A LIMITED FEEDBACK SCHEME FOR LINEAR DISPERSION CODES OVER CORRELATED MIMO CHANNELS

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ABSTRACT

With partial Channel State Information (CSI) at the transmitter, the design of space-time codes for frequency flat, spatially correlated MIMO fading channels is considered. The focus of the paper is on the class of space-time block codes known as Linear Dispersion (LD) codes, introduced by Hassibi and Hochwald. For perfect CSI at the transmitter, the LD codes are optimized with respect to the instantaneous mutual information between the inputs to the space-time encoder and the output of the channel. An equivalent optimization problem is proposed and can that be solved by standard convex optimization algorithms. It is then conjectured that the LD codes obtained by maximizing the instantaneous mutual information converge to that obtained from maximizing the averaged mutual information in the large antenna asymptote. Based on the insights drawn from the conjecture, a limited feedback scheme for LD codes is proposed assuming a common codebook at the transmitter and receiver. The numerical result for a scattering environment suggests that the proposed feedback scheme achieves high performance at low complexity.

Index Terms— Array signal processing, Channel coding, Fading channels, Feedback communication, Linear antennas.

1. INTRODUCTION

The problem of designing coding schemes that exploit the full potential of the multi-input multi-output (MIMO) antenna system has sparked considerable research interest in recent years. Practical approaches to coding for MIMO channels typically separate the encoding into an outer code concatenated with an inner block code, a space-time code, which is matched to the MIMO channel.

Recent work on Low-Density Parity Check Codes (see, e.g, [1]) has shown that it is possible to construct outer codes

that come close to achieving the mutual information between the input and output of the inner space-time code. This justifies the use of mutual information as a design criterion for space-time block codes if soft decisions are allowed at the space-time decoder. The mutual information criterion was first proposed by Hassibi and Hochwald [2]. Much of the existing work on space-time block code design using the mutual information criterion has focused on the i.i.d. Rayleigh fading model for the channel. In [3], the problem of spacetime code design for spatially correlated MIMO channel was addressed and the optimal LD codes were derived according to the channel statistics. However, no CSI at the transmitter is exploited in [3]. If partial CSI is available at the transmitter, communication at a higher data rate or/and with lower error probability is possible.

Recent works [4], [6] have shown promising improvements in terms of error probability when partial CSI is exploited at the transmitter by using a common codebook at both the transmitter and receiver. These works, however, have mostly focused on linear precoders, which do not code across time. Moreover, spatial correlation in the channel is generally ignored (notable exceptions are [5] and [6]). In this paper, we proposed a limited feedback scheme for LD codes over a correlated MIMO channel. Simulation results demonstrate that significant gains can result from exploiting partial CSI at the transmitter.

2. CHANNEL AND SYSTEM MODEL

2.1. Channel Model

Consider a frequency flat, multiple-antenna communication system with n_t transmit antennas and n_r receive antennas. In a discrete-time, complex baseband model, the transmitted signal matrix $\boldsymbol{X} \in \mathbb{C}^{n_t \times T}$ and the received signal matrix $\boldsymbol{Y} \in \mathbb{C}^{n_r \times T}$ are related by

$$oldsymbol{Y} = \sqrt{rac{\Gamma}{n_t}}oldsymbol{H}oldsymbol{X} + oldsymbol{W}$$

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Fig. 1. System Model

where $\boldsymbol{H} \in \mathbb{C}^{n_r \times n_t}$ denotes the channel matrix, and $\boldsymbol{W} \in$ $\mathbb{C}^{n_r \times T}$ denotes the complex additive white Gaussian noise with i.i.d. entries $\boldsymbol{W}_{ij} \sim \mathcal{CN}(0,1)$. We assume that the channel remains constant over the blocklength of T. We also assume that the channel changes in an ergodic fashion from block to block, and an average input power constraint of $E[Tr{XX^{\dagger}}] \leq n_t T$. If we further assume that each entry of the channel matrix H is identically distributed and normalized such that $E[|H_{i,j}|^2] = 1$, then Γ represents the effective signal-to-noise ratio (SNR) at each receive antenna. We assume that the receiver knows the realization of the channel matrix H while the transmitter only has partial Channel State Information (CSI). The partial CSI is characterized by a zero-delay and noiseless feedback channel with limited feedback rate B (bits/sec/Hz) [4]. We consider general correlation model for the channel. The model and assumptions for the distribution of *H* are the same as that in [3].

