FULL DIVERSITY DETECTION IN MIMO SYSTEMS WITH A FIXED-COMPLEXITY SPHERE DECODER

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ABSTRACT

The fixed-complexity sphere decoder (FSD) has been previously proposed for multiple input-multiple output (MIMO) detection to overcome the two main drawbacks of the original sphere decoder (SD), namely its variable complexity and sequential structure. As such, the FSD is highly suitable for hardware implementation and has shown remarkable performance through simulations. Herein, we explore the theoretical aspects of the algorithm and prove that the FSD achieves the same diversity order as the maximum likelihood detector (MLD). Further, we show that the coding loss can be made negligible in the high signal to noise ratio (SNR) regime with a significantly lower complexity than that of the MLD.

***Index Terms*—** sphere decoder, MIMO, diversity order, signal detection

1. INTRODUCTION

We consider a spatially-multiplexed multiple input-multiple output (MIMO) system with n_{T} transmit and n_{R} receive antennas in the context of wireless communications [1]. The vector of received symbols $\mathbf{r} \in \mathbb{C}^{n_{\mathrm{R}} \times 1}$ can be modeled as

$$\mathbf{r} = \mathbf{Hs} + \mathbf{v}\,, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{n_{\mathrm{T}} \times 1}$ denotes the vector of transmitted symbols taken independently from an arbitrary constellation $\mathcal{O}$ of M points with $\mathrm{E}[|s_i|^2] = 1/n_{\mathrm{T}}$ and where $\mathbf{v} \in \mathbb{C}^{n_{\mathrm{R}} \times 1}$ is the vector of independent complex Gaussian noise samples $v_i \sim \mathcal{CN}(0, \sigma^2)$. The channel matrix $\mathbf{H} \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{T}}}$ has independent elements $h_{ij} \sim \mathcal{CN}(0, 1)$ representing a wireless propagation environment with uncorrelated Rayleigh fading. We assume that the channel is perfectly known at the receiver and that $n_{\mathrm{R}} \geq n_{\mathrm{T}}$.

The optimum detector in such a scheme is the maximum likelihood detector (MLD), given by

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg \min_{\mathbf{s} \in \mathcal{O}^{n_{\mathrm{T}}}} \|\mathbf{r} - \mathbf{Hs}\|^2\,.$$

However, it suffers from an exponential complexity with the number of transmit antennas $O(M^{n_{\mathrm{T}}})$, making it unfeasible for high-dimensional MIMO systems. The same maximum likelihood (ML) performance can also be achieved by the sphere decoder (SD) [2], although it was shown to have an exponential complexity (in the worst case as well as in the average case) of $O(M^{\gamma n_{\mathrm{T}}})$ with $\gamma \in (0, 1]$ [3]. Herein, we study a detector that maintains the diversity order of the MLD with a fixed complexity $O(M^{\sqrt{n_{\mathrm{T}}}})$ if $n_{\mathrm{R}} = n_{\mathrm{T}}$, which represents an advantage over the sphere decoder (SD). Specifically, we consider the fixed-complexity sphere decoder (FSD) previously proposed in [4] and implemented in real-time on a field-programmable gate array (FPGA) platform in [5], [6]. This paper also proves that the error probability of this detector has a negligible degradation compared to that of the MLD in the high signal to noise ratio (SNR) regime. It is shown that

$$\lim_{\sigma^2 \to 0} \frac{\mathrm{P}\left(\hat{\mathbf{s}}_{\mathrm{FSD}} \neq \mathbf{s}\right)}{\mathrm{P}\left(\hat{\mathbf{s}}_{\mathrm{ML}} \neq \mathbf{s}\right)} = 1\,, \tag{2}$$

which indicates that the FSD, in addition to having the same diversity as the MLD, has a (de)coding loss in terms of SNR which tends to zero in the high SNR limit.

2. FIXED-COMPLEXITY SPHERE DECODER

The FSD has been previously proposed for the detection in uncoded MIMO systems using quadrature amplitude modulation (QAM) constellations [4]. It overcomes the two main drawbacks of the SD from an implementation point of view, i.e., its variable complexity depending on the noise level and the channel conditions and the sequential nature of its tree search phase.

