf{S}_m^{*(i)})$ to the carrier under consideration $(\mathbf{S}^{(i)})$ leading to Inter-Carrier-Interference (ICI).

In the case when cyclic prefix is not long enough ($\nu < L_t + L + L_r - 2$), then in addition to ICI there is also interference from the adjacent OFDM symbol carriers $\mathbf{S}^{(i-1)}$, leading to IBI. This IBI can be compensated by a PTEQ [6], which is obtained by transforming a time domain equalizer (TEQ) into the frequency domain. This is explained in the next section.

3. IQ IMBALANCE COMPENSATION

In the case ($\nu < L_t + L + L_r - 2$), we first propose a compensation scheme based on two TEQs \mathbf{w}_1 and \mathbf{w}_2 each with L' taps. \mathbf{w}_1 is applied to the received signal ($\mathbf{z}_1 = \mathbf{z}$) and \mathbf{w}_2 to the conjugated version of the received signal ($\mathbf{z}_2 = \mathbf{z}^*$). The second TEQ is needed to compensate for the image symbol. Now \mathbf{z} in equation (5) is of size ($N + L' - 1 \times 1$), where $\mathbf{O}_1 = \mathbf{0}_{(N+L'-1\times N+2\nu-L-L_T-L_r-L'+4)}$, \mathbf{T}_{rd} (for d = 1, 2) is of size ($N + L' - 1 \times N + L_r + L' - 2$) with first column $[\mathbf{g}_{rd(L_r-1)}, \mathbf{0}_{(1\times N+L'-2)}]^T$ and first row $[\mathbf{g}_{rd(L_r-1)}, \ldots, \mathbf{g}_{rd(0)}, \mathbf{0}_{(1\times N+L'-2)}]$. The output of the TEQs can be given as:

$$\mathbf{z}_t = \mathbf{W}_1^H \mathbf{z}_1 + \mathbf{W}_2^H \mathbf{z}_2 \tag{8}$$



Fig. 1. PTEQ for OFDM with IQ imbalance

where \mathbf{z}_t is of size $(N \times 1)$ and \mathbf{W}_d (for d = 1, 2) is an (N + 1) $L' - 1 \times N$) Toeplitz matrix with first column $[\mathbf{w}_{d,L'-1}, \ldots,$ $\mathbf{w}_{d,0}, \mathbf{0}_{(1 \times N-1)}$ and first row $[\mathbf{w}_{d,L'-1}, \mathbf{0}_{(1 \times N-1)}]$. In addition to the TEQs, a one tap frequency-domain equalizer (FEQ) is applied to the received sequence to recover the transmitted OFDM symbol. The estimate of the transmitted OFDM symbol $\overset{\sim^{(i)}}{\mathbf{S}}$ is then obtained as:

$$\widetilde{\mathbf{S}}^{(i)}[l] = \frac{1}{\mathbf{d}[l]} (\mathbf{F}[l] \mathbf{W}_1^H \mathbf{z}_1 + \mathbf{F}[l] \mathbf{W}_2^H \mathbf{z}_2)$$
(9)

where d[l] is the 1-tap FEO operating on the *l*th sub-carrier, and $\mathbf{F}[1]$ is the *l*th row of the DFT matrix \mathbf{F} .

Following the derivation in [6], the 2 TEQs can be transformed to the frequency domain resulting in 2 PTEQs each employing one DFT and L' - 1 difference terms. Equation (9) is then modified as follows:

$$\widetilde{\mathbf{S}}^{(i)}[l] = \mathbf{v}_1^H[l]\mathbf{F}_i[l]\mathbf{z}_1 + \mathbf{v}_2^H[l]\mathbf{F}_i[l]\mathbf{z}_2$$
(10)

where \mathbf{v}_d (for d = 1, 2) are PTEQ coefficient vectors of size $(L' \times N)$. $\mathbf{F}_i[l]$ is defined as:

$$\mathbf{F}_{i}[l] = \frac{\begin{bmatrix} \mathbf{I}_{L'-1} & \mathbf{0}_{L'-1 \times N - L'+1} & -\mathbf{I}_{L'-1} \\ \hline \mathbf{0}_{1 \times L'-1} & \mathbf{F}[l] \end{bmatrix}}{\mathbf{F}[l]}$$

where the first block row in $\mathbf{F}_{i}[l]$ extracts the difference terms, while the last row corresponds to the single DFT. As \mathbf{z}_2 = $\mathbf{z}_1^* = \mathbf{z}^*$, the PTEQ structure is further simplified by taking only one DFT whose conjugated output in reverse order corresponds to $\mathbf{Z}_2 = \mathbf{F}\{\mathbf{z}_2\} = \mathbf{Z}_{1m}^* = \mathbf{F}\{\mathbf{z}_1^*\}$. The resulting block scheme is shown in **Figure 1**. The PTEQ coefficients \mathbf{v}_1 and \mathbf{v}_2 for the *l*th subcarrier can be obtained based on the following MSE minimization function:

$$min_{(\mathbf{v}_{1}[l],\mathbf{v}_{2}[l])}\mathbb{E}\left\{\left|\widetilde{\mathbf{S}}^{(i)}[l] - \begin{bmatrix}\mathbf{v}_{1}^{H}[l] & \mathbf{v}_{2}^{H}[l]\end{bmatrix}\begin{bmatrix}\mathbf{F}_{i}[l]\mathbf{z}_{1}\\\mathbf{F}_{i}[l]\mathbf{z}_{2}\end{bmatrix}\right|^{2}\right\}$$
(11)

where $\mathbb{E}\{.\}$ is the expectation operator. A training based RLS algorithm [7] is considered to initialize the PTEQ as this provides optimal convergence and achieves initialization with an acceptably small number of training symbols.

