NONINTERSECTING SUBSPACE OSTBCS FOR BLIND ML DETECTION

Wing-Kin Ma

Institute Commun. Eng. & Dept. Elect. Eng.
National Tsing Hua University, Hsinchu, Taiwan
E-mail: wkma@ieee.org

ABSTRACT

Recently there has been growing interest in employing the orthogonal space-time block codes (OSTBCs) for blind maximum-likelihood (ML) detection. Several independent works have suggested that OSTBCs are favorable space-time codes from a blind receiver implementation standpoint. In this work we turn our attention to blind ML identifiability, with an emphasis on a special class of codes called the nonintersecting subspace (NIS) OSTBCs. We show a powerful property that NIS-OSTBCs are uniquely identifiable up to a sign for any nonzero channel. However, many existing OSTBCs are not NIS. We propose a code construction procedure that can convert an existing OSTBC to an NIS-OSTBC. Simulation result are provided to support our theoretical findings.

Index Terms— orthogonal space-time block code, blind identifiability, blind and semiblind detection, maximum likelihood detection, noncoherent detection

1. INTRODUCTION

This paper considers the problem of blind maximum-likelihood (ML) detection of orthogonal space-time block codes (OSTBCs). The OSTBC scheme has been well known for its full spatial diversity and low receiver complexity, given channel state information (CSI) at the receiver. Recently it has been found that in the blind scenario (also known as the noncoherent or no CSI scenario), OSTBCs are also attractive. Specifically the special code characteristics of OSTBCs can be exploited to facilitate the implementation of a blind ML receiver, and this has resulted in a variety of suboptimal and optimal blind ML receiver implementations. Those implementations include a particularly simple closed-form method [1] (also [2]) that also has an interesting relationship to a blind subspace receiver [3], an efficient iterative method called cyclic ML [1, 4], a quasi-ML convex optimization based method called semidefinite relaxation (SDR) [2], and an exact ML solver using sphere decoding [2]. All these methods are structurally effective, which would not be possible for the case of a general space-time code. Some performance and complexity comparison of the various blind OSTBC methods can be found in [2].

The focus of this paper is on a special class of OSTBCs, which we call the nonintersecting subspace (NIS) OSTBCs. The reason for this investigation is that NIS-OSTBCs exhibit very relaxed blind ML identifiability conditions, as we will explain soon. The concept of NIS space-time codewords was introduced in the noncoherent space-time coding context [5] for achieving the full noncoherent spatial diversity[1]. To our best knowledge, no work has been done on exploring connections between OSTBCs and NIS. We show that from a blind signal processing viewpoint, an NIS-OSTBC is 'perfect' in the sense that it achieves unique identifiability up to a sign for any nonzero channel. This powerful result motivates us to study the properties of NIS-OSTBCs. Our experience was that many existing OSTBCs are not NIS; e.g., the Alamouti code. To fill this gap, we propose a construction procedure that can convert an existing OSTBC to an NIS-OSTBC. Simulation results are presented to support our theoretical findings.

2. BACKGROUND

We review some key concepts essential to the ensuing development. The first subsection considers OSTBCs and their structures. The second subsection describes blind ML OSTBC detection.

2.1. Orthogonal Space-Time Block Codes

This work considers OSTBCs with binary PSK (BPSK) or quaternary PSK (QPSK) constellations, for which the transmitted code matrix can be expressed as

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} s_k \mathbf{X}_k \in \mathbb{C}^{M_t \times T}, \tag{1}$$

where M_t is the number of transmitter antennas, T is the time length of the code, $\mathbf{s} \in \{\pm 1\}^K$ is a bit vector, K is the number of bit symbols, and $\mathbf{X}_k \in \mathbb{C}^{M_t \times T}$ are code basis matrices which satisfy [7–9]

$$\mathbf{X}_k \mathbf{X}_\ell^H = \begin{cases} \mathbf{I}, & k = \ell \\ -\mathbf{X}_\ell \mathbf{X}_k^H, & k \neq \ell \end{cases} \tag{2}$$

with $T \geq M_t$. Equation (1) provides a natural formulation for real-valued BPSK OSTBCs. Through a simple reformulation, complex-valued QPSK OSTBCs can also be expressed as (1); see [2] for example. An OSTBC is row orthogonal: from (1) and (2) one can show that

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|_2^2 \mathbf{I} = K\mathbf{I} \tag{3}$$

for any $\mathbf{s} \in \{\pm 1\}^K$. Here $\|.\|_2$ denotes the 2-norm. In the coherent detection scenario, the code properties in (1), (2), and (3) result in the well-known advantages of simple ML detection structure and the maximum spatial diversity [7].