The FSD achieves quasi-ML performance combining a specific channel matrix ordering with a search over only a fixed number of points $\mathbf{s}$, generated by a small subset $\mathcal{S} \subset \mathcal{O}^{n_{\mathrm{T}}}$, around the received vector $\mathbf{r}$. The transmitted vector $\mathbf{s} \in \mathcal{S}$ with the smallest Euclidean distance is then selected as the solution. The process can be written as

$$\hat{\mathbf{s}}_{\mathrm{FSD}} = \arg \min_{\mathbf{s} \in \mathcal{S}} \|\mathbf{r} - \mathbf{Hs}\|^2\,. \tag{3}$$

The FSD, analogously to the SD, can be seen as a constrained tree search through a tree with n_{T} levels where M branches originate from each node [2]. The paths in the tree followed by the FSD are determined by defining the number of branches per node that are expanded in each level. In [4], it was shown that quasi-ML performance can be achieved by performing the following two-stage constrained tree search:

- Initially, a full search is performed in p levels, expanding all M branches per node. This will herein be denoted as the full expansion (FE) stage of the algorithm.
- Secondly, a single search is performed in the remaining $n_{\mathrm{T}} - p$ levels, expanding only one branch per node following the decision-feedback equalization (DFE) path. This will be denoted as the single expansion (SE) stage of the algorithm.

An example is given in Fig. 1 for the constrained tree search required in a 4×4 system with 4-QAM modulation. Here, the FE stage

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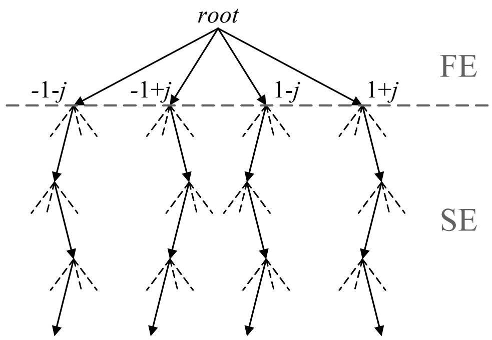

Fig. 1. FE and SE stages in the FSD tree search applied to a 4×4 system with 4-QAM modulation.

corresponds to only one level, i.e. $p = 1$. In Section 3, we show that this scheme will still maintain the diversity of the MLD.

The two-stage constrained tree search of the FSD is independent of the noise level and the channel conditions, resulting in a fixed complexity detector as opposed to the variable complexity of the SD. The total number of Euclidean distances calculated in the FSD is M^p, and simulations show that quasi-ML performance is achieved with $M^p \ll M^{n_\mathrm{T}}$, i.e. $\mathcal{S}$ is a very small subset of $\mathcal{O}^{n_\mathrm{T}}$ [4].

In order to achieve the aforementioned quasi-ML performance, the FSD uses a channel matrix ordering based on the two stages of the algorithm [4]. The n_T columns of $\mathbf{H}$ are ordered iteratively so that the signals with the *largest* post-processing noise amplification, as defined in [7], are detected in the FE stage. On the other hand, the signals with the smallest post-processing noise amplification are detected in the SE stage.

3. ERROR ANALYSIS

The error probability of the FSD, $p_{e\mathrm{FSD}}$, is defined as the probability that the estimate obtained by the FSD, $\hat{\mathbf{s}}_\mathrm{FSD}$, is not equal to the transmitted message, $\mathbf{s}$. Note that

$$\begin{aligned} p_{e\mathrm{FSD}} &\triangleq \mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s}\right) \\ &= \mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s} \cap \mathbf{s} \in \mathcal{S}\right) + \mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s} \cap \mathbf{s} \notin \mathcal{S}\right) \end{aligned} \quad (4)$$

where $\mathcal{S}$ is given as in (3). The first term in (4) asserts that the transmitted message belongs to the subset $\mathcal{S}$ considered by the FSD and that it does not correspond to the message with the minimum metric according to (3). This directly implies that the ML detector will also make an error in this case and it follows that

$$\mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s} \cap \mathbf{s} \in \mathcal{S}\right) \leq \mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{ML} \neq \mathbf{s}\right). \quad (5)$$

The second term in (4) asserts that the transmitted message does not belong to $\mathcal{S}$. In this case, it is impossible for the FSD to obtain the transmitted message which directly implies

$$\mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s} \cap \mathbf{s} \notin \mathcal{S}\right) = \mathrm{P}\left(\mathbf{s} \notin \mathcal{S}\right). \quad (6)$$