For the case ($\nu \ge L_t + L + L_r - 2$), the PTEQ is reduced to order L' = 1 and the desired signal $\mathbf{\tilde{S}}^{(i)}$ is then obtained as: $\sim(i)$

S
$$[l] = \mathbf{v}_1^*[l].\mathbf{Z}_1[l] + \mathbf{v}_2^*[l].\mathbf{Z}_2[l]$$
 (12)

The coefficients can again be estimated based on an MSE minimization: <u>م</u>

$$\min_{(\mathbf{v}_1[l],\mathbf{v}_2[l])} \mathbb{E} \left\{ \left| \widetilde{\mathbf{S}}^{(i)}[l] - \begin{bmatrix} \mathbf{v}_1^*[l] & \mathbf{v}_2^*[l] \end{bmatrix} \begin{bmatrix} \mathbf{Z}_1[l] \\ \mathbf{Z}_2[l] \end{bmatrix} \right|^2 \right\}$$
(13)

As in this case low complexity equalizer with only two taps per bin is sufficient for compensation with optimal performance, the taps v_d (for d = 1, 2) can be updated according to the LMS rule:

$$\mathbf{v}_{\mathbf{d}}^{(i+1)}[l] = \mathbf{v}_{\mathbf{d}}^{(i)}[l] + \mu.\mathbf{e}^{*(i)}[l] \star \mathbf{Z}_{\mathbf{d}}^{(i)}[l];$$
(14)

where $\mathbf{e}^{(i)}[l] = \mathbf{D}^{(i)}[l] - \mathbf{S}^{(i)}[l]$ is the error signal generated using a training symbol $\mathbf{D}^{(i)}[l]$. μ is the LMS step-size parameter. In the case of IEEE 802.11a where data is transmitted on 48 out of 64 tones, 48 equalizers will be needed. The table below compares the computational load for Weights Update (WU) and Data Correction (DC) between our algorithm and the one in [5] for such systems. From the table we observe that the proposed algorithm has a significantly reduced complexity because fewer taps are needed per equalizer. This also allows for a faster convergence as shown for a typical scenario in Figure 2 where the proposed equalizer weights converge after only 30-50 training symbols as compared to [5] where convergence takes much longer.

	Algorithm in [5]		Proposed algorithm	
WU	288 mul	288 mul	144 mul	144 mul
DC	384 add	288 mul	96 add	48 mul



Equalizer coefficients convergence for 64QAM Fig. 2. OFDM (noiseless scenario). The dark curves represent equalizer weights of the proposed 2 tap LMS adaptive scheme and the dotted curves represent equalizer weights in the scheme of [5]. Frequency independent amplitude imbalance of $g_t, g_r =$ 3% and phase imbalance of $\phi_t, \phi_r = 3^\circ$. The filter impulse response are \mathbf{h}_{ti} = \mathbf{h}_{ri} = $[1, 0.5], \mathbf{h}_{tq}$ = \mathbf{h}_{rq} = $[0.9, 0.2], N = 64, \nu = 8.$

4. SIMULATION RESULTS

A typical OFDM system (similar to IEEE 802.11a) is simulated to evaluate the performance of the compensation scheme.



(a) 4-tap complex Gaussian channel (fading)



The parameters used in the simulation are as follows: OFDM symbol length N = 64, cyclic prefix length $\nu = 8$. Similar results can be obtained for $\nu = 16$ when the combined filter impulses are longer. There are 2 different channel profiles: 1) a multipath channel with L = 4, thus $(L \ll \nu)$ so that a 2 tap LMS equalizer can be used for compensation. The step size μ is initially set to 0.2 and is dynamically reduced as the simulation progresses. 2) A multipath channel with L = 10 taps $(L > \nu)$. In this case an RLS based adaptive PTEQ is used. The taps of the multipath channel are chosen independently with complex Gaussian distribution. The convergence can be improved further by estimating the channel separately on initial training symbols as is done normally in 802.11a. But an adaptive equalizer is still needed to completely equalize the channel distortions and IQ imbalances. Figure 3 shows the performance curves for an uncoded 64QAM OFDM system. The performance comparison is made with an ideal system with no front-end distortion and with a system with no IQ compensation algorithm included. The BER results depicted are obtained by averaging the BER curves over 10^4 independent channels. With no compensation scheme in place, the OFDM system is completely unusable. Even for the case when there is only frequency independent IQ imbalance, the BER is very high. For the case where the compensation scheme is employed, the curves are very close to the ideal situation with no front-end distortion. For the case $(\nu < L_t + L + L_r - 2)$, the PTEQ is essential to shorten the combined channel, transmitter and receiver filter and to compensate for the channel and IQ imbalance distortions.

5. CONCLUSION

In this paper the joint effect of transmitter and receiver frequency selective IO imbalance along with channel distortion has been studied and algorithms have been developed to compensate for such distortions in the digital domain. The algorithms provide a very efficient, post-FFT adaptive equaliza-



(b) 10-tap complex Gaussian channel (fading)

Fig. 3. BER vs SNR for 64QAM constellation. Frequency independent amplitude imbalance of $g_t, g_r = 5\%$ and phase

tion which leads to near ideal compensation.

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