PEP ANALYSIS OF THE SDP BASED JOINT CHANNEL ESTIMATION AND SIGNAL DETECTION

M. Stojnic, B. Hassibi, and H. Vikalo

California Institute of Technology, Pasadena, CA

ABSTRACT

In multi-antenna communication systems, channel information is often not known at the receiver. To fully exploit bandwidth resources of the system and ensure practical feasibility of the receiver, channel parameters are often estimated blindly and then employed in the design of signal detection algorithms. Instead of separating channel estimation from signal detection, in this paper we focus on the joint channel estimation and signal detection problem in a single-input multiple-output (SIMO) system. It is well known that finding solution to this optimization requires solving an integer maximization of a quadratic form and is, in general, an NP hard problem. To solve it, we propose an approximate algorithm based on the semi-definite program (SDP) relaxation. We derive a bound on the pairwise probability of error (PEP) of the proposed algorithm and show that, the algorithm achieves the same diversity as the exact maximum-likelihood (ML) decoder. The computed PEP implies that, over a wide range of system parameters, the proposed algorithm requires moderate increase in the signal-to-noise ratio (SNR) in order to achieve performance comparable to that of the ML decoder but with often significantly lower complexity.

Index Terms— Probability,Signal detection,Estimation, Noise,Communication systems

1. INTRODUCTION

Multi-antenna wireless communication systems are capable of providing reliable data transmission at very high rates. The channel in those systems is, in principle, unknown to the receiver and needs to be estimated either prior to or concurrently with the detection of the transmitted signal. One way of obtaining the channel parameters is by sending a training sequence known to both the transmitter and the receiver. Alternatively, to save the bandwidth, one may resort to blind estimation techniques which, in general, learn the channel by exploiting the known properties of the transmitted symbols. In this paper, we study the latter and focus on the *joint* channel estimation and signal detection in systems that have single transmit and multiple receive antennas, a frequent cellular systems uplink scenario.

We assume a standard flat-fading channel model for multi-antenna systems,

$$X = \sqrt{\frac{\rho T}{M}}\mathbf{s}\mathbf{h} + W \tag{1}$$

where T denotes the number of time intervals during which the channel remains constant, $M = 1$ is the number of the transmitted antennas, N is the number of the received antennas, ρ is the signal-to-noise ratio (SNR), X is a $T \times N$ matrix of the received symbols, $\mathbf{s}$ is a $T \times 1$ transmitted symbol vector comprised of components s_i for which it holds that $|s_i|^2 = \frac{1}{T}$, $\mathbf{h}$ is an $1 \times N$ channel matrix whose components are independent, identically distributed (i.i.d.) zero-mean, unit-variance complex Gaussian random variables, and W is an $N \times T$ noise matrix whose components are i.i.d. zero-mean, unit-variance complex Gaussian random variables. Furthermore, we assume that the components of $\mathbf{h}$ and W are uncorrelated and that $T \geq N$, which is often the case in practice.

In the next section, we review the joint channel estimation and signal detection problem and propose an efficient algorithm for finding its approximate solution.

2. JOINT CHANNEL ESTIMATION AND SIGNAL DETECTION

The optimal, joint maximum-likelihood (ML) channel estimator and signal decoder of the system (1) solves the optimization

$$\min_{\mathbf{s}\in\{-\frac{1}{\sqrt{T}},\frac{1}{\sqrt{T}}\}^T,\mathbf{h}} \|X - \sqrt{\rho T}\mathbf{s}\mathbf{h}\|^2. \tag{2}$$

It is easy to see (e.g., [5]) that the optimal $\mathbf{h}$ can be found as

$$\hat{\mathbf{h}} = \frac{1}{\sqrt{\rho T}}\mathbf{s}^* X.$$

Substituting this value of $\hat{\mathbf{h}}$ in (2), we can write

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$$\min_{\mathbf{s}\in\mathcal{S}} \|X - \sqrt{\rho T}\mathbf{s}\mathbf{h}\|^2 = \min_{\mathbf{s}\in\mathcal{S}} \|X - \mathbf{s}\mathbf{s}^* X\|^2$$
$$= \min_{\mathbf{s}\in\mathcal{S}} \{-\text{Tr}\,(\mathbf{s}^* X X^* \mathbf{s})\} = \max_{\mathbf{s}\in\mathcal{S}} \text{Tr}\,(X^* \mathbf{s}(X^*\mathbf{s})^*),$$

where we denoted $\mathcal{S} = \{-\frac{1}{\sqrt{T}}, \frac{1}{\sqrt{T}}\}^T$. Therefore, the integer optimization problem one needs to solve can be written as

$$\max_{\mathbf{s}\in\mathcal{S}} \text{Tr}\,(XX^*\mathbf{s}\mathbf{s}^*) \qquad (3)$$