By applying (5) and (6) to (4), it follows that

$$\underbrace{\mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{FSD} \neq \mathbf{s}\right)}_{p_{e\mathrm{FSD}}} \leq \underbrace{\mathrm{P}\left(\hat{\mathbf{s}}_\mathrm{ML} \neq \mathbf{s}\right)}_{p_{e\mathrm{ML}}} + \underbrace{\mathrm{P}\left(\mathbf{s} \notin \mathcal{S}\right)}_{p_{e\mathrm{SE}}}. \quad (7)$$

The first term on the right hand side of (7), $p_{e\mathrm{ML}}$, is the error probability of the MLD and is clearly independent of the detection ordering of the FSD. The second term, $p_{e\mathrm{SE}}$, may be interpreted as an error in the SE stage and does depend on the detection ordering. Thus, by selecting the ordering in such a way that $p_{e\mathrm{SE}}$ is small in comparison with $p_{e\mathrm{ML}}$, quasi-ML performance can be achieved by the FSD.

We consider the error probability in the high SNR regime which is characterized by the diversity order of the detector [1]. The diversity order of MLD under the assumed model is well known to be equal to n_R [8] which implies that the error probability in the high SNR limit tends to zero with a rate given by

$$\lim_{\sigma^2 \to 0} \frac{\log p_{e\mathrm{ML}}}{\log \sigma^2} = n_\mathrm{R}. \quad (8)$$

It can be shown that, under the natural (or any fixed) detection ordering, the diversity order of the second term, $p_{e\mathrm{SE}}$, is equal to $(n_\mathrm{R} - n_\mathrm{T} + 1) + p$ for $1 \leq p \leq n_\mathrm{T} - 1$. This indicates that the FSD will have a strictly larger diversity than the ML-DFE detector proposed in [9]. The difference between the detectors lies on the fact that the ML-DFE would correspond to a search where only one path in the FE stage is expanded through the SE stage, as opposed to the FSD, where all paths in the FE stage are expanded.

However, the main advantage of the FSD becomes more apparent when ordering is considered. By properly selecting the detection ordering a much higher diversity can be obtained. Specifically, we will show that there exists a (channel dependent) detection ordering for which

$$\lim_{\sigma^2 \to 0} \frac{\log p_{e\mathrm{SE}}}{\log \sigma^2} \geq d \triangleq (n_\mathrm{R} - n_\mathrm{T})(p+1) + (p+1)^2. \quad (9)$$

Further, we will also argue that the detection ordering originally proposed in [4] satisfies (9).

Therefore, by combining (7), (8) and (9) it can be seen that the diversity of the FSD is lower bounded by

$$\lim_{\sigma^2 \to 0} \frac{\log p_{e\mathrm{FSD}}}{\log \sigma^2} \geq \min(n_\mathrm{R}, d)$$

which implies that maximal diversity is obtained whenever $d \geq n_\mathrm{R}$. From (9) it can be seen that:

- d grows *quadratically* in p under the optimal ordering (as opposed to linearly for the natural ordering) which implies that p can be selected much smaller than n_R while maintaining the diversity of the MLD.
- If $d > n_\mathrm{R}$ the second term in (7) has strictly larger diversity than the MLD and becomes negligible at high SNR in the sense indicated by (2).

In particular, if $n_\mathrm{R} = n_\mathrm{T}$ we obtain that $p = \lfloor\sqrt{n_\mathrm{T}}\rfloor$ is sufficient to achieve $d > n_\mathrm{R}$. Thus, near ML detection can be achieved with a complexity $O(M^{\sqrt{n_\mathrm{T}}})$ as stated in the introduction. Although this is, strictly speaking, larger than polynomial, it does not pose a problem from an implementation point of view [5], [6]. Additionally, due to the non-sequential structure of the algorithm, its implementation can be fully pipelined and highly parallelized.