2.2. System Model and Linear Dispersion Codes

As in [3], we assume that there are K streams of input symbols $x_1[t], \ldots, x_K[t]$ (See Figure 1) for the space-time encoder at a given symbol time t and that they satisfy the following assumption.

Assumption 1 The streams of input symbols $x_1[t], \ldots, x_K[t]$ satisfy:

- (i) For each stream, input symbols $x_k[t]$ are i.i.d. across time and are drawn from some real constellations with marginal distribution $p(x_k)$.
- (ii) Different streams are independent from each other.

Since the input symbols $\boldsymbol{x}_k[t]$ are i.i.d. across time, we will drop the time index t for the rest of this paper. As in [3], we focus on applying LD codes as our inner space-time codes. The definition of a LD code involves a set of dispersion matrices $\{A_k\} \in \mathbb{C}^{n_t \times T}$ such that our space-time code \boldsymbol{X} is

given by

$$\boldsymbol{X} = \sum_{k=1}^{K} \boldsymbol{x}_k \boldsymbol{A}_k \tag{1}$$

where the symbols $\{x_k\}_{k=1}^K$ satisfy Assumption 1. Some further discussions can be found in [3].

3. OPTIMAL LD CODES ACCORDING TO CHANNEL STATISTICS

To motivate the limited feedback scheme proposed in Section 4, it is necessary to introduce the optimal LD codes that adapts to the channel statistics [3], where no CSI is exploited at the transmitter. In [3], optimal LD codes are derived by maximizing the averaged mutual information between the input and output of the effective channel $I(x_1, \ldots, x_K; \hat{Y} | H)$. (See Figure 1) To capture the scattering environment in general spatial correlation, the virtual representation ([7], [8]) of the channel matrix is exploited, yielding the following channel model in the virtual domain

$$\tilde{\boldsymbol{Y}} = \sqrt{\frac{\Gamma}{n_t}} \sum_{k=1}^{K} \boldsymbol{x}_k \tilde{\boldsymbol{H}} \tilde{A}_k + \tilde{\boldsymbol{W}}$$
(2)

where $\tilde{A}_k = S_t^{\dagger} A_k$, $\tilde{Y} = S_r^{\dagger} Y$, and $\tilde{W} = S_r^{\dagger} W$. S_r and S_t are unitary spatial Fourier transform matrices. Since S_t is unitary, the power constraint in the virtual domain remains the same and is given by $\sum_{k=1}^{K} \text{Tr}\{\tilde{A}_k \tilde{A}_k^{\dagger}\} \leq n_t T$. We also define V as the variance matrix such that $V_{k,\ell} = \text{Var}(\tilde{H}_{k,\ell})$. For simplicity, we further assume equal power constraint for each symbol stream, i.e., $\text{Tr}\{\tilde{A}_k \tilde{A}_k^{\dagger}\} \leq \frac{n_t T}{T} = n_t$, $\forall k = 1, \ldots, K$. We then have the following theorem [3].

Theorem 1 Let $\tilde{\mathbf{X}}$ be a LD code (1) with K symbols and corresponding dispersion matrices $\{\tilde{A}_k\}_{k=1}^K \in \mathbb{C}^{n_t \times T}$, and assume that input symbols $\mathbf{x}_1, \ldots, \mathbf{x}_K$ satisfy Assumption 1. If there exist LD codes satisfying the set of Generalized Orthogonal Conditions (GOC's):

- (i) $\tilde{A}_k \tilde{A}_j^{\dagger} + \tilde{A}_j \tilde{A}_k^{\dagger} = 0 \ \forall k, j \ k \neq j.$
- (ii) $\tilde{A}_k \tilde{A}_k^{\dagger} = \Lambda^{st}, \quad k = 1, \dots, K$, where Λ^{st} is a diagonal matrix that maximizes $I\left(\boldsymbol{x}_k; \sqrt{\frac{\Gamma}{n_t}} \boldsymbol{x}_k \tilde{\boldsymbol{H}} \tilde{A}_k + \tilde{\boldsymbol{W}} \middle| \tilde{\boldsymbol{H}} \right)$ and satisfies the power constraints $Tr\{\Lambda^{st}\} \leq \frac{n_t T}{K}$.

then such LD codes maximize the mutual information $I(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K; \tilde{\boldsymbol{Y}} | \tilde{\boldsymbol{H}})$ and are thus optimal under the mutual information criterion.

Since only the statistical information of the channel is used in deriving the optimal LD codes, the optimal power allocation matrix Λ^{st} represents the optimal coding strategy (under the LD constraint) if no CSI is exploited at the transmitter. In the following section, we propose a limited feedback scheme that exploits both the partial CSI at the transmitter and the optimal coding strategy Λ^{st} according to channel statistics.

