LOW-COMPLEXITY MIMO DATA DETECTION USING SEYSEN'S LATTICE REDUCTION ALGORITHM

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ABSTRACT

Lattice reduction (LR) is a powerful technique for improving the performance of suboptimum MIMO data detection methods. For LR-assisted data detection, the LLL algorithm has been considered almost exclusively so far. In this paper, we propose and develop the application of *Seysen's algorithm* (SA) to LR-assisted MIMO detection, and we show that the SA is a promising alternative to the LLL algorithm. Specifically, the SA outperforms the LLL algorithm in that it finds better lattice bases for MIMO systems of practical interest, which is reflected by an improved performance of SA-assisted detectors relative to their LLL-assisted counterparts. We present an efficient implementation of the SA whose per-iteration complexity is linear in the number of antennas, and we demonstrate that the SA requires significantly fewer iterations than the LLL algorithm.

***Index Terms*—**MIMO data detection, spatial multiplexing, lattice reduction, LLL algorithm, Seysen's algorithm.

1. INTRODUCTION

It is well known that the diversity offered by multiple-input multiple-output (MIMO) fading channels cannot be fully exploited by conventional suboptimum detectors (like linear equalization and decision-feedback schemes [1]), and thus the performance of such detectors is significantly inferior to that of maximum likelihood (ML) detection [2]. ML detection exploits all of the available diversity but tends to be computationally intensive.

A powerful preprocessing technique for improving the performance of suboptimum data detectors is *lattice reduction* (LR) [3–6]. The channel realization is regarded as a basis of a lattice, and one attempts to find a better (i.e., more orthogonal) basis for the same lattice. Suboptimum detectors can then be applied to this better basis, which results in improved performance. So far, almost exclusively the *LLL algorithm* [7] has been considered for LR-assisted data detection. The LLL algorithm allows suboptimum detectors to exploit all of the available diversity [8].

In this paper, we propose *Seysen's algorithm* (SA) [9,10] for LR-assisted MIMO detection. By simultaneously reducing the lattice basis and its dual, the SA attains a (local) minimum of Seysen's orthogonality measure in an efficient manner. Seysen's orthogonality measure, like the orthogonality defect, is minimal if and only if the basis is orthogonal. It is shown in [9,10] that the SA can achieve very good results—in the sense of efficiently finding the shortest lattice basis vector—for lattices of moderate size.

This work was supported by the STREP project MASCOT (IST-026905) within the Sixth Framework of the European Commision.

Here, we will demonstrate that, for MIMO detection, the SA outperforms the LLL algorithm in that it finds better bases for MIMO channels of practical interest. Indeed, our simulation results show a significantly improved performance of SA-assisted detectors compared to their LLL-assisted counterparts. We also describe an efficient implementation of the SA whose computational complexity per iteration is linear in the number of antennas (as for the LLL algorithm), and we demonstrate that the SA requires significantly fewer iterations and basis updates than the LLL algorithm.

This paper is organized as follows. In the remainder of this section, we discuss the system model and we review the principle of LR-assisted data detection. In Section 2, we describe the SA in the MIMO detection setting. An efficient implementation of the SA and a complexity assessment are provided in Section 3. Finally, simulation results are presented in Section 4.

1.1. System Model

We consider a MIMO channel with M transmit antennas and $N \geq M$ receive antennas, and a spatial multiplexing system such as V-BLAST [1] where the mth data symbol d_m is directly transmitted by the mth transmit antenna. For a given time instant, this leads to the well-known baseband model

$$\mathbf{r} = \mathbf{H}\mathbf{d} + \mathbf{w}, \tag{1}$$

with the transmitted data vector $\mathbf{d} \triangleq (d_1 \cdots d_M)^T$, the $N \times M$ channel matrix $\mathbf{H}$, the received vector $\mathbf{r} \triangleq (r_1 \cdots r_N)^T$, and the noise vector $\mathbf{w} \triangleq (w_1 \cdots w_N)^T$. The data symbols d_m are drawn from a symbol alphabet $\mathscr{A}$ that is properly scaled and shifted such that it consists of complex-valued integers (see, e.g., [4,5]). The noise components w_i are assumed independent and circularly symmetric complex Gaussian with variance $\sigma_{\mathbf{w}}^2$.