3.1. The DFE Error Probability

We start by analyzing the error probability $p_{e\mathrm{SE}}$ for some given ordering o, before discussing the specific ordering in the following section. Thus, let the ordered channel matrix $\mathbf{H}_o$ be given by $\mathbf{H}_o \triangleq \mathbf{H}\mathbf{\Pi}_o$ where $\mathbf{\Pi}_o$ is the permutation matrix corresponding to the ordering. Taking into account the two stages of the FSD, we can partition $\mathbf{H}_o$ according to

$$\mathbf{H}_o \triangleq \mathbf{H}\mathbf{\Pi}_o = \begin{bmatrix}\mathbf{H}_{o1} & \mathbf{H}_{o2}\end{bmatrix}$$

where $\mathbf{H}_{o1} \in \mathbb{C}^{n_\mathrm{R}\times(n_\mathrm{T}-p)}$ and $\mathbf{H}_{o2} \in \mathbb{C}^{n_\mathrm{R}\times p}$ correspond to the SE and the FE stage of the FSD, respectively. The same partitioning can be applied to the (ordered) transmitted message, $\mathbf{s}_o$, i.e.

$$\mathbf{s}_o^\mathrm{T} \triangleq \mathbf{s}^\mathrm{T}\mathbf{\Pi}_o = \begin{bmatrix}\mathbf{s}_{o1}^\mathrm{T} & \mathbf{s}_{o2}^\mathrm{T}\end{bmatrix}$$

where $\mathbf{s}_{o1} \in \mathcal{O}^{n_\mathrm{T}-p}$ and $\mathbf{s}_{o2} \in \mathcal{O}^p$.

By the nature of the algorithm, the path in the SE stage extending from the path corresponding to $\mathbf{s}_{o2}$, denoted as $\hat{\mathbf{s}}_{o1\mathrm{SE}}$, is given by the DFE estimate of $\mathbf{s}_{o1}$ under the perfect feedback assumption, i.e.

$$\mathbf{r}_{o1} \triangleq \mathbf{r} - \mathbf{H}_{o2}\mathbf{s}_{o2} = \mathbf{H}_{o1}\mathbf{s}_{o1} + \mathbf{v}\,. \qquad (10)$$

Note also that the event $\mathbf{s} \in \mathcal{S}$ is satisfied if and only if $\hat{\mathbf{s}}_{o1\mathrm{SE}} = \mathbf{s}_{o1}$. Therefore, $\mathrm{P}\left(\mathbf{s} \notin \mathcal{S}\right)$ is equal to the probability of an error in a DFE detector applied to the (partial) channel matrix $\mathbf{H}_{o1}$.

The error probability of DFE in the high SNR regime is governed by the outage probability. This is defined as the probability that the minimum post-processing SNR drops below a given threshold. This minimum SNR is lower bounded according to

$$\mathrm{SNR}_{\min} \geq \frac{\lambda_1(\mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1})}{n_\mathrm{T}\sigma^2} \qquad (11)$$

where $\lambda_1(\mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1})$ denotes the *smallest* eigenvalue of $\mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1}$ [10]. This bound holds regardless of whether zero forcing (ZF) or minimum mean square error (MMSE)-DFE is considered.

Specifically, if the cumulative distribution function (CDF) of $\lambda_1 \triangleq \lambda_1(\mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1})$ satisfies

$$\mathrm{P}\left(\lambda_1 \leq x\right) \leq \alpha x^d \qquad (12)$$

for some $\alpha \in \mathbb{R}$ and $d > 0$, it follows that

$$\lim_{\sigma^2\to 0}\frac{\mathrm{P}\left(\mathbf{s}\notin\mathcal{S}\right)}{\log\sigma^2} = \lim_{\sigma^2\to 0}\frac{\mathrm{P}\left(\hat{\mathbf{s}}_{o1\mathrm{SE}}\neq\mathbf{s}_{o1}\right)}{\log\sigma^2} \geq d\,. \qquad (13)$$

The rigorous proof of this observation can be obtained in a fashion similar to [11].

Thus, in light of the above, it makes sense to choose the ordering that maximizes $\lambda_1(\mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1})$ among all possible orderings. In the next section, we will first show that there exists a detection ordering satisfying (12) for d given in (9). Next, we will also argue that the detection ordering originally proposed in [4] achieves this diversity order.

