RANDOMIZED SCHEDULER FOR TEMPORALLY-CORRELATED CHANNELS

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ABSTRACT

In a slowly time-varying fading broadcast channel, a proposed randomized scheduler achieves multi-user diversity gain while reducing the amount of feedback. The scheduler requests feedback of signal-to-noise ratios (SNR) from a random subset of users in conjunction with the previously scheduled user, and then selects the user with the largest SNR. With temporal correlation, this scheduler achieves near optimal sum-rate even with feedback from a small subset of users, which considerably reduces the amount of feedback.

Index Terms— Broadcast channels, Feedback communication, Multi-user diversity, Randomized algorithm

1. INTRODUCTION

Multi-user diversity (MUD) methods achieve the sum-rate capacity in fading scalar broadcast channels (BC) and multiple access channels by selecting the user with the most favorable channel condition [1], [2]. However, MUD generally requires signal-to-noise ratio (SNR) feedback from all users, and scales poorly to large population. Fortunately, the MUD gain can be efficiently achieved even with a coarse quantization of the SNR because only the high SNR region needs to be quantized [3]. Moreover, the amount of feedback can be reduced by obtaining feedback only from users having relatively large SNRs [4], [5]. The aforementioned methods use only the instantaneous SNR to reduce the amount of feedback; thus, temporal correlation of the SNR has not yet been exploited for that purpose.

This temporal correlation has been exploited recently in packet switch design. Although the maximum weight matching (MWM) algorithm [6] achieves 100% throughput, it is unsuitable for high bandwidth switch because of its high complexity and poor scalability. Thus, [7],[8] proposed a randomized algorithm that achieves 100% throughput utilizing temporal correlation of queue states. This temporal correlation mainly results from a characteristic of arrival and departure process of queueing systems. Since only one packet can arrive at or depart from a queue during a time slot, the queue states change slowly in time, therefore the best matching at a certain time slot would be the best matching in the next time slot. Thus, temporal correlation is exploited by memorizing the previous best matching and comparing it with the next random matchings. Similarly in a slow fading channel, the best channel at a time slot would be the best channel at the subsequent time slots. Moreover, finding the best user requires a large amount of feedback just like the MWM algorithm requires a large amount of time. These similarities motivate the use of randomized scheduler with memory in wireless channels.

This paper proposes a randomized scheduler that exploits temporal correlations in slow fading channels. Since the scheduler that polls all users would consecutively select a same user with high probability, the proposed scheduler polls U_s users in a roster set that contains the previously selected user as well as $U_s - 1$ randomly selected users, where U_s can be much smaller than the total number of users. Then, it selects the user with the largest SNR among U_s users as a destination for a packet. Since the memory boosts the probability of selecting a good user, the sum-rate of the proposed method is comparable to that of the optimal method, which polls all users, except with much less feedback. This reduction in the amount of feedback is measured by an *effective number of users*, which is derived by both simulations and analysis.

Notation: $\mathbb{E}$ denotes statistical expectation. $f_X(x)$ is the probability density function (pdf) of a continuous random variable X; $F_X(x) = \int_0^x f_X(\xi)d\xi$ is its cumulative distribution function (cdf). $J_0(x)$ is the zeroth-order Bessel function and $\Re(x)$ is the real part of a complex variable x.

2. SYSTEM MODEL

In a wireless BC, a base station communicates to U users through slow fading channels. It receives the feedback messages containing the SNR from users and then selects a user as a destination of an information-bearing packet based on the feedback. The received signal of user u at time t, denoted as $y_{u,t}$, is given by

$$y_{u,t} = h_{u,t}s_t + n_t, \quad u = 1, 2, \cdots, U, \tag{1}$$

where s_t is the transmitted signal from the base station, $h_{u,t}$ denotes the channel gain from the base station to user u, and

n_t is an additive white complex Gaussian noise with a variance σ^2 for all users. Throughout the paper, $h_{u,t}, u = 1 \cdots U$ are zero mean circularly symmetric Gaussian random variables. The channel gains from different users are assumed to be independent and identically distributed (i.i.d.), which is reasonable with a sufficient spatial separation of users. The channel gains are temporally correlated according to Jake's model. Suppose that the mobile speed is v and the carrier frequency is f_c, then the maximum doppler frequency is

$$f_m = \frac{v}{c} f_c, \tag{2}$$

where c is the speed of light. Then, the autocorrelation between two channel realizations temporally separated by τ is

$$R(\tau) = \mathbb{E}\left[\Re(h_{u,t})\Re(h_{u,t+\tau})\right] = J_0(2\pi f_m \tau). \tag{3}$$

3. RANDOMIZED SCHEDULER

In a time-division multiple-access (TDMA) system, the scheduler selects the user with the largest channel gain, therefore increasing the sum-rate as the number of users grows. This increase is called multi-user diversity gain. If the scheduler knows channel gains of all users, the sum-rate is

$$R(U) = \log_2\left(1 + \frac{|h^*(U)|^2}{\sigma^2}\right), \tag{4}$$

where $|h^*(U)| = \max\{|h_1|, \cdots, |h_U|\}$. With temporal channel correlation, the sum-rate is unchanged since channels are still independent between users; however, the scheduler would consecutively select a same user.