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[1] We should point out that the definitions of spatial diversity in the coherent and noncoherent cases are quite different; see [6] for example.

In principle, any matrix function satisfying (1) and (2) is said to be an OSTBC. In practice, most existing OSTBCs are based on *generalized orthogonal designs* (GODs) [7–9], which have additional restrictions on the code structures. In real GODs, for instance, the entries of $\mathbf{C}(\mathbf{s})$ are constrained to be drawn from $\{0, \pm s_1, \ldots, \pm s_K\}$ thereby $\mathbf{X}_k \in \{0, \pm 1\}^{M_t \times T}$ for all k. For example, the BPSK Alamouti code

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix} \tag{4}$$

is a GOD. The analysis here is established from (1) and (2), and GODs will not be assumed unless specified. Hence our analysis is also applicable to non-GOD codes (e.g., the 'sporadic' codes [7]).

2.2. Blind ML Detection and Blind Identifiability

We consider a standard scenario [2] where a sequence of OSTBCs is transmitted over a frequency-flat, quasi-static channel. The respective received signal model is given by

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{s}_p) + \mathbf{V}_p, \qquad p = 1, \ldots, P, \tag{5}$$

where $\mathbf{Y}_p \in \mathbb{C}^{M_r \times T}$ is the received code matrix at pth code block, $\mathbf{s}_p \in \{\pm 1\}^K$ is the transmitted bit vector at the pth code block, $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the multiple-input-multiple-output (MIMO) channel, M_r is the number of receiver antennas, P is the frame length or number of code blocks in which the channel remains static, and $\mathbf{V}_p \in \mathbb{C}^{M_r \times T}$ is an additive white Gaussian noise (AWGN) matrix. Let

$$\mathbf{S} = [\, \mathbf{s}_1, \ldots, \mathbf{s}_P \,] \in \{\pm 1\}^{K \times P}, \tag{6}$$

and assume that $\mathbf{H}$ is a deterministic unknown. The blind ML detector is given by [2, 4]

$$\{\hat{\mathbf{H}}, \hat{\mathbf{S}}\} = \arg \min_{\substack{\tilde{\mathbf{H}} \in \mathbb{C}^{M_r \times M_t}, \\ \tilde{\mathbf{S}} \in \{\pm 1\}^{K \times P}}} \sum_{p=1}^{P} \| \mathbf{Y}_p - \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}}_p) \|_F^2 \tag{7}$$

where the unknown $\mathbf{H}$ and $\mathbf{S}$ are estimated jointly. Here $\|.\|_F$ stands for the Frobenius norm.

If $\mathbf{C}(.)$ is simply a linear dispersion code satisfying (1), then solving (7) would be difficult. It has been shown [2] that by exploiting the code orthogonality in (3), Problem (7) can simplified to a Boolean quadratic program

$$\hat{\mathbf{S}} = \arg \max_{\tilde{\mathbf{S}} \in \{\pm 1\}^{K \times P}} [\, \tilde{\mathbf{s}}_1^T, \ldots, \tilde{\mathbf{s}}_P^T \,] \begin{bmatrix} \mathbf{G}_{11} & \ldots & \mathbf{G}_{1P} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{P1} & \ldots & \mathbf{G}_{PP} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_1 \\ \vdots \\ \tilde{\mathbf{s}}_P \end{bmatrix} \tag{8}$$

where $\mathbf{G}_{pq} \in \mathbb{R}^{K \times K}$ with $[\mathbf{G}_{pq}]_{k\ell} = \mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_\ell \mathbf{Y}_q^H\}\}$ (While (8) leads to $\hat{\mathbf{S}}$ only, $\hat{\mathbf{H}}$ can be computed directly from $\hat{\mathbf{S}}$ if necessary; see [2, 4]). Problem (8) can then be handled suboptimally by either the closed-form method [1, 3] or the quasi-optimal SDR method [2]. Alternatively, (8) can be solved optimally by a sphere decoding algorithm [2]. Some performance and complexity aspects of the various algorithms have been reported in [2].