1.2. LR-Assisted Data Detection

In LR-assisted data detection [3–6] (see also Babai's approximation [11]), the columns $\mathbf{h}_m$ of the channel matrix $\mathbf{H} = [\mathbf{h}_1 \cdots \mathbf{h}_M]$ are viewed as a basis for an M-dimensional lattice $\mathscr{L}$ in[1] $\mathbb{C}^N$,

$$\mathscr{L} \triangleq \left\{ \mathbf{H}\mathbf{z} = \sum_{m=1}^{M} \mathbf{h}_m z_m : \ \mathbf{z} \in \mathbb{CZ}^M \right\}.$$

The goal now is to transform the lattice basis $\mathbf{H}$ into a "better" (i.e., more orthogonal) basis $\widetilde{\mathbf{H}}$ for the same lattice $\mathscr{L}$. This allows sub-

[1] We remain in the complex domain although LR with the LLL algorithm is usually discussed for an equivalent real $2M$-dimensional lattice [4,5].

optimum detectors (such as the zero-forcing (ZF) detector) to perform more reliably. The "reduced" and original bases are related as $\widetilde{\mathbf{H}} = \mathbf{HB}$, where $\mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_M]$ is an $M \times M$ unimodular matrix (i.e., it has complex integer entries and $\det(\mathbf{B}) = \pm 1$).

Data detection based on the reduced basis $\widetilde{\mathbf{H}}$ is now performed as follows. We can rewrite our system model (1) as

$$\mathbf{r} = \mathbf{HBB}^{-1}\mathbf{d} + \mathbf{w} = \widetilde{\mathbf{H}}\tilde{\mathbf{d}} + \mathbf{w},$$

where $\tilde{\mathbf{d}} = \mathbf{B}^{-1}\mathbf{d}$ is in $\mathbb{CZ}^M$ since $\mathbf{B}^{-1}$ is again integer-valued. Hence, instead of detecting $\mathbf{d} \in \mathscr{A}^M$ based on $\mathbf{H}$, one can detect $\tilde{\mathbf{d}} \in \mathbb{CZ}^M$ based on $\widetilde{\mathbf{H}}$. Let us denote the result as $\hat{\tilde{\mathbf{d}}} \in \mathbb{CZ}^M$. The final detection result $\hat{\mathbf{d}} \in \mathscr{A}^M$ is then obtained as

$$\hat{\mathbf{d}} = Q_{\mathscr{A}}\{\mathbf{B}\hat{\tilde{\mathbf{d}}}\},$$

where $Q_{\mathscr{A}}\{\cdot\}$ denotes componentwise quantization with respect to the symbol alphabet $\mathscr{A}$.

2. SEYSEN'S ALGORITHM FOR MIMO DETECTION

2.1. Preliminaries

For LR-assisted data detection or precoding, the LLL algorithm has been considered almost exclusively [4–6, 11, 12]. By using an upper bound on the *orthogonality defect* of the reduced basis $\widetilde{\mathbf{H}}$ obtained with the LLL algorithm, it has been shown [8] that the usual suboptimum detectors applied after LLL preprocessing can achieve full diversity. We note that the orthogonality defect of $\widetilde{\mathbf{H}}$ is defined by

$$\delta(\widetilde{\mathbf{H}}) \triangleq \frac{1}{|\mathscr{L}|} \prod_{m=1}^{M} \|\widetilde{\mathbf{h}}_m\|^2, \tag{2}$$

where $|\mathscr{L}| = \det(\mathbf{H}^H\mathbf{H}) = \det(\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}})$ is the volume of a fundamental cell of the lattice $\mathscr{L}$. We have $\delta(\widetilde{\mathbf{H}}) \geq 1$, with $\delta(\widetilde{\mathbf{H}}) = 1$ if and only if the columns of $\widetilde{\mathbf{H}}$ are orthogonal.