In packet switches, a randomized algorithm with memory is throughput-optimal because it efficiently uses the temporal correlation of the queue states. Similarly, to take advantage of this repetitive selection of the same user, a simple randomized scheduler with memory is proposed as follows:

Algorithm 1 *Randomized scheduler:*

- *Step I: Randomly select U_s users, and generate a set $\mathcal{A}$, called a roster, containing these users.*
- *Step II: Request the users in $\mathcal{A}$ to feedback their SNRs, and then receive SNRs.*
- *Step III: Transmit a packet to the user u^* having the largest SNR among U_s SNRs.*
- *Step IV: Generate a set $\mathcal{C}$ containing $U_s - 1$ randomly selected users, and then $\mathcal{A} = \mathcal{C} \cup \{u^*\}$. Go to step II.*

For this random scheduler, the sum-rate is defined as follows:

$$R(U_s, U) = \log_2\left(1 + \frac{|h^*(U_s, U)|^2}{\sigma^2}\right), \tag{5}$$

where $|h^*(U_s, U)| = \max\{|h_i| \, | i \in \mathcal{A}\}$.

As an example, Fig. 1 shows how the proposed algorithm operates with $U_s = 5$. In the first time slot, a packet is transmitted to user 1, so the roster set $\mathcal{A}$ is composed of four random users and user 1. After receiving the SNRs from the users in set $\mathcal{A}$, the base station transmits the second packet to user 7 who has the largest SNR. In the subsequent slot, the base station polls user 7 along with four more random users, and learns that user 7 still has the largest SNR among the polled users even though user 6, who was outside the roster, has the largest SNR among all U users. Since the memorized user tends to have a reasonably large SNR because of the temporal correlation, this failure to select user 6 insignificantly reduces the sum-rate. As seen in this example, memory plays a vital role in exploiting the temporal correlation. Without memory, the base station polls U_s random users in every slot; therefore, the sum-rate of this purely randomized algorithm is $R(U_s)$, which is the sum-rate of a BC with only U_s users. Since the proposed method exploits the temporal correlations, $R(U_s) \leq R(U_s, U) \leq R(U)$. In Section 4, the effective number of users U_e is defined to describe $R(U_s, U)$ as $R(U_e)$.

The first benefit of this randomized scheduler is the reduction in the amount of feedback because the roster size is shrunk to U_s instead of U. In addition, this proposed scheduler can be implemented without a central polling by employing a voluntary round-robin feedback. As an example of round-robin feedback, if $U_s - 1 = U/2$, then odd-numbered users and even-numbered users will feedback their SNRs respectively at odd and even numbered slots, while the previously selected user will feedback in the first feedback slot. In this way, explicit controls from the base station are unnecessary. Thus, this distributed implementation saves the resources used for the down-link control.

4. EFFECTIVE NUMBER OF USERS

As suggested in Section 3, the effect of the memory in the proposed scheduler can be interpreted as an increase of the roster size of a purely random scheduler. This section defines the effective number of users to quantify this effect.

The SNR of the memorized user at time t is denoted by $X_{m,t}$, and the largest SNR among those of U_s users in the roster is denoted as $X_{s,t}$. At time $t+1$, the memorized user is the one with the largest SNR among U_s users at time t. In a steady state, X_m and X_s are stationary, i.e., their distributions will be independent of t. Since $X_{s,t}$ is a random variable obtained by selecting the maximum values among many SNRs, an ordered distribution of the SNR can closely approximate the distribution of $X_{s,t}$. The *effective number of users U_e* is defined as the parameter of the ordered distribution of SNRs that is closest to the actual distribution of $X_{s,t}$, while the distance is measured by the relative entropy [9].

Definition 1 *The effective number of users U_e is defined as*

$$U_e = \arg\min_{n \geq U_s} \int_0^\infty f_{X_{s,t}}(x) \log \frac{f_{X_{s,t}}(x)}{f_{\tilde{X}(x)}} dx, \tag{6}$$

where $f_{\tilde{X}(x)}$ is $dF_X(x)^n/dx$.