The implementation simplicity of blind ML OSTBC detection motivates us to investigate the blind identifiability aspects, stated as follows: Suppose that the true channel $\mathbf{H}$ and data matrix $\mathbf{S}$ is a solution of the blind ML problem in (7). This desired solution is unique only when we cannot find another solution, denoted by $\{\tilde{\mathbf{H}}, \tilde{\mathbf{S}}\}$, such that

$$\mathbf{H}\mathbf{C}(\mathbf{s}_p) = \tilde{\mathbf{H}}\mathbf{C}(\tilde{\mathbf{s}}_p), \qquad p = 1, \ldots, P. \tag{9}$$

An obvious situation leading to (9) is when $\{\tilde{\mathbf{H}}, \tilde{\mathbf{S}}\} = \{-\mathbf{H}, -\mathbf{S}\}$. Sign ambiguity is an inherent problem, but can be fixed easily by a number of ways; e.g., setting one element of $\mathbf{S}$ to be a pilot. Given $\{\mathbf{H}, \mathbf{S}\}$, we say that $\mathbf{S}$ is uniquely identifiable up to a sign if (9) does not hold for any $\{\tilde{\mathbf{H}}, \tilde{\mathbf{S}}\} \neq \pm\{\mathbf{H}, \mathbf{S}\}$. In the next section, we will consider the class of the NIS-OSTBCs and its blind identifiability.

3. NONINTERSECTING SUBSPACE OSTBCS

This section contains the main results of this paper. In the first subsection we define the NIS-OSTBCs and study their properties. The second subsection proposes an NIS-OSTBC construction procedure. The proof leading to our construction procedure is detailed in the third subsection.

3.1. Nonintersecting Subspace Codes and Their Properties

Let $\mathcal{R}(\mathbf{A})$ denote the range space of $\mathbf{A}$. The following is our definition for a nonintersecting subspace (NIS) OSTBC:

Definition 1 *An OSTBC is said to be a NIS-OSTBC if*

$$\mathcal{R}(\mathbf{C}^T(\mathbf{s})) \cap \mathcal{R}(\mathbf{C}^T(\tilde{\mathbf{s}})) = \{\mathbf{0}\} \tag{10}$$

for every $\mathbf{s}, \tilde{\mathbf{s}} \in \{\pm 1\}^K$, $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$.

The NIS concepts were introduced in the noncoherent space-time coding literature [5] for achieving the full *noncoherent spatial diversity* in an i.i.d. Rayleigh channel. Up to this point there is no study regarding the existence and construction of NIS-OSTBCs, which is the subject of this paper. From a blind identifiability standpoint, NIS-OSTBCs are 'perfect' blind space-time codes:

Theorem 1 *Given every nonzero channel* $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ *and data matrix* $\mathbf{S} \in \{\pm 1\}^{K \times P}$, $\mathbf{S}$ *is uniquely identifiable up to a sign if and only if* $\mathbf{C}(.)$ *is an NIS-OSTBC.*

Proof: One can verify easily that to avoid the channel-code ambiguity in (9) for any $\mathbf{S} \in \{\pm 1\}^{K \times P}$, $\mathbf{H} \in \mathbb{C}^{M_r \times M_t} \setminus \{\mathbf{0}\}$, it is sufficient and necessary that the following statement holds: For every pair of bit vectors $(\mathbf{s}, \tilde{\mathbf{s}})$ with $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$, the condition

$$\mathbf{h}^T\mathbf{C}(\mathbf{s}) \neq \tilde{\mathbf{h}}^T\mathbf{C}(\tilde{\mathbf{s}}) \tag{11}$$

holds for any $\mathbf{h}, \tilde{\mathbf{h}} \in \mathbb{C}^{M_t} \setminus \{\mathbf{0}\}$. By noting that

$$\mathcal{R}(\mathbf{C}^T(\mathbf{s})) \cap \mathcal{R}(\mathbf{C}^T(\tilde{\mathbf{s}})) = \left\{ \mathbf{y} \;\middle|\; \mathbf{y} = \mathbf{C}^T(\mathbf{s})\mathbf{h} = \mathbf{C}^T(\tilde{\mathbf{s}})\tilde{\mathbf{h}}, \text{ for some } \mathbf{h}, \tilde{\mathbf{h}} \in \mathbb{C}^{M_t} \right\} \tag{12}$$

and by comparing (12) and (11), we conclude that (11) is equivalent to have $\mathcal{R}(\mathbf{C}^T(\mathbf{s})) \cap \mathcal{R}(\mathbf{C}^T(\tilde{\mathbf{s}})) = \{\mathbf{0}\}$ for any $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$. □

There is a price for using NIS-OSTBCs, however.