The SA is based on a measure of orthogonality that is different from the orthogonality defect in (2), namely, *Seysen's orthogonality measure* [9]

$$S(\widetilde{\mathbf{H}}) \triangleq \sum_{m=1}^{M} \|\widetilde{\mathbf{h}}_m\|^2 \|\widetilde{\mathbf{h}}_m^\#\|^2. \tag{3}$$

Here, $\widetilde{\mathbf{h}}_m^\#$ is the mth basis vector of the *dual lattice* $\mathscr{L}^\#$, i.e., $\widetilde{\mathbf{H}}^{\#H}\widetilde{\mathbf{H}} = \mathbf{I}$, where $\widetilde{\mathbf{H}}^\# \triangleq [\widetilde{\mathbf{h}}_1^\# \cdots \widetilde{\mathbf{h}}_M^\#]$ denotes the dual basis. $S(\widetilde{\mathbf{H}})$ assumes its minimum, $S(\widetilde{\mathbf{H}}) = M$, if and only if the basis $\widetilde{\mathbf{H}}$ is orthogonal. The SA finds a (local) minimum of $S(\widetilde{\mathbf{H}}) = S(\mathbf{HB})$ in an iterative way. In view of (3), we can say that the basis *and its dual* are reduced simultaneously.

2.2. Basic Principle of the SA

The following discussion of the basic principle of the SA adapts the SA (as presented in [9, 10]) to the complex-valued case.

Structure of the SA. With the initialization $\mathbf{B} = \mathbf{I}$ and $\widetilde{\mathbf{H}} = \mathbf{H}$, the SA repeats the following steps until $\widetilde{\mathbf{H}}$ is SA-reduced (see below):

1. Based on $\widetilde{\mathbf{H}} = \mathbf{HB}$, an index pair (s,t) with $s,t \in \{1,\ldots,M\}$ is selected and a corresponding update value $\lambda_{s,t} \in \mathbb{CZ}$ is calculated.
2. Basis update:
$$\mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_{s-1}\, \mathbf{b}_s'\, \mathbf{b}_{s+1} \cdots \mathbf{b}_M] \quad \text{with} \quad \mathbf{b}_s' = \mathbf{b}_s + \lambda_{s,t}\mathbf{b}_t \tag{4}$$
or, equivalently,
$$\widetilde{\mathbf{H}} = \left[\widetilde{\mathbf{h}}_1 \cdots \widetilde{\mathbf{h}}_{s-1}\, \widetilde{\mathbf{h}}_s'\, \widetilde{\mathbf{h}}_{s+1} \cdots \widetilde{\mathbf{h}}_M\right] \quad \text{with} \quad \widetilde{\mathbf{h}}_s' = \widetilde{\mathbf{h}}_s + \lambda_{s,t}\widetilde{\mathbf{h}}_t. \tag{5}$$
Note that $\widetilde{\mathbf{h}}_s' = \mathbf{H}\mathbf{b}_s'$.

At each iteration, $\widetilde{\mathbf{H}}$ is again a valid basis for $\mathscr{L}$. If fact, any basis for $\mathscr{L}$ can be achieved by a sequence of updates according to (4) or (5) (also the LLL algorithm uses such updates).