For a given $F_{X_{s,t}}(x)$, U_e can be found from the above definition, and $F_{X_{s,t}}(x)$ can be approximately expressed in terms of $F_X(x)$ as follows:

$$F_{X_{s,t}}(x) \approx F_X(x)^{U_e}. \tag{7}$$

However, since $F_{X_{s,t}}(x)$ is analytically unknown, U_e needs to be found in an indirect way. Using (7), $F_{X_{m,t+1}}(x)$ can be expressed in two different forms of distributions. We will find U_e that minimizes the distance of two forms of distributions of $F_{X_{m,t+1}}(x)$ as follows:

First, for $U \gg U_s$ or $U = \infty$, $X_{m,t}$ can be assumed to be independent of the SNRs of the randomly selected $U_s - 1$ users. Then, $F_{X_{s,t}}(x)$ can be derived from $F_X(x)$, and $X_{m,t}$:

$$F_{X_{s,t}}(x) = F_{X_{m,t}}(x) F_X(x)^{U_s - 1}, x \geq 0. \tag{8}$$

From the stationarity of $X_{m,t}$,

$$\begin{aligned} F_{X_{m,t+1}}(x) &= F_{X_{m,t}}(x) \\ &= F_{X_{s,t}}(x) F_X(x)^{-U_s+1} \\ &\approx F_X(x)^{U_e - U_s + 1}, x \geq 0. \end{aligned} \tag{9}$$

Second, a linear approximation of the temporal correlation between $X_{m,t+1}$ and $X_{s,t}$ yields the following relationship :

$$\begin{aligned} X_{m,t+1} &= \rho(X_{s,t} - \mathbb{E}[X_{s,t}]) + \mathbb{E}[X_{m,t+1}], \\ &= \rho X_{s,t} + e_t, \end{aligned} \tag{10}$$

where ρ is the correlation coefficient in (3) and $e_t = -\rho\mathbb{E}[X_{s,t}] + \mathbb{E}[X_{m,t+1}]$. From (7) and (10),

$$\begin{aligned} F_{X_{m,t+1}}(x) &= P(\rho X_{s,t} + e_t < x) = P\left(X_{s,t} < \frac{x - e_t}{\rho}\right) \\ &\approx \begin{cases} F_X\left(\frac{x-e_t}{\rho}\right)^{U_e}, x \geq e_t & ; \\ 0, x \leq e_t & . \end{cases} \end{aligned} \tag{11}$$

From (9) and (11), U_e that minimizes the distance of two distributions can be numerically found.

5. NUMERICAL RESULTS

This section shows simulations of the sum-rate using the proposed scheduler for Rayleigh fading channels with 0dB average SNR for all the users. The carrier frequency f_c is 2.1GHz, and the packet duration τ is 0.5msec. The mobile speed v is between 0.5m/sec (a walking speed) and 9m/s (an urban vehicle speed). The corresponding maximum Doppler frequencies range from 3.5Hz to 63Hz.

In Fig. 2, the number of users in a cell is 1000[1], while the roster size is varied from 2 to 200. Fig. 2 shows that the expected sum-rate of the proposed method is comparable to that of the optimal scheduler, which polls all the users, when v is small and U_s is sufficiently large. For instance, at a walking speed (0.5m/s), the proposed scheduler with $U_s = 2$ increases the sum-rate by 94% compared with $R(2)$, the sum-rate without memory. Compared to the optimal method, the loss is only 24%, which is insignificant considering the 99.8% reduction in the amount of feedback. With $U_s = 50$, the sum-rate loss is even less than 2%.

The sum-rate of the proposed method with $U = 100$, $R(U_s, 100)$, is shown in Fig. 3. With small U_s, $R(U_s, 100) \simeq R(U_s, 1000)$ because U_s is so small that the randomly polled users are almost independent of the previously polled users even with $U = 100$. However, as U_s increases, the previously polled users will often reappear in the roster without sufficient temporal separations, therefore, the sum-rates with $U = 100$ are smaller than those with $U = 1000$.

The efficiency of the proposed scheduler can be explained by the effective number of users U_e. In Fig. 2, the effective number of users U_e is shown along the graph. For example, with $v = $ 1m/s and $U_s = 2$, U_e is about 36.5, i.e., $R(2, 1000) = R(36.5)$. As suggested in the previous paragraph, the effective number of users is the same for $U = 1000$ and $U = 100$ when U_s is small.

Fig. 4 shows the effective number of users from the Section 4. This figure corresponds to the case of $U = \infty$ or $U \gg U_s$, i.e., $R(U_e) = R(U_s, \infty)$. For small U_s, $U = 1000$ is large enough to satisfy this condition. With 9m/s and $U_s = 2$, U_e is 11, which is almost twice of U_e in Fig. 2. If $U_s = 100$, U_e is 163, which is almost half of U_e in Fig. 2. This discrepancy may rise from the linear temporal-correlation model. However, the overall tendency of analytical results coincides with that of the simulation results.