4. A LIMITED FEEDBACK SCHEME FOR LD CODES

4.1. Perfect CSI at the Transmitter

If perfect CSI is available at the transmitter, the optimal coding scheme involves maximizing the instantaneous mutual information $I(\tilde{X}; \tilde{Y} | \tilde{H} = \tilde{H})$. In this case, *Theorem* 1 of [3] can be easily extended and we have the following upperbound

$$I(\tilde{\boldsymbol{X}}; \tilde{\boldsymbol{Y}} | \tilde{\boldsymbol{H}} = \tilde{H}) = I(\boldsymbol{x}_1, \dots, \boldsymbol{x}_K; \tilde{\boldsymbol{Y}} | \tilde{\boldsymbol{H}} = \tilde{H})$$

$$\leq \sum_{k=1}^{K} I\left(\boldsymbol{x}_k; \sqrt{\frac{\Gamma}{n_t}} \boldsymbol{x}_k \tilde{\boldsymbol{H}} \tilde{A}_k + \tilde{\boldsymbol{W}} \left| \tilde{\boldsymbol{H}} = \tilde{H} \right.\right)$$
(3)

Equality holds if and only if $\tilde{A}_k \tilde{A}_j^{\dagger} + \tilde{A}_j \tilde{A}_k^{\dagger} = 0$, $\forall k, j$ such that $k \neq j$. A natural next step would be to apply the similar techniques in [3] to maximize each term on the right-hand side of the inequality. For simplicity, assume equal power constraint for each symbol stream

$$\tilde{A}^{o} = \arg \max_{\mathrm{Tr}\{\tilde{A}\tilde{A}^{\dagger}\} \leq \frac{n_{t}T}{K}} I\left(\boldsymbol{x}; \sqrt{\frac{\Gamma}{n_{t}}}\boldsymbol{x}\tilde{\boldsymbol{H}}\tilde{A} + \tilde{\boldsymbol{W}} \middle| \tilde{\boldsymbol{H}} = \tilde{H}\right)$$
(4)

Note that the subscript k is ignored since the optimization problem is the same for all k. Finding the optimal LD codes in (4) is equivalent to finding Q^o that solves the optimization problem [3]

$$Q^{o} = \arg \max_{\operatorname{Tr}\{Q\} \leq \frac{n_{t}T}{K}} \varphi\left(\frac{\Gamma}{n_{t}} \operatorname{Tr}\{\tilde{\boldsymbol{H}}Q\tilde{\boldsymbol{H}}^{\dagger}\}\right)$$
(5)

where $Q = \tilde{A}\tilde{A}^{\dagger}$, $\varphi(a) = h(\sqrt{a}x + \tilde{n}|\tilde{H} = \tilde{H})$ with $h(\cdot)$ denoting the differential entropy, and \tilde{n} is a real zero-mean Gaussian random variable of variance 1/2. The above optimization problem can be solved using convex optimization algorithms with linear constraints since φ is concave. However, implementing a convex optimization algorithm for each realization of the channel (each block for our channel model) is computationally infeasible in practice. Moreover, feeding back the whole Q^o from the receiver to the transmitter poses a challenging task for the design of feedback schemes. Therefore, we propose the following limited feedback scheme as a possible remedy.

4.2. A Limited Feedback Scheme with partial CSI at transmitter

In [6], Raghavan, et al. have shown that the optimal beamformer for correlated MIMO channel with perfect CSI at transmitter converges to the statistical beamformer in the large antenna asymptote. Although the results in [6] is restricted to beamforming, we expect analogous results to hold for our problem and we have the following conjecture. **Conjecture 1** Let Q^o be defined as

$$Q^{o} = \arg \max_{Tr\{Q\} \le \frac{n_{t}T}{K}} \varphi\left(\frac{\Gamma}{n_{t}} Tr\{\tilde{\boldsymbol{H}}Q\tilde{\boldsymbol{H}}^{\dagger}\}\right)$$

where $\varphi(a) = h(\sqrt{a}x + \tilde{n}|\tilde{H} = \tilde{H})$ and \tilde{n} is a real zeromean Gaussian random variable of variance 1/2. We conjecture that

$$Q^o \to \Lambda^{st} \text{ a.s. as } \frac{n_t}{n_r} \to 0 \text{ and } n_r, n_t \to \infty$$
 (6)

where Λ^{st} is defined in Theorem 1.