SA-Reduced Basis. Let us consider a basis vector update (5) for a given index pair (i,j) (not necessarily the selected index pair (s,t)):

$$\widetilde{\mathbf{H}}_{i,j} \triangleq [\widetilde{\mathbf{h}}_1 \cdots \widetilde{\mathbf{h}}_{i-1}\, \widetilde{\mathbf{h}}_i'\, \widetilde{\mathbf{h}}_{i+1} \cdots \widetilde{\mathbf{h}}_M] \quad \text{with} \quad \widetilde{\mathbf{h}}_i' = \widetilde{\mathbf{h}}_i + \lambda_{i,j}\widetilde{\mathbf{h}}_j.$$

Following the derivation in [9, 10], the best update value $\lambda_{i,j}$ such that $S(\widetilde{\mathbf{H}}_{i,j})$ is minimized is obtained as

$$\lambda_{i,j} = \left\lfloor \frac{1}{2}\left(\frac{\widetilde{\mathbf{h}}_j^{\#H}\widetilde{\mathbf{h}}_i^\#}{\|\widetilde{\mathbf{h}}_i^\#\|^2} - \frac{\widetilde{\mathbf{h}}_j^H\widetilde{\mathbf{h}}_i}{\|\widetilde{\mathbf{h}}_j^\#\|^2}\right)\right\rceil, \tag{6}$$

where $\lfloor\cdot\rceil$ denotes rounding to the nearest integer. It can be shown [9] that $S(\widetilde{\mathbf{H}}_{i,j}) < S(\widetilde{\mathbf{H}})$ if and only if $\lambda_{i,j} \neq 0$. We call the basis $\widetilde{\mathbf{H}}$ *SA-reduced* if no decrease of $S(\widetilde{\mathbf{H}})$ can be achieved for any (i,j), i.e., $\lambda_{i,j} = 0$ for all possible (i,j). Thus, to obtain an SA-reduced basis, one simply has to repeat Step 1 and Step 2 until no decrease of $S(\widetilde{\mathbf{H}}_{i,j})$ is observed; this corresponds to a local minimum of Seysen's orthogonality measure.

Selection of (s,t). For determination of the index pair (s,t), we adopt a greedy selection procedure as proposed in [10]. At each iteration, one selects (s,t) such that the decrease in Seysen's orthogonality measure is maximized, i.e.,

$$(s,t) = \arg\max_{(i,j)} \Delta_{i,j}, \quad \text{with } \Delta_{i,j} \triangleq S(\widetilde{\mathbf{H}}) - S(\widetilde{\mathbf{H}}_{i,j}). \tag{7}$$

That is, one tests all $M^2 - M$ potential basis updates with respect to their achieved reduction of Seysen's orthogonality measure and the best basis update is retained. If $\lambda_{s,t} = 0$ (which happens after a finite number of iterations), the algorithm is converged and a local minimum of Seysen's orthogonality measure has been found.

3. EFFICIENT IMPLEMENTATION

While the basic principle of SA reduction is rather simple, the computational complexity appears to be high because at each iteration $M^2 - M$ different λ and Δ values have to calculated. However, the complexity can be reduced significantly by an efficient implementation of the SA (foreshadowed in [10]) that is presented next.

3.1. Algorithm Statement

Input. The input of the algorithm is given by the original basis of $\mathscr{L}$, i.e., the channel matrix $\mathbf{H}$; the basis of the dual lattice $\mathscr{L}^\#$, i.e., $\mathbf{H}^\# = \mathbf{P}^H$, where $\mathbf{P} \triangleq (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ is the pseudo-inverse (or ZF equalizer) of $\mathbf{H}$; and the corresponding Gram matrices $\mathbf{G} = \mathbf{H}^H\mathbf{H}$ and $\mathbf{G}^\# = \mathbf{H}^{\#H}\mathbf{H}^\# = (\mathbf{H}^H\mathbf{H})^{-1}$.