6. CONCLUSION

This paper proposes a randomized scheduler that requires SNR feedback from partial users. When the temporal correlation is high, simulation and analysis results show that the proposed method exhibits little loss compared with the optimal method, which requires the SNR feedback from all users. Moreover, the proposed method can be implemented in a distributed manner and with much less feedback.

Although this paper considers only Rayleigh fading channel, the proposed method can be applied to other channels even in the presence of multiple antennas without significant modification. However, the proposed method only considers the rate as its criteria; thus, it disregards QoS requirements and short-term fairness among users. To use the proposed

[1] This large population is considered to show the performance in an extreme case. This paper also considers the population size of 100, which is typical in cellular systems.

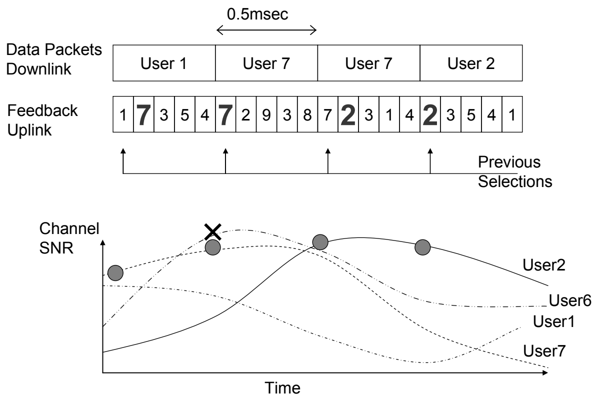

Fig. 1. The operation of the randomized scheduler.

method in conjunction with time sensitive services, the fairness issues should also be addressed.

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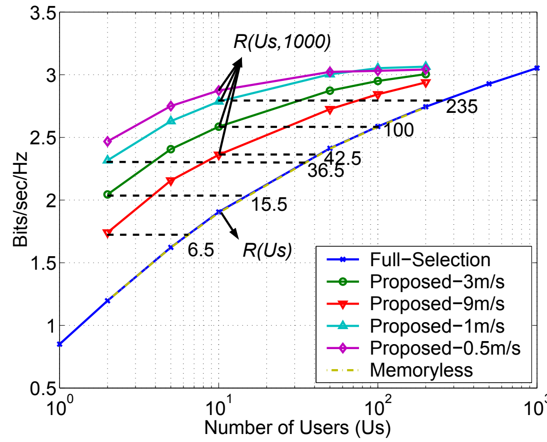

Fig. 2. The expected sum-rate with 1000 users. The numbers along the graph denote the effective number of users U_e.

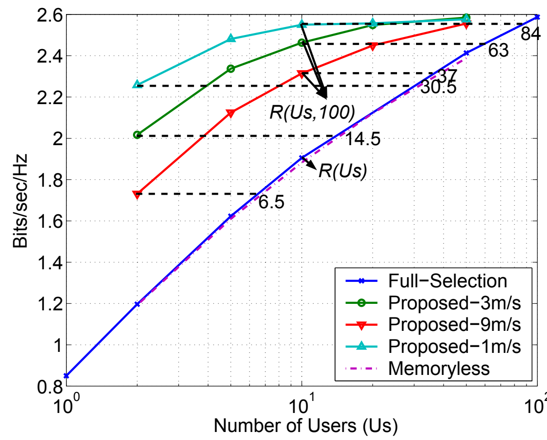

Fig. 3. The expected sum-rate with 100 users. The numbers along the graph denote the effective number of users U_e.

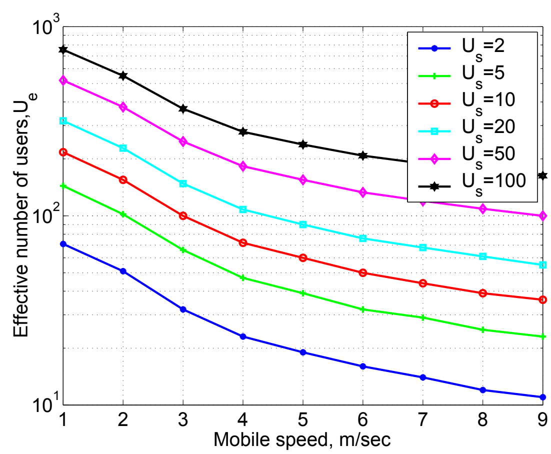

Fig. 4. The effective number of users from the analysis.