Optimization (3) is a very difficult problem. In [5], the sphere decoder algorithm is employed to solve (3) *exactly* which, for some parameters, may be computationally costly. In this paper, we focus on finding a computationally efficient approximate solution to (3). In particular, we relax (3) and instead solve

$$\max_{Q\geq 0, Q_{ii}=1} \text{Tr}\,(XX^*Q). \qquad (4)$$

(This is a well-known semi-definite programming (SDP) relaxation, often used for obtaining approximate solutions to difficult combinatorial problems. Interesting reader can find more on that in [1] and its applications in communications in excellent references [6],[7]). Let $\hat{Q}$ and $\mathbf{s}_{\text{ML}}$ denote the solutions to (4) and (3), respectively. It can be shown (see [2]) that

$$\alpha \text{Tr}(XX^*\hat{Q}) \leq \text{Tr}(XX^*\mathbf{s}_{\text{ML}}\mathbf{s}_{\text{ML}}^*), \qquad (5)$$

where $\alpha = \frac{2}{\pi}$. Furthermore, for $\hat{\mathbf{s}} = \text{sgn}(L\mathbf{r})$, where L is any matrix such that $LL^* = \hat{Q}$ and $\mathbf{r}$ is a vector with Gaussian i.i.d. components, one can write

$$\alpha \text{Tr}\,(XX^*\mathbf{s}_{\text{ML}}\mathbf{s}_{\text{ML}}^*) \leq E_{|r}\text{Tr}(XX^*\hat{\mathbf{s}}\hat{\mathbf{s}}^*). \qquad (6)$$

Therefore, one can construct a suboptimal solution to (3) which has a guaranteed performance. Of course, strictly speaking, the performance is guaranteed only in the expected sense. However, if we repeat the randomized procedure sufficiently many times, we are very likely to obtain an instance with a cost whose value is greater than the true expectation. In fact, it was shown in [3] that, with certain modifications, the expectation in (6) can indeed be omitted.

Hence, there is a polynomial time algorithm which provides a suboptimal solution to (3), $\hat{\mathbf{s}}$, such that

$$\alpha \text{Tr}\,(XX^*\mathbf{s}_{\text{ML}}\mathbf{s}_{\text{ML}}^*) \leq \text{Tr}\,(XX^*\hat{\mathbf{s}}\hat{\mathbf{s}}^*). \qquad (7)$$

Now, in order to provide sound proofs in the following section we will slightly modify the SDP relaxation. Let $\bar{\mathbf{s}}$ be the solution of the following optimization problem

$$\bar{s} = \arg\max_{\mathbf{s},(\mathbf{s}^*\hat{\mathbf{s}})^2\geq\alpha} \text{Tr}XX^*\mathbf{s}\mathbf{s}^* \qquad (8)$$

We refer later in the paper to this way (based on a slight modification of the standard SDP-relaxation randomized algorithm) of generating a solution $\bar{\mathbf{s}}$ as Algorithm 1.

3. COMPUTING PEP

The probability of error can be written as

$$P_e = \sum_{i=1}^{2^T} P(\text{error}|\mathbf{s}_t \text{is sent})P(\mathbf{s}_t \text{is sent}). \qquad (9)$$

In the remainder of this section, we derive an upper bound on the $P(\text{error}|\mathbf{s}_t\text{is sent})$. To facilitate this derivation, let us assume that there is a Genie who can tell us if $\hat{\mathbf{s}}$ found in the first part of our algorithm is such that $(\hat{\mathbf{s}}^*\mathbf{s}_t)^2 < \alpha$. We formulate a slightly modified version of the algorithm and refer to it as the *Genie*. Its solution is $\hat{\mathbf{s}}_1$ such that

$$\begin{aligned} &\text{if} \quad (\hat{\mathbf{s}}^*\mathbf{s}_t)^2 < \alpha \qquad \hat{\mathbf{s}}_1 = \hat{\mathbf{s}} \\ &\text{if} \quad (\hat{\mathbf{s}}^*\mathbf{s}_t)^2 \geq \alpha \qquad \hat{\mathbf{s}}_1 = \bar{\mathbf{s}} \end{aligned} \qquad (10)$$