Based on the conjecture, we propose a codebook-based ([4] limited feedback scheme given that a semi-unitary codebook C is known at both the transmitter and receiver:

At the receiver,

- 1. Compute $Q_i = V_i \overline{\Lambda}^{st} V_i^{\dagger}$ for each $i = 1, ..., 2^B$, where $\overline{\Lambda}^{st}$ is the $M \times M$ principal submatrix of Λ^{st} with only nonzero diagonal elements, $V_i \in \mathbb{C}^{n_t \times M}$ is a codeword from \mathcal{C} , and M is the number of nonzero diagonal elements.
- 2. Given $\{Q_i\}$, compute $I_i = \varphi\left(\frac{\Gamma}{n_t} \operatorname{Tr}\{\tilde{\boldsymbol{H}} Q_i \tilde{\boldsymbol{H}}^{\dagger}\}\right)$ for each $i = 1, \dots, 2^B$.
- 3. Find Q_j with the maximal I_j and feedback the index j from the receiver to the transmitter.

The transmitter then uses the index to select V_j , the corresponding Q_j , and the LD codes associated with it. The explicit construction of $\{A_k\}_{k=1}^K$ from Q_j is similar to that proposed in [3]. The codebook $\{V_i\}$ is constructed using similar techniques from [4]. Namely, we construct semi-unitary codebook that is distributed as uniformly as possible in the Grassmanian manifold [4].

The structure of $Q_i = V_i \overline{\Lambda}^{st} V_i^{\dagger}$ is proposed for the following reason: Since Q_i lies within a local perturbation cone centered around Λ^{st} [6], the directions of its eigenvectors convey more information than its eigenvalues. The structure, together with the conjecture, also suggests that V_i converges to the identity matrix in the large antenna asymptote. Moreover, the statistical information of the channel is completely captured in $\overline{\Lambda}^{st}$. Both of these motivate our uniform quantization of the semi-unitary matrices $\{V_i\}$ in the Grassmanian manifold.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the limited feedback scheme proposed in Section 4.2. We let K = T, where T is chosen to be large enough such that there exists LD codes satisfying the first GOC and $\tilde{A}\tilde{A}^{\dagger} = Q_j$, where



Fig. 2. Comparison of the averaged mutual information using the optimal power allocation, beamforming, equal power allocation, and limited feedback schemes with 1, 2, 3, and 4 bits of feedback for scattering environment I using BPSK

 Q_j is the codeword selected by the algorithm mentioned earlier. We consider the scattering environment from [3] and use BPSK symbols for our simulation example. In the scattering environment, we have a system of 5 transmit and 5 receive antennas, where each element of \tilde{H} is a zero-mean proper complex Gaussian random variable with the following variance matrix,

$$V = \frac{25}{5.05} \begin{pmatrix} 1 & 0 & 0.21 & 0 & 0\\ 0 & 1 & 0.21 & 0 & 0\\ 0 & 0 & 0.21 & 0 & 0\\ 0 & 0 & 0.21 & 0.5 & 0\\ 0 & 0 & 0.21 & 0.5 & 1 \end{pmatrix}$$

The entries of V are normalized so that $\sum_{k,\ell} V_{k,\ell} = n_t n_r = 25$. In Figure 2, we compare two sets of mutual information. In the first set, no CSI is exploited at the transmitter. The LD codes we consider include the optimal power allocation Λ^{st} , beamforming, and equal power allocation LD codes. In the second set, partial CSI is exploited in the form of the limited feedback channel. Our limited feedback schemes with 1, 2, 3, and 4 bits of feedback are presented and shown to improve the mutual information significantly even with 1 bit of feedback. Similar trends are observed for other scattering environments and that gains can be obtained even for channels that are close to i.i.d. channels.

6. CONCLUSIONS

We have considered the design of LD codes that maximize the instantaneous mutual information in spatially correlated MIMO channels with partial CSI at the transmitter. For perfect CSI at the transmitter, we proposed an equivalent optimization problem that can be solved for each realization of the channel using convex optimization algorithms. However, the intensive computational requirement makes it infeasible in practice. We further conjectured that in the large antenna asymptote, the LD codes obtained by maximizing the instantaneous mutual information converge to that obtained from [3], where only statistical information is exploited at the transmitter. We then proposed a low-complexity limited feedback scheme for LD codes using a common codebook at the transmitter and receiver. Finally, we presented numerical simulations for the scattering environment adopted from [3]. Our results indicate significant improvements in the averaged mutual information even at a low feedback rate.

7. REFERENCES

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