3.2. The Detection Ordering

Consider the positive semi-definite (PSD) matrix $\mathbf{Q} \triangleq \mathbf{H}^\mathrm{H}\mathbf{H} \in \mathbb{C}^{n_\mathrm{T}\times n_\mathrm{T}}$, where $\mathbf{H}$ is the full channel matrix, and let $\lambda_1(\mathbf{Q}) \leq \ldots \leq \lambda_{n_\mathrm{T}}(\mathbf{Q})$ denote the ordered eigenvalues of $\mathbf{Q}$. It then follows that the CDF of $\lambda_k(\mathbf{Q})$ is bounded according to

$$\mathrm{P}\left(\lambda_k(\mathbf{Q}) \leq x\right) \leq \beta x^{(n_\mathrm{R}-n_\mathrm{T})k+k^2} \qquad (14)$$

for some $\beta \in \mathbb{R}$ [12]. In addition we denote by $\mathbf{Q}_{op} \triangleq \mathbf{H}_{o1}^\mathrm{H}\mathbf{H}_{o1} \in \mathbb{C}^{(n_\mathrm{T}-p)\times(n_\mathrm{T}-p)}$ a (possibly permuted) PSD principal submatrix of $\mathbf{Q}$, obtained by removing p rows and columns from $\mathbf{Q}$.

From [13, Page 189], it is known that given an arbitrary PSD matrix, $\mathbf{A} \in \mathbb{C}^{n\times n}$, there exists (at least) one principal submatrix, $\mathbf{A}_1 \in \mathbb{C}^{(n-1)\times(n-1)}$, obtained by removing a row and a column from $\mathbf{A}$ satisfying

$$\lambda_k(\mathbf{A}_1) \geq \frac{k}{n}\lambda_{k+1}(\mathbf{A}) \qquad (15)$$

for $k = 1, \ldots, n-1$.

By repeated application of (15), it follows that there is a principal submatrix, $\mathbf{A}_p \in \mathbb{C}^{(n-p)\times(n-p)}$, obtained by removing p rows and columns from $\mathbf{A}$, satisfying

$$\lambda_1(\mathbf{A}_p) \geq \binom{n}{p}^{-1}\lambda_{p+1}(\mathbf{A})\,. \qquad (16)$$

This implies that there must exist an ordering o for which

$$\lambda_1(\mathbf{Q}_{op}) \geq \binom{n}{p}^{-1}\lambda_{p+1}(\mathbf{Q})\,. \qquad (17)$$

Inserting (17) into (14) for this ordering yields

$$\mathrm{P}\left(\lambda_1(\mathbf{Q}_{op}) \leq x\right) \leq \mathrm{P}\left(\lambda_{p+1}(\mathbf{Q}) \leq \binom{n}{p}x\right) \leq \beta\binom{n}{p}^d x^d$$

where d is given in (9). Applying this to the result obtained in (12) and (13) for $\alpha = \beta\binom{n}{p}^d$ shows that as long as $d \geq n_\mathrm{R}$ there is an ordering under which the FSD achieves the same diversity as the MLD. The preceding discussion is summarized by the following theorem.

Theorem 1 *There exists a detection ordering that makes the FSD achieve the same diversity as the MLD if p levels are examined in the FE stage, with p satisfying*

$$(n_\mathrm{R} - n_\mathrm{T})(p+1) + (p+1)^2 \geq n_\mathrm{R}\,. \qquad (18)$$

Further, if (18) *is satisfied with strict inequality the loss due to suboptimality is negligible in the high SNR regime.*

Naturally, an optimal ordering in the sense that it maximizes $\lambda_1(\mathbf{Q}_{op})$ can be found by simply searching over all $(n_\mathrm{T}-p)$ by $(n_\mathrm{T}-p)$ principal submatrices of $\mathbf{Q}$. However, as there are $\binom{n_\mathrm{T}}{p}$ such matrices, this approach becomes impractical when n_T and p are large. Instead, [4] suggested finding $\mathbf{H}_{o1}$ by successively removing the symbols in (1) which would experience the largest noise amplification (or equivalently smallest SNR) in a ZF detector. Note that this corresponds to a *reversed* vertical-Bell Labs layered space time (V-BLAST) ordering for the first p layers. The motivation was that under such an ordering the worst symbols would be detected in the (more robust) FE stage of the algorithm, thereby improving the performance.