The probability of error for the *Genie* algorithm is given by

$$P_e^g = \sum_{i=1}^{2^T} P_g(\text{error}|\mathbf{s}_t \text{is sent})P(\mathbf{s}_t \text{is sent}). \qquad (11)$$

Clearly, our original algorithm will have smaller probability of error than the *Genie* since in the case when they differ, the original algorithm can work only better. Hence, we concentrate on bounding the probability of the *Genie*, i.e., on bounding $P_g(\text{error}|\mathbf{s}_t\text{is sent})$. To this end, note that

$$\begin{aligned} &P_g(\text{error}|\mathbf{s}_t\text{is sent}) = P(\hat{\mathbf{s}}_1 \neq \mathbf{s}_t) \\ &= P(\exists i:\ \hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t) \leq \sum_{\mathbf{s}_i\neq\mathbf{s}_t} P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t) \\ &\leq \sum_{(\mathbf{s}_i^*\mathbf{s}_t)^2<\alpha} P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t) + \sum_{(\mathbf{s}_i^*\mathbf{s}_t)^2\geq\alpha} P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t). \end{aligned} \qquad (12)$$

Let us consider $P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, (\mathbf{s}_i^*\mathbf{s}_t)^2 < \alpha)$ in more details. (For the brevity of notation, in the following expressions we omit that everything is conditioned on $\mathbf{s}_t$ being transmitted, and that $(\mathbf{s}_i^*\mathbf{s}_t)^2 < \alpha$.) So,

$$\begin{aligned} P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t) &= P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t|\hat{\mathbf{s}}_1 = \hat{\mathbf{s}})P(\hat{\mathbf{s}}_1 = \hat{\mathbf{s}})) \\ &+ P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, \hat{\mathbf{s}}_1 \neq \hat{\mathbf{s}}). \end{aligned} \qquad (13)$$

Let us define function C as $C(\mathbf{s}) = \text{Tr}XX^*\mathbf{s}\mathbf{s}^*$. Furthermore, let E denote the event that $(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, \hat{\mathbf{s}}_1 \neq \hat{\mathbf{s}})$. Clearly, E implies that $C(\mathbf{s}_i) = C(\hat{\mathbf{s}}_1) \geq C(\hat{\mathbf{s}}) \geq \alpha C(\mathbf{s}_t)$, which further means that $C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t)$. Using this, we obtain $P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, \hat{\mathbf{s}}_1 \neq \hat{\mathbf{s}}) \leq P(C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t))$. Also, following the similar argument, it is not difficult to see that $P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t | \hat{\mathbf{s}}_1 = \hat{\mathbf{s}}) P(\hat{\mathbf{s}}_1 = \hat{\mathbf{s}})) \leq P(C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t))$. Replacing the obtained inequalities in (13) we have

$$P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, (\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha) \leq 2P(C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t)). \tag{14}$$

Now, let us consider $P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, (\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha)$. It is easy to see that

$$P(\hat{\mathbf{s}}_1 = \mathbf{s}_i \neq \mathbf{s}_t, (\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha) \leq P(C(\mathbf{s}_i) \geq C(\mathbf{s}_t)). \tag{15}$$

Substituting (14) and (15) in (13), we finally obtain

$$\begin{aligned} P_g(\text{error}|\mathbf{s}_t \text{is sent}) \leq & \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 \leq \alpha} 2P(C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t)) \\ & + \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha} P(C(\mathbf{s}_i) \geq C(\mathbf{s}_t)). \end{aligned} \tag{16}$$

In the remainder of this section, we compute bounds on $P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} = P(C(\mathbf{s}_i) \geq \alpha C(\mathbf{s}_t) | \mathbf{s}_t \text{is sent}, (\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha)$, $P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha} = P(C(\mathbf{s}_i) \geq \mathcal{C}(\mathbf{s}_t) | \mathbf{s}_t \text{ is sent}, (\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha)$,

$$\begin{aligned} P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} = & P(\text{Tr}(X^* \mathbf{s}_i)(X^* \mathbf{s}_i)^* \geq \\ & \alpha \text{Tr}(X^* \mathbf{s}_t)(X^* \mathbf{s}_t)^* | \mathbf{s}_t \text{ is sent}). \end{aligned} \tag{17}$$