Initialization. Set $\widetilde{\mathbf{H}} = \mathbf{H}$ and $\widetilde{\mathbf{H}}^\# = \mathbf{H}^\#$, and calculate all possible update values $\lambda_{i,j}$ with their corresponding reduction $\Delta_{i,j}$ of Seysen's orthogonality measure. These values will be used for the updates performed at the various iterations (see below). From (6),

$$\lambda_{i,j} = \lfloor x_{i,j} \rceil , \quad \text{with } x_{i,j} \triangleq \frac{1}{2}\left(\frac{G^{\#}_{j,i}}{G^{\#}_{i,i}} - \frac{G_{j,i}}{G_{j,j}}\right). \tag{8}$$

Using this expression, $\Delta_{i,j}$ can be efficiently calculated as follows. The update of the ith basis vector of $\widetilde{\mathbf{H}}$ according to (5), $\widetilde{\mathbf{h}}'_i = \widetilde{\mathbf{h}}_i + \lambda_{i,j}\widetilde{\mathbf{h}}_j$, corresponds to the update of the jth basis vector of $\widetilde{\mathbf{H}}^{\#}$ according to

$$\widetilde{\mathbf{H}}^{\#}_{i,j} = [\widetilde{\mathbf{h}}^{\#}_1 \cdots \widetilde{\mathbf{h}}^{\#}_{j-1}\, \widetilde{\mathbf{h}}^{\#\prime}_j\, \widetilde{\mathbf{h}}^{\#}_{j+1} \cdots \widetilde{\mathbf{h}}^{\#}_M] \;\; \text{with} \;\; \widetilde{\mathbf{h}}^{\#\prime}_j = \widetilde{\mathbf{h}}^{\#}_j - \lambda^*_{i,j}\widetilde{\mathbf{h}}^{\#}_i . \tag{9}$$

We then have

$$\begin{aligned}\Delta_{i,j} &= S(\widetilde{\mathbf{H}}) - S(\widetilde{\mathbf{H}}_{i,j})\\ &= \|\widetilde{\mathbf{h}}_i\|^2\|\widetilde{\mathbf{h}}^{\#}_i\|^2 + \|\widetilde{\mathbf{h}}_j\|^2\|\widetilde{\mathbf{h}}^{\#}_j\|^2 - \|\widetilde{\mathbf{h}}'_i\|^2\|\widetilde{\mathbf{h}}^{\#}_i\|^2 - \|\widetilde{\mathbf{h}}_j\|^2\|\widetilde{\mathbf{h}}^{\#\prime}_j\|^2 .\end{aligned}$$

Inserting $\widetilde{\mathbf{h}}'_i = \widetilde{\mathbf{h}}_i + \lambda_{i,j}\widetilde{\mathbf{h}}_j$ and $\widetilde{\mathbf{h}}^{\#\prime}_j = \widetilde{\mathbf{h}}^{\#}_j - \lambda^*_{i,j}\widetilde{\mathbf{h}}^{\#}_i$, we obtain further

$$\begin{aligned}\Delta_{i,j} &= -2\big(|\lambda_{i,j}|^2\|\widetilde{\mathbf{h}}_j\|^2\|\widetilde{\mathbf{h}}^{\#}_i\|^2 + \|\widetilde{\mathbf{h}}^{\#}_i\|^2\,\mathrm{Re}\{\lambda_{i,j}\widetilde{\mathbf{h}}^H_i\widetilde{\mathbf{h}}_j\}\\ &\qquad\qquad - \|\widetilde{\mathbf{h}}_j\|^2\,\mathrm{Re}\{\lambda^*_{i,j}\widetilde{\mathbf{h}}^{\#H}_j\widetilde{\mathbf{h}}^{\#}_i\}\big)\\ &= -2\big(|\lambda_{i,j}|^2 G_{j,j}G^{\#}_{i,i} + G^{\#}_{i,i}\,\mathrm{Re}\{\lambda_{i,j}G_{i,j}\} - G_{j,j}\,\mathrm{Re}\{\lambda^*_{i,j}G^{\#}_{j,i}\}\big)\\ &= 2\,G_{j,j}G^{\#}_{i,i}\left[\mathrm{Re}\left\{\lambda^*_{i,j}\left(\frac{G^{\#}_{j,i}}{G^{\#}_{i,i}} - \frac{G^*_{i,j}}{G_{j,j}}\right)\right\} - |\lambda_{i,j}|^2\right]\\ &= 2\,G_{j,j}\,G^{\#}_{i,i}\big(2\,\mathrm{Re}\{\lambda^*_{i,j}\,x_{i,j}\} - |\lambda_{i,j}|^2\big). \end{aligned} \tag{10}$$