The symbol with the largest noise amplification is given by the largest diagonal entry of $\mathbf{Q}^{-1}$ and it is in fact possible to derive a result similar to (15) for the principal submatrix obtained by removing the row and column corresponding to the largest diagonal value in $\mathbf{Q}^{-1}$. Specifically, it is possible to show the following:

Theorem 2 *Let* $\mathbf{A} \in \mathbb{C}^{n\times n}$ *be PSD, let* k *be given by*

$$k \triangleq \arg\max_{i=1,\ldots,n}[\mathbf{A}^{-1}]_{ii}\,,$$

and let $\mathbf{A}_1$ *be the principal submatrix obtained by removing the kth column and row of* $\mathbf{A}$*. Then*

$$\lambda_k(\mathbf{A}_1) \geq \frac{1}{n}\lambda_{k+1}(\mathbf{A})$$

Proof: Given in [14].

Repeated application of Theorem 2 yields, similarly to (17),

$$\lambda_1(\mathbf{Q}_{op}) \geq \frac{(n-p)!}{n!}\lambda_{p+1}(\mathbf{Q})$$

if the reversed V-BLAST ordering is applied in the first p layers. Analogous to the previous analysis, this yields an equivalent of Theorem 1 for the ordering proposed in [4].

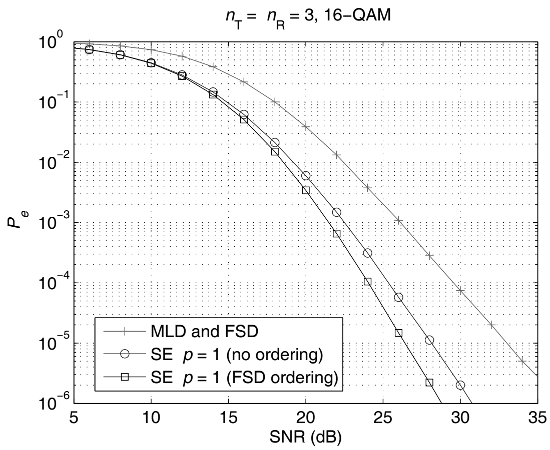

Fig. 2. Error probability of the MLD and the SE stage of the FSD as a function of the SNR.

4. NUMERICAL EXAMPLE

In this section a numerical example is given to corroborate the diversity result in (18). For that purpose, a 3×3 MIMO system using 16-QAM modulation has been considered. Fig. 2 shows the error probability of the MLD compared to that of the SE stage of the FSD as a function of the SNR $= 1/\sigma^2$. The error probability curve of the FSD is superimposed with that of the MLD showing its ML performance (the degradation is of only 0.008 dB at SNR = 20 dB). The FSD has been simulated with $p = 1$ so that the signal with the *largest* post-processing noise amplification is detected in the FE stage. The error probability of the SE stage has been obtained for two different orderings under the perfect feedback assumption given in (10). In the no ordering case, the signals in the SE stage are detected according to the natural detection ordering. On the other hand, in the FSD ordering case, the signals in the SE stage are detected according to the FSD ordering proposed in [4].

Initially, the diversity increase in the performance of the SE stage can be observed compared to that of the MLD. In particular, applying (18), the diversity of the SE stage is expected to be $d \geq 4$, which is greater than the diversity $n_{\mathrm{R}} = 3$ of the MLD. In addition, if the FSD ordering is applied, a further improvement in the error probability can be observed. Thus, although the diversity of the MLD can be achieved by the FSD by choosing p according to (18), the performance of the detector can be further improved by ordering the remaining levels in increasing order of post-processing noise amplification. It should be noted that, although the analytical results presented in this paper refer to the high SNR regime, the effect is already noticeable at relevant SNRs as shown in Fig. 2.

5. CONCLUSION

This paper proves that the FSD maintains the diversity of the MLD while searching over only a very small number of candidates compared to the MLD. It also has a negligible coding loss in the high SNR regime. In particular, it has been shown that, by properly selecting the signals to be detected in the FE stage of the algorithm, the diversity of the SE stage grows beyond the diversity order of the MLD. The specific increase in diversity depends on the number of signals (i.e. levels) detected in the FE stage.

It has been argued also that an ordering which selects the signals with the *largest* post-processing noise amplification in the FE stage is sufficient for the diversity increase in the SE stage. In addition, by ordering the signals to be detected in the SE stage, the FSD is shown to provide an improved performance.

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