Since we assume that $\mathbf{s}_t$ was transmitted, it holds that $X = \sqrt{\rho T} \mathbf{s}_t \mathbf{h} + W$. To make writing easier let $k = \rho T$.Replacing this value for X in (17), we obtain

$$P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} = P(\text{Tr}(\begin{bmatrix} \mathbf{h} \\ W \end{bmatrix}^* Q_n \begin{bmatrix} \mathbf{h} \\ W \end{bmatrix} \geq 0 | \mathbf{s}_t \text{ is sent}), \tag{18}$$

where

$$\begin{aligned} Q_n &= \begin{bmatrix} \sqrt{k} \mathbf{s}_t^* \\ I \end{bmatrix} (\mathbf{s}_i \mathbf{s}_i^* - \alpha \mathbf{s}_t \mathbf{s}_t^*) \begin{bmatrix} \sqrt{k} \mathbf{s}_t & I \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{k} \mathbf{s}_t^* \\ I \end{bmatrix} \begin{bmatrix} \mathbf{s}_i & \mathbf{s}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} \mathbf{s}_i^* \\ \mathbf{s}_t^* \end{bmatrix} \begin{bmatrix} \sqrt{k} \mathbf{s}_t & I \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{k} \psi_{it}^* & \sqrt{k} \\ \mathbf{s}_i & \mathbf{s}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} \sqrt{k} \psi_{it} & \mathbf{s}_i^* \\ \sqrt{k} & \mathbf{s}_t^* \end{bmatrix}, \end{aligned}$$

and $\psi_{it} = \mathbf{s}_i^* \mathbf{s}_t$. Although it is possible to compute explicitly the probability in (18), we will find that it is sufficient to find its Chernoff bound. In particular,

$$\begin{aligned} P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} \leq \min_{\mu} E e^{\mu(\text{Tr}(\begin{bmatrix} \mathbf{h} \\ W \end{bmatrix}^* Q_n \begin{bmatrix} \mathbf{h} \\ W \end{bmatrix}))} = \\ = \int \frac{e^{-\text{Tr}(\begin{bmatrix} \mathbf{h} \\ W \end{bmatrix}^* (I - \mu Q_n) \begin{bmatrix} \mathbf{h} \\ W \end{bmatrix})} d\mathbf{h} dW}{\pi^N} = \frac{1}{\det(I - \mu Q_n)^N} \end{aligned} \tag{19}$$

We first simplify the determinant in the denominator as

$$\det(I - \mu Q_n) = \det(I - \mu \begin{bmatrix} k \psi_{it} \psi_{it}^* + 1 & (k+1) \psi_{it} \\ -\alpha (k+1) \psi_{it}^* & -\alpha (k+1) 1 \end{bmatrix}).$$

After some further algebraic transformations we obtain

$$\det(I - \mu Q_n) = (k+1) \alpha (V^{(it)} - 1)(-\mu + \xi^{(1)})(-\mu + \xi^{(2)}) \tag{20}$$

with

$$\xi^{(1)} = \frac{V^{(it)} - \alpha + \frac{1-\alpha}{k} + \sqrt{(V^{(it)} - \alpha + \frac{1-\alpha}{k})^2 + \frac{4\alpha(1 - V^{(it)})(k+1)}{k^2}}}{2\alpha (V^{(it)} - 1) \frac{k+1}{k}},$$

$$\xi^{(2)} = \frac{V^{(it)} - \alpha + \frac{1-\alpha}{k} - \sqrt{(V^{(it)} - \alpha + \frac{1-\alpha}{k})^2 + \frac{4\alpha(1 - V^{(it)})(k+1)}{k^2}}}{2\alpha (V^{(it)} - 1) \frac{k+1}{k}},$$

and $V^{(it)} = \psi_{it} \psi_{it}^*$. Although our results will hold for any SNR, to make writing less tedious in the rest of the paper we consider only the case of large SNR. Therefore, the previous results simplify to

$$P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} \leq \frac{1}{(k \frac{(\alpha - V^{(it)})^2}{4(1 - V^{(it)})})^N}. \tag{21}$$

To compute the bound on $P(C(\mathbf{s}_i) \geq \mathcal{C}(\mathbf{s}_t) | \mathbf{s}_t \text{is sent}, (\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha)$ we will use a well known result from the literature (see e.g.,[4])

$$P_{it|(\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha} \leq \frac{1}{(k \frac{(1 - V^{(it)})}{4})^N}. \tag{22}$$

Now we can substitute the results from (21) and (22) in (16) and obtain

$$\begin{aligned} P_g(\text{error}|\mathbf{s}_t \text{is sent}) \leq & \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} 2 \frac{1}{(k \frac{(\alpha - V^{(it)})^2}{4(1 - V^{(it)})})^N} \\ & + \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha} \frac{1}{(k \frac{(1 - V^{(it)})}{4})^N}. \end{aligned} \tag{23}$$