Iteration. Set $\mathbf{B} = \mathbf{I}$ and repeat the following steps until $\widetilde{\mathbf{H}}$ is SA-reduced (i.e., $\lambda_{i,j} = 0$ for all (i,j)).

1. Select (s,t) according to (7) and update $\mathbf{B}$ (see (4)), $\widetilde{\mathbf{H}}$ (see (5)), and $\widetilde{\mathbf{H}}^{\#}$ (see (9)) using $\lambda_{s,t}$ (see (6)).

2. Compute corresponding updates of $\mathbf{G}$ and $\mathbf{G}^{\#}$. Because the update of $\widetilde{\mathbf{H}}$ just changes the sth column, only the sth row and column of $\mathbf{G}$ have to be updated. This can be performed according to

$$\begin{aligned} G'_{s,j} &= (\widetilde{\mathbf{h}}_s + \lambda_{s,t}\widetilde{\mathbf{h}}_t)^H\widetilde{\mathbf{h}}_j = G_{s,j} + \lambda^*_{s,t}G_{t,j}, \quad j \neq s\\ G'_{s,s} &= \|\widetilde{\mathbf{h}}'_s\|^2\\ G'_{j,s} &= G'^*_{s,j}. \end{aligned}$$

   Similarly, the tth row and column of $\mathbf{G}^{\#}$ are updated according to

$$\begin{aligned} G^{\#\prime}_{t,j} &= (\widetilde{\mathbf{h}}^{\#}_t - \lambda^*_{s,t}\widetilde{\mathbf{h}}^{\#}_s)^H\widetilde{\mathbf{h}}^{\#}_j = G^{\#}_{t,j} - \lambda_{s,t}G^{\#}_{s,j}, \quad j \neq t\\ G'_{t,t} &= \|\widetilde{\mathbf{h}}^{\#\prime}_t\|^2\\ G^{\#\prime}_{j,t} &= G^{\#\prime *}_{t,j}. \end{aligned}$$

3. Calculate new $\lambda_{i,j}$ values (see (8)) and $\Delta_{i,j}$ values (see (10)) for all index pairs corresponding to updated elements of $\mathbf{G}$ and $\mathbf{G}^{\#}$. These are the index pairs (i,s), (j,t), (s,i), and (t,j) for $i = 1,\ldots,M$ and $j = 1,\ldots,M$ with $i \neq s$, $j \neq t$.

Output. The output of the algorithm is given by the unimodular transformation matrix $\mathbf{B}$, the SA-reduced basis $\widetilde{\mathbf{H}} = \mathbf{HB}$, and the associated reduced dual basis $\widetilde{\mathbf{H}}^{\#} = \widetilde{\mathbf{P}}^H$.

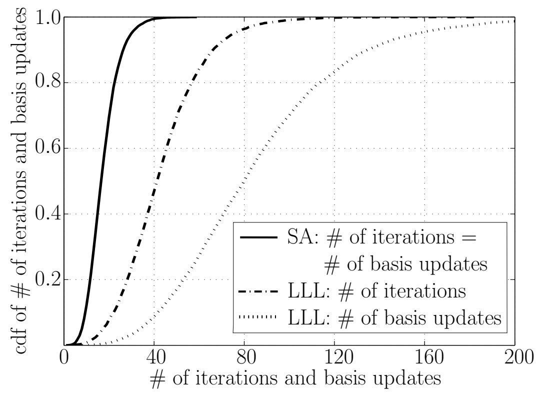

Figure 1. *cdf of the number of iterations and basis updates required by the SA and the LLL algorithm.*

3.2. Computational Complexity

The complexity of an LR algorithm for MIMO detection depends on the hardware implementation and platform used. However, to a large extent, it will be determined by the required number of iterations (which will be assessed by means of simulation results in Section 4), since across the iterations parallel or pipelined hardware structures are hardly possible. For a rough picture of the per-iteration complexity, we now provide corresponding asymptotic $\mathcal{O}(\cdot)$ results.