Lemma 1 *Suppose that* $\mathbf{C}(.)$ *is a real or complex GOD. If* $\mathbf{C}(.)$ *is also an NIS-OSTBC, then it does not achieve the full rate; i.e.,* $K < T$ *for real GODs and* $K/2 < T$ *for complex GODs.*

The proof of Lemma 1 is omitted here due to lack of space, but it is available in [10]. Here we give an example verifying Lemma 1. For $M_t = 3$, a BPSK full-rate OSTBC is given by [7]

$$\mathbf{C}(\mathbf{s}) = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \end{bmatrix} \tag{13}$$

where $T = K = 4$. If this code were an NIS-OSTBC, then

$$\mathbf{h}^T\mathbf{C}(\mathbf{s}) = \tilde{\mathbf{h}}^T\mathbf{C}(\tilde{\mathbf{s}}) \tag{14}$$

must not hold for $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$. Now, suppose that $\mathbf{h} = [\,1, 0, 0\,]^T$ and $\tilde{\mathbf{h}} = [\,0, 1, 0\,]^T$. Then (14) becomes

$$\begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \end{bmatrix} = \begin{bmatrix} \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & -\tilde{s}_3 \end{bmatrix} \tag{15}$$

which can definitely be satisfied by some $(\mathbf{s}, \tilde{\mathbf{s}})$, $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$.

NIS-OSTBCs have a constraint on the code length:

Property 1 *All NIS-OSTBCs have $2M_t \leq T$.*

The above property can be proven easily by using matrix analysis results [11]. Many existing OSTBCs essentially do not satisfy Property 1 [e.g., (13)], let alone falling into the NIS class.

3.2. Code Construction

We use the hints provided by Property 1 and Lemma 1 to come up with the following OSTBC construction:

Construction I:

Given an OSTBC function $\mathbf{C}_o(\mathbf{s}) = \sum_{k=1}^{K} s_k\mathbf{X}_k \in \mathbb{C}^{M_t\times T}$, where K is even.

Step 1. Set $\mathbf{C}_1(\mathbf{s}) = \sum_{k=1}^{K-1} s_k\mathbf{X}_k$.

Step 2. Output $\mathbf{C}_{new}(\mathbf{s}) = [\,\mathbf{C}_1(\boldsymbol{\mu})\ \mathbf{C}_o(\boldsymbol{\nu})\,] \in \mathbb{C}^{M_t\times 2T}$ as the new code, where $\mathbf{s} = [\,\boldsymbol{\mu}^T\boldsymbol{\nu}^T\,]^T$, $\boldsymbol{\mu} \in \{\pm1\}^{K-1}$, $\boldsymbol{\nu} \in \{\pm1\}^K$.

In the above construction we concatenate two OSTBCs, thereby forming a longer code that satisfies Property 1. Moreover, we drop 1 bit so as not to enable full rate; cf., Lemma 1. Surprisingly, by doing so it is sufficient to obtain an NIS-OSTBC:

Theorem 2 *Given any OSTBC $\mathbf{C}_o : \mathbb{R}^K \to \mathbb{C}^{M_t\times T}$ where K is even, the code generated by Construction I is an NIS-OSTBC.*

The proof of Theorem 2 is given in the next subsection. We should stress that the NIS conversion in Construction I is applicable to all OSTBCs having an even K (almost all existing BPSK/QPSK OSTBCs have an even K). The resultant NIS-OSTBCs have a data rate of $(2K-1)/(2T)$ bits per channel use (bpcu). This rate is relatively lower than that of the original code; that is, K/T bpcu. To reduce the rate loss, the following modified transmission scheme may be used:

Modified OSTBC Transmission Scheme

Given an OSTBC $\mathbf{C}_o(\mathbf{s}) = \sum_{k=1}^{K} s_k\mathbf{X}_k \in \mathbb{C}^{M_t\times T}$ where K is even, and a frame length $P \geq 2$.

Step 1. Set $\mathbf{C}_1(\mathbf{s}) = \sum_{k=1}^{K-1} s_k\mathbf{X}_k$.

Step 2. For $p = 1$, transmit $\mathbf{C}_1(\mathbf{s}_1)$ where $\mathbf{s}_1 \in \{\pm1\}^{K-1}$.