Recall that in the case of the exact ML decoding, which requires algorithms none of which is of polynomial complexity, we have for the same probability of error

$$\begin{aligned} P_{ML}(\text{error}|\mathbf{s}_t \text{is sent}) \leq & \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 < \alpha} \frac{1}{(k \frac{(1 - V^{(it)})}{4})^N} \\ & + \sum_{(\mathbf{s}_i^* \mathbf{s}_t)^2 \geq \alpha} \frac{1}{(k \frac{(1 - V^{(it)})}{4})^N}. \end{aligned} \tag{24}$$

Clearly, comparing (23) and (24) it follows that the algorithm based on the well known SDP relaxation (slightly refined here for the purposes of the valid proof) has the same diversity as the exact ML solution. Of course, since the SDP-relaxation algorithm is only an approximation, the exact ML solution still has an advantage of $(\frac{1-V^{(it)}}{\alpha-V^{(it)}})^2$ in the coding gain.

We summarize the previous results in the following theorem.

Theorem 1 *Consider a problem of joint channel estimation and signal detection for a SIMO system described in (1). Assume that any codeword $\mathbf{s}_t$ was transmitted. Then the probability that an error occurred if Algorithm 1 was applied to solve (3), as a part of the joint channel estimation and signal detection process, can be upper bounded in the following way*

$$\begin{aligned} P(\mathit{error}|\mathbf{s}_t \mathit{is\ sent}) \quad & \leq \quad \sum_{(\mathbf{s}_i^*\mathbf{s}_t)^2<\alpha} 2\frac{1}{(\rho T\frac{(\alpha-V^{(it)})^2}{4(1-V^{(it)})})^N} \\ & + \quad \sum_{(\mathbf{s}_i^*\mathbf{s}_t)^2\geq\alpha} \frac{1}{(\rho T\frac{(1-V^{(it)})}{4})^N}. \end{aligned}$$

Proof: Follows from the previous discussion.

At the end, let us elaborate briefly on the complexity of the algorithm that we proposed. By carefully inspecting it, one can note that due to the modification of the conventional SDP randomized algorithm, our algorithm is strictly speaking no longer polynomial. However, for the cases where $T < 60$, the additional amount of operations on top of the basic SDP core of the algorithm is of effectively negligible complexity (although, strictly speaking, this additional amount is what makes our algorithm being exponential). To see this, note that the additional complexity is equal to the number of the vectors $\mathbf{s}$ which satisfy inequality $(\mathbf{s}^*\hat{\mathbf{s}})^2 \geq \alpha = \frac{2}{\pi}$, $|S^c|$. Clearly, this number can be upper-bounded as

$$|S^c| \leq \left\lfloor \frac{T(1-\sqrt{\alpha})}{2} \right\rfloor \binom{T}{\lfloor \frac{T(1-\sqrt{\alpha})}{2} \rfloor} \leq T^{4.2}, \text{if} \quad T < 60, \tag{25}$$

where we have assumed that for $T < 60$ complexity of solving an SDP is $60^{4.2}$.

However, using the fact that for large T and small k, $\binom{T}{k} \approx 2^{TH(k/T)}$, (where $H(k/T)$ is the entropy function evaluated at k/T), one can show that

$$\left\lfloor \frac{T(1-\sqrt{\alpha})}{2} \right\rfloor \binom{T}{\lfloor \frac{T(1-\sqrt{\alpha})}{2} \rfloor} \approx 2^{TH(\lfloor(1-\sqrt{\alpha})/2\rfloor)} = 2^{0.47T}. \tag{26}$$

The previous expression implies that the additional amount of computation introduced to ensure validity of our proof is indeed exponential, while of course in the limit of large T the complexity of solving SDP becomes $T^{3.5}$. However, the exponential constant is two times smaller than in the exhaustive search. Therefore, in communications, where the dimension of practical SIMO systems is rarely bigger than 60, the complexity of our algorithm is of the same order as the complexity of the SDP.

4. DISCUSSION AND CONCLUSION

We proposed a modification of the SDP relaxation for solving the joint channel estimation and data detection problem in single-input multiple-output communication systems. The computed PEP implies that the performance of the algorithm is comparable to that of the optimal ML solution, but is obtained at potentially significantly lower computational complexity. Of course, it would be of a great interest if one could construct a provably polynomial algorithm which has the same PEP performance as the one we analyzed in this paper. That will be subject of a future work.

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