The initialization of the SA requires calculation of M^2-M different λ values according to (8) and at most (if the corresponding $\lambda_{i,j}$'s are all nonzero) M^2-M different Δ values according to (10). Thus, the initialization step has a computational complexity of $\mathcal{O}(M^2)$. At each iteration, the update of $\mathbf{B}$, $\widetilde{\mathbf{H}}$, and $\widetilde{\mathbf{H}}^{\#}$ has a complexity of $\mathcal{O}(M)$. Furthermore, the update of $\mathbf{G}$ and $\mathbf{G}^{\#}$ involves M elements, which gives a complexity of $\mathcal{O}(M)$. Finally, the calculation of $4M-4$ λ values and at most $4M-4$ Δ values in Step 3 above again results in a complexity of $\mathcal{O}(M)$. Thus, per iteration the computational complexity of the SA is just linear in M.

The LLL algorithm, too, has a complexity of $\mathcal{O}(M)$ per iteration. However, it is important to note that one LLL iteration (i.e., column swap with Givens rotations) usually comprises several basis updates using size reduction (see, e.g., [5] for details). This is different from the SA, where one iteration corresponds to exactly one basis update. We note, at this point, that the per-iteration complexity of the SA is larger than that of the LLL algorithm whereas its required number of iterations is significantly smaller (see Section 4). Furthermore, the individual update operations performed at each SA iteration are to a large part independent of each other, which allows the use of parallel hardware structures to increase the throughput.

4. SIMULATION RESULTS

We will now assess the performance of the SA and compare it to that of the LLL algorithm by means of simulation results. We considered a MIMO channel with $M = N = 8$ antennas and iid Gaussian entries. The SA was directly applied to the complex channel while the LLL algorithm was applied to its equivalent real form (cf. [4–6]).

Number of Iterations. First, we compare the number of iterations and basis updates required by the SA and the LLL algorithm. Fig. 1 shows the corresponding cumulative density functions (cdf's). It is seen that the SA requires significantly fewer iterations and basis