Step 3. For $p = 2, \ldots, P$, transmit $\mathbf{C}_o(\mathbf{s}_p)$ where $\mathbf{s}_p \in \{\pm1\}^K$.

The difference between the original and modified transmission schemes lies in the first transmitted code block only, where the modified scheme transmits a 1-bit-reduced OSTBC $\mathbf{C}_1(.)$ in place of $\mathbf{C}_o(.)$ in the original case. Since the first two code blocks $\mathbf{C}_1(\mathbf{s}_1)$ and $\mathbf{C}_o(\mathbf{s}_2)$ can be seen as one NIS-OSTBC, perfect identification of $\mathbf{H}$ (up to a sign) is guaranteed and it follows that the rest of the code blocks $\mathbf{C}_o(\mathbf{s}_3), \ldots, \mathbf{C}_o(\mathbf{s}_P)$ are also perfectly identifiable. Alternatively, the code frame $[\,\mathbf{C}_1(\mathbf{s}_1), \mathbf{C}_o(\mathbf{s}_2), \ldots, \mathbf{C}_o(\mathbf{s}_P)\,]$ can be seen as a single OSTBC, with the NIS characteristic inherited from $[\,\mathbf{C}_1(\mathbf{s}_1), \mathbf{C}_o(\mathbf{s}_2)\,]$. The rate of the modified scheme is $(2KP-1)/(2TP)$ bpcu. Hence, for large P, the rate of the modified scheme approaches that of the original.

It is interesting to mention that for the special case of the BPSK Alamouti code in (4), the modified scheme is essentially a pilot-code-assisted or semiblind [2] scheme. To see this, we note that the corresponding $\mathbf{C}_1(\mathbf{s}_1)$ is

$$\mathbf{C}_1(\mathbf{s}_1) = \begin{bmatrix} s_{11} & 0 \\ 0 & s_{11} \end{bmatrix} \tag{16}$$

If s_{11} is also chosen to be the pilot bit for solving sign ambiguity, then it is equivalent that $\mathbf{C}_1(\mathbf{s}_1)$ serves as a pilot space-time code. However, for $K > 2$ it is no longer true that the modified scheme is a pilot-code-assisted scheme.

3.3. Proof of Theorem 2

Theorem 2 is proven by contradiction. Suppose that $\mathbf{C}_{new}(.)$ is not an NIS-OSTBC such that for some distinct pair of bit vectors $(\mathbf{s}, \tilde{\mathbf{s}})$, there exist $\mathbf{h}, \tilde{\mathbf{h}} \in \mathbb{C}^{M_t} \setminus \{\mathbf{0}\}$ such that

$$\mathbf{h}^T\mathbf{C}_{new}(\mathbf{s}) = \tilde{\mathbf{h}}^T\mathbf{C}_{new}(\tilde{\mathbf{s}}). \tag{17}$$

From Construction I, Eq. (17) can be decomposed to 2 equations

$$\mathbf{h}^T\mathbf{C}_1(\boldsymbol{\mu}) = \tilde{\mathbf{h}}^T\mathbf{C}_1(\tilde{\boldsymbol{\mu}}), \tag{18}$$

$$\mathbf{h}^T\mathbf{C}_o(\boldsymbol{\nu}) = \tilde{\mathbf{h}}^T\mathbf{C}_o(\tilde{\boldsymbol{\nu}}), \tag{19}$$

where $\mathbf{s} = [\,\boldsymbol{\mu}^T\ \boldsymbol{\nu}^T\,]^T$ and $\tilde{\mathbf{s}} = [\,\tilde{\boldsymbol{\mu}}^T\ \tilde{\boldsymbol{\nu}}^T\,]^T$. Postmultiplying (18) and (19) by $\mathbf{C}_1(\boldsymbol{\mu})$ and $\mathbf{C}_o(\boldsymbol{\nu})$ respectively, we obtain

$$\mathbf{h}^T = \tilde{\mathbf{h}}^T\mathbf{Q}_1, \quad \mathbf{h}^T = \tilde{\mathbf{h}}^T\mathbf{Q}_2, \tag{20}$$

where

$$\mathbf{Q}_1 = \tfrac{1}{K-1}\mathbf{C}_1(\tilde{\boldsymbol{\mu}})\mathbf{C}_1^H(\boldsymbol{\mu}), \quad \mathbf{Q}_2 = \tfrac{1}{K}\mathbf{C}_o(\tilde{\boldsymbol{\nu}})\mathbf{C}_o^H(\boldsymbol{\nu}). \tag{21}$$

