A NEAR-OPTIMUM TECHNIQUE USING LINEAR PRECODING FOR THE MIMO BROADCAST CHANNEL

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ABSTRACT

We consider the MIMO broadcast channel (MIMO-BC) where an array equipped with M antennas transmits distinct information to K users, each equipped with N antennas. We propose a linear precoding technique, called multiuser eigenmode transmission (MET), based on the block diagonalization precoding technique. MET addresses the shortcomings of previous ZF-based beamformers by transmitting to each user on one or more eigenmodes chosen using a greedy algorithm. We consider both the typical sum-power constraint (SPC), and a per-antenna power constraint (PAPC) motivated by array architectures where antennas are powered by separate amplifiers and are either co-located or spatially separated. Numerical results show that the proposed MET technique outperforms previous linear techniques with both SPC and PAPC. Asymptotically as the number of user K increases without bound, we show that block diagonalization with receive antenna selection under PAPC and SPC are asymptotically optimal.

***Index Terms*—** Array signal processing, MIMO systems.

1. INTRODUCTION

We consider the multiple-input multiple-output broadcast channel (MIMO-BC) where the transmitter equipped with M antennas sends distinct information to K users, each equipped with N antennas. It has recently been shown [1] that the capacity region of the MIMO-BC can be achieved by means of a nonlinear transmission technique known as "dirty paper coding" (DPC) . Linear precoding (beamforming) techniques with lower complexity have also been proposed in which the transmitted signal is a linear combination of the users' data signals. One class of beamforming techniques for the case of a single-antenna receiver ($N = 1$) is based on zero-forcing [2] where each user receives only its desired signal with no interference. The most straightforward extensions of the zero-forcing technique to the case of $N > 1$ appear in [3], [4], [5], where multiple spatial streams (or eigenmodes) are transmitted to each user with no interuser interference, resulting in a block diagonal (BD) covariance matrix. Further improvements of the original schemes have been proposed in [6, 7]. In this paper, we propose a linear transmission technique based on the BD technique using joint coordinated transmit-receive processing; as in [4] we use a receiver beamformer to select a subset of the eigenmodes of a given user, and a transmitter beamformer in order to guarantee the orthogonality between the different users. Reference [4] does not address the case of $K > M$, and it suggests transmitting on the dominant eigenmodes for each user. This technique for eigenmode selection is not optimum since the dominant eigenmodes for different users could be highly correlated, resulting in poor performance under ZF. In constrast to [6], we make full use of each user's multiple antennas. Our proposal uses a joint eigenmode-user selection scheme where the set of active users and active eigenmodes is selected in a greedy manner to maximize the sum-rate of the system. It is a generalization of the greedy algorithm proposed for the case $N = 1$ [8] and is not restricted in the number of users (K can be larger than M) nor in the assignment of dominant eigenmodes.

We consider both the classic case of sum power constraint (SPC) on the antennas and the case of per-antenna power constraint (PAPC) as [9] (see also reference therein). We emphasize that PAPC is applicable in systems where each antenna is powered by its own amplifier and is limited by the linearity of that amplifier. PAPC is further motivated by future wireless networks where base stations with spatially separated antennas transmit in a coordinated fashion to the mobile users.

We show that the the sum rate of the of BD algorithm with antenna selection grows at the same rate as the optimum DPC sum rate, with both SPC and PAPC, when the number of users K increases asymptotically. Moreover, we give numerical results that show how the proposals achieve a significant fraction of the DPC sum rate for practical systems with finite K, and outperform previous BD schemes [3, 4, 5, 6]

2. SYSTEM MODEL

We consider a narrowband multiantenna downlink channel modeled as a MIMO-BC with flat fading, where K users, each equipped with N receive antennas, request service from the transmitter which has M antennas. The discrete-time complex baseband received signal by the kth user is

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k, \quad k = 1, \ldots, K \tag{1}$$

where $\mathbf{H}_k \in \mathbb{C}^{N \times M}$ is the kth user's channel matrix, $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the transmitted signal vector, and $\mathbf{n}_k \sim \mathcal{CN}(0, 1)$ is the complex additive white Gaussian noise at the kth user. We assume that $\mathbf{H}_1, \ldots, \mathbf{H}_K$ are known to the transmitter. On a given symbol period, the base serves a subset of users $\mathcal{S} \subseteq \{1, \ldots, K\}$. Under a per-antenna average power constraint, the transmitted signal must satisfy

$$\mathbb{E}\left[|x_m|^2\right] \leq P_m. \quad m = 1, \ldots, M \tag{2}$$

where P_m is the power constraint for the mth antenna. The sum-power constraint can be written as

$$\mathbb{E}\left[tr\left[\mathbf{x}\mathbf{x}^{\mathbf{H}}\right]\right] \leq \sum_{m=1}^{M} P_m = P. \tag{3}$$