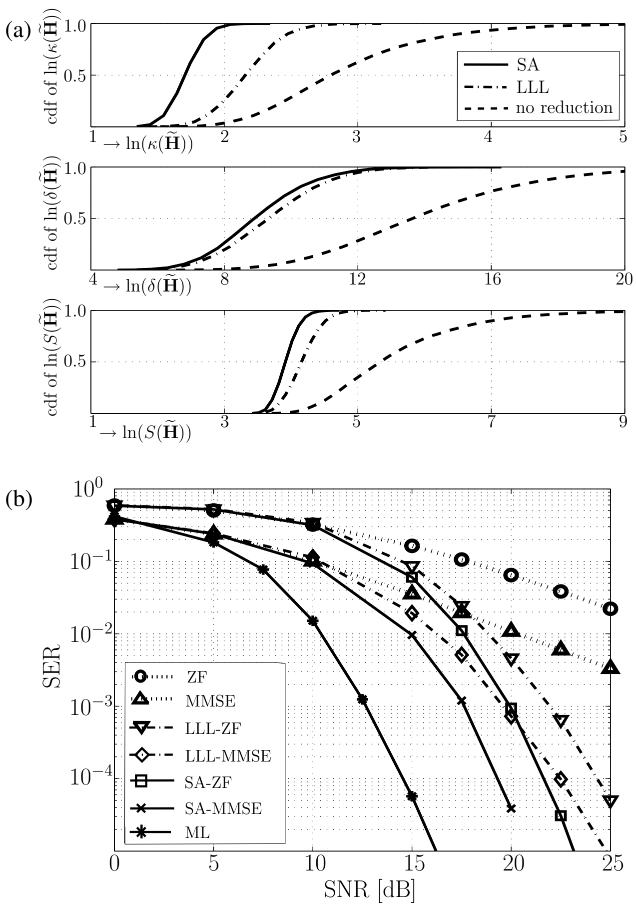

Figure 2. *Performance of the SA and the LLL algorithm: (a) cdf of* $\ln(\kappa(\widetilde{\mathbf{H}}))$ *(*$\kappa(\widetilde{\mathbf{H}})$ *denotes the condition number of* $\widetilde{\mathbf{H}}$*),* $\ln(\delta(\widetilde{\mathbf{H}}))$ *in* (2)*, and* $\ln(S(\widetilde{\mathbf{H}}))$ *in* (3) *for the SA-reduced, LLL-reduced, and original bases, (b) SER-versus-SNR performance of SA-assisted and LLL-assisted ZF and MMSE detection.*

updates than the LLL algorithm. Furthermore, the variability of the number of iterations and basis updates is smaller.

Quality of Reduced Basis. Next, we compare the performance of the SA and the LLL algorithm in terms of the quality (orthogonality) of the reduced basis $\widetilde{\mathbf{H}}$. Besides the orthogonality defect $\delta(\widetilde{\mathbf{H}})$ in (2) and Seysen's orthogonality measure $S(\widetilde{\mathbf{H}})$ in (3), we also consider the *condition number*, i.e. the ratio of the maximum and minimum singular values of $\widetilde{\mathbf{H}}$, denoted as $\kappa(\widetilde{\mathbf{H}})$. This is another popular measure of the quality of a basis for data detection (e.g., [5, 13]). Fig. 2(a) compares the cdf's of the natural logarithm $\ln(\cdot)$ of these quantities for the SA-reduced, LLL-reduced, and original bases. It is seen that both the SA and the LLL algorithm achieve strong reductions of all three orthogonality measures. Furthermore, the SA outperforms the LLL algorithm significantly in terms of $\kappa(\widetilde{\mathbf{H}})$, less strongly in terms of $S(\widetilde{\mathbf{H}})$, and slightly in terms of $\delta(\widetilde{\mathbf{H}})$.

Performance of Detectors. Finally, we compare the performance of SA-assisted and LLL-assisted data detectors for 4-QAM modulation. Fig. 2(b) shows the symbol-error rate (SER) versus the signal-to-noise ratio (SNR) for the ZF and minimum mean-square error (MMSE) detectors and their LR-assisted versions (cf. [5]). As a performance reference, the result of the ML detector is also provided. It is seen that LR strongly improves the performance of ZF and MMSE detection. Furthermore, the SA-assisted detectors perform significantly better than their LLL-assisted counterparts.

5. CONCLUSIONS

We proposed *Seysen's algorithm* (SA) for efficient lattice-reduction assisted MIMO detection. The SA is different from the more widely known LLL algorithm in that it simultaneously reduces the lattice basis and its dual. This was seen to lead to a conceptually simple procedure for finding "more orthogonal" lattice bases. We presented an efficient implementation of the SA whose complexity per iteration is linear in the number of antennas (as for the LLL algorithm). Our simulations showed that the SA requires significantly fewer iterations than the LLL algorithm. We also observed that the bases obtained with the SA tend to be better than those obtained with the LLL algorithm, and that SA-assisted MIMO detectors outperform their LLL-assisted counterparts.f

While MIMO detection has been considered in this paper, we note that the SA can also be used to assist suboptimum vector perturbation techniques for *MIMO precoding* (see, e.g., [12] for the application of the LLL algorithm in this context).

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