Eqs. (20) lead to

$$\tilde{\mathbf{h}}^T(\mathbf{Q}_1 - \mathbf{Q}_2) = \mathbf{0}, \tag{22}$$

implying that $\mathbf{Q}_1 - \mathbf{Q}_2$ is singular. We now show that $\mathbf{Q}_1 - \mathbf{Q}_2$ cannot be singular. The matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$ can be expressed as

$$\mathbf{Q}_1 = \frac{\alpha_1}{K-1}\mathbf{I} + \mathbf{B}_1, \quad \mathbf{Q}_2 = \frac{\alpha_2}{K}\mathbf{I} + \mathbf{B}_2 \tag{23}$$

where

$$\alpha_1 = \sum_{k=1}^{K-1}\mu_k\tilde{\mu}_k \in \{\pm1, \pm3, \ldots, \pm(K-1)\}, \tag{24}$$

$$\alpha_2 = \sum_{k=1}^{K}\nu_k\tilde{\nu}_k \in \{0, \pm2, \pm4, \ldots, K\}, \tag{25}$$

and $\mathbf{B}_1 = \frac{1}{K-1}\sum_k \sum_{\ell\neq k} \tilde{\mu}_k \mu_\ell \mathbf{X}_k \mathbf{X}_\ell^H$ and $\mathbf{B}_2 = \frac{1}{K}\sum_k \sum_{\ell\neq k} \tilde{\nu}_k \nu_\ell \mathbf{X}_k \mathbf{X}_\ell^H$ are skew-Hermitian [cf., Eq. (2)]. Hence,

$$\mathbf{Q}_1 - \mathbf{Q}_2 = \gamma \mathbf{I} + (\mathbf{B}_1 - \mathbf{B}_2), \tag{26}$$

where

$$\gamma = \frac{\alpha_1}{K-1} - \frac{\alpha_2}{K}. \tag{27}$$

If $\mathbf{Q}_1 - \mathbf{Q}_2$ is singular, then at least one of its eigenvalues has to be 0. From (26), the eigenvalues of $\mathbf{Q}_1 - \mathbf{Q}_2$ are given by $\lambda_i(\mathbf{Q}_1 - \mathbf{Q}_2) = \gamma + \lambda_i(\mathbf{B}_1 - \mathbf{B}_2)$, $i = 1, \ldots, M_t$. Since $\mathbf{B}_1 - \mathbf{B}_2$ is skew-Hermitian, its eigenvalues $\lambda_i(\mathbf{B}_1 - \mathbf{B}_2)$ are either pure imaginary or zero [11]. Hence, to have a singular $\mathbf{Q}_1 - \mathbf{Q}_2$ it is necessary that $\gamma = 0$. Since K is even, we can write $K = 2m$ for some integer m. Likewise, α_2 can be represented by $\alpha_2 = 2c$ where $c \in \{0, \pm 1, \ldots, \pm m\}$. The condition $\gamma = 0$ implies that

$$\alpha_1 = \frac{K-1}{K}\alpha_2 = \frac{(2m-1)c}{m} = 2c - \frac{c}{m}. \tag{28}$$

From (24) α_1 is an odd number, but Eq. (28) indicates that α_1 is not an integer unless $c = 0$ or $c = \pm m$. For $c = 0$ we have $\alpha_1 = 0$, a contradiction. For $c = \pm m$, we have $\alpha_1 = \pm(K-1)$ and $\alpha_2 = \pm K$. Such a condition can only be satisfied when $[\, \boldsymbol{\mu}^T \; \boldsymbol{\nu}^T \,]^T = \pm[\, \tilde{\boldsymbol{\mu}}^T \; \tilde{\boldsymbol{\nu}}^T \,]^T$, a contradiction to $\mathbf{s} \neq \pm\tilde{\mathbf{s}}$.

4. SIMULATION RESULTS

This simulation example compares the performance of the original and modified OSTBC transmission schemes. The numbers of transmitter and receiver antennas are $M_t = 3$ and $M_r = 1$, respectively. The code matrix is the one in (13). In the simulation, the channel is i.i.d. zero-mean Gaussian distributed. The blind receiver employed here is the SDR-ML [2] (note that the other blind ML implementations could also be used, but with the page limit only SDR-ML is considered). The sign ambiguity effect is eliminated by assuming that one of the bit symbols is known at the receiver.

The results were plotted in Fig. 1. In the figure, we also show the performance of differential OSTBC [12, 13], another effective noncoherent space-time technique. We see that the modified scheme has a substantially better bit error performance than the original scheme. It also yields better performance than the differential scheme for $P \geq 8$, by about 1.5dB when $P = 16$.

Fig. 1 also reveals that the modified scheme achieves the full noncoherent spatial diversity, as promised by its NIS characteristic. In essence, the bit error probability of the modified scheme is observed to be decaying at the same rate as that of the coherent ML. However, such a decaying rate is lower in the original scheme.

5. CONCLUSION

The contributions of this paper are twofold. First, we have examined the impacts of NIS-OSTBCs on blind ML space-time coding. In particular, our analysis has shown that NIS-OSTBCs are an attractive class of blind space-time codes, in the sense that they are uniquely identifiable up to a sign for any nonzero channel. Second, we have derived an NIS-OSTBC construction procedure. The procedure works by applying a simple modification to an existing OSTBC, and is very easy to use.

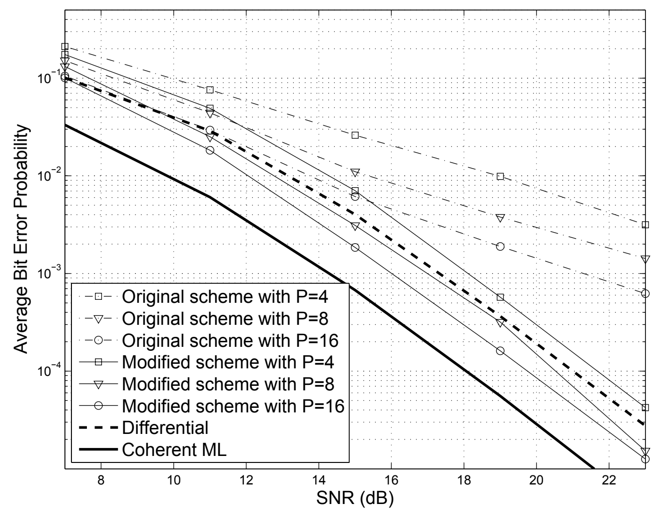

Fig. 1. Bit error rate of the the original and modified OSTBC schemes.

6. REFERENCES

[1] P. Stoica and G. Ganesan, “Space-time block codes: trained, blind, and semi-blind detection,” *Digital Signal Processing*, vol. 13, pp. 93–105, 2003.

[2] W.-K. Ma, B.-N. Vo, T. N. Davidson, and P. C. Ching, “Blind ML detection of orthogonal space-time block codes: Efficient high-performance implementations,” *IEEE Trans. Signal Process.*, pp. 738–751, Feb. 2006.

[3] S. Shahbazpanahi, A. Gershman, and J. Manton, “Closed-form blind MIMO channel estimation for orthogonal space-time block codes,” *IEEE Trans. Signal Processing*, vol. 53, no. 12, pp. 4506–4517, Dec. 2005.

[4] E. G. Larsson, P. Stoica, and J. Li, “Orthogonal space-time block codes: Maximum likelihood detection for unknown channels and unstructured interferences,” *IEEE Trans. Signal Processing*, vol. 51, no. 2, pp. 362–372, 2003.

[5] F. E. Oggier, N. J. A. Sloane, S. N. Diggavi, and A. R. Calderbank, “Nonintersecting subspaces based on finite alphabets,” *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4320–4325, 2005.

[6] E. G. Larsson and P. Stoica, *Space-Time Coding for Wireless Communications*. Cambridge University Press, May 2003.

[7] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[8] X.-B. Liang, “Orthogonal designs with maximal rates,” *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2468–2503, 2003.

[9] W. Su and X.-G. Xia, “On space-time codes from complex orthogonal designs,” *Wireless Personal Commun.*, vol. 25, pp. 1–26, 2003.

[10] W.-K. Ma, “Blind ML detection of orthogonal space-time block codes: Identifiability and code construction,” to appear in *IEEE Trans. Signal Process.*, 2007.

[11] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, Cambridge, U.K., 1990.

[12] G. Ganesan and P. Stoica, “Differential modulation using space-time block codes,” *IEEE Signal Processing Lett.*, vol. 9, no. 2, pp. 57–60, 2002.

[13] X.-B. Liang and X.-G. Xia, “Fast differential unitary space-time demodulation via square orthogonal designs,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1331–1336, 2005.