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3. THE MULTIUSER EIGENMODE TRANSMISSION (MET) METHOD

We fix the set of served users $\mathcal{S}$ and assign indices $k = 1, \ldots, |S|$. For the kth user, we fix the set of transmitted eigenmodes S_k and assume they are indexed from 1 to $|S_k|$. We note that if we transmit to a single user, the number of eigenmodes is limited to $|S_k| \leq \min(M, N)$. The transmitted signal after precoding can be written as

$$\mathbf{x} = \sum_{k=1}^{|S|} \mathbf{G}_k \mathbf{d}_k, \tag{4}$$

where $\mathbf{G}_k \in \mathbb{C}^{M \times |S|}$ is the precoding matrix for user k and $\mathbf{d}_k = [d_{k,1} \ldots d_{k,|S_k|}]^T$ is the $|S_k|$-dimensional vector of symbols.

The channel of the kth user can be decomposed using the singular value decomposition (SVD) as $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$, where the eigenvalues in $\mathbf{\Sigma}_k$ are arranged so that the ones associated with the allocated set S_k appear in the leftmost $|S_k|$ columns. We denote these eigenvalues as $\Sigma_{k,1}, \ldots, \Sigma_{k,|S_k|}$ The kth user's receiver is a linear detector given by the Hermitian transposition of the leftmost $|S_k|$ columns of $\mathbf{U}$ which we denote as $\mathbf{u}_{k,1} \ldots \mathbf{u}_{k,|S_k|}$. Likewise, we denote the leftmost $|S_k|$ columns of the right eigenvector matrix $\mathbf{V}_k$ as $\mathbf{v}_{k,1} \ldots \mathbf{v}_{k,|S_k|}$. The signal for the kth user after this detector can be written as

$$\begin{aligned} \mathbf{r}_k &= \left[\mathbf{u}_{k,1} \ldots \mathbf{u}_{k,|S_k|}\right]^H \mathbf{y}_k & (5) \\ &= \mathbf{\Gamma}_k \mathbf{G}_k \mathbf{d}_k + \mathbf{\Gamma}_k \sum_{j \in \mathcal{S}, j \neq k} \mathbf{G}_j \mathbf{d}_j + \mathbf{n}'_k & (6) \end{aligned}$$

where $\mathbf{y}_k$ is the received signal given by (1), $\mathbf{n}'_k$ is the processed noise, and $\mathbf{\Gamma}_k = [\Sigma_{k,1}\mathbf{v}_{k,1} \ldots \Sigma_{k,|S_k|}\mathbf{v}_{k,|S_k|}]^H$ is a $|S_k| \times M$ matrix. By defining

$$\tilde{\mathbf{H}}_k = \left[\mathbf{\Gamma}_1^H \ldots \mathbf{\Gamma}_{k-1}^H \, \mathbf{\Gamma}_{k+1}^H \ldots \mathbf{\Gamma}_{|\mathcal{S}|}^H\right]^H, \tag{7}$$

our zero-forcing constraint requires that $\mathbf{G}_k$ lie in the null space of $\tilde{\mathbf{H}}_k$. Hence $\mathbf{G}_k$ can be found by considering the SVD of $\tilde{\mathbf{H}}_k$:

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \tilde{\mathbf{\Sigma}}_k \left[\tilde{\mathbf{V}}_k^{(1)} \, \tilde{\mathbf{V}}_k^{(0)}\right]^H, \tag{8}$$

where $\tilde{\mathbf{V}}_k^{(0)}$ corresponds to the right eigenvectors associated with the null modes. From the relation between the dimension of the null space and rank of $\tilde{\mathbf{V}}^{(k)}$, the following constraint has to be satisfied in order to build the set of precoding matrices for the selected users $\mathcal{S}$:

$$\sum_{j \in \mathcal{S}, j \neq k} |S_j| < M \quad \forall k \in \mathcal{S}. \tag{9}$$

The number of modes allocated to the kth user satisfies

$$|S_k| \leq M - \sum_{j \in \mathcal{S}, j \neq k} |S_j| \tag{10}$$

It follows that the number of allocated modes is upperbounded by the number of transmit antennas: $\sum_{k \in \mathcal{S}} |S_k| \leq M$. We note that it is possible to allocate all the modes if the channels are statistically independent.

We recall that in the block diagonalization scheme [3, 4, 5] the following constraints have to be satisfied in the construction of the precoding matrices

$$\sum_{j \in \mathcal{S}, j \neq k} N = N(|\mathcal{S}| - 1) < M \tag{11}$$

whereas in the block diagonalization scheme with receive antenna selection [6] the constraints become less restrictive

$$\sum_{j \in \mathcal{S}, j \neq k} N'_j < M \quad \forall k \in \mathcal{S} \tag{12}$$

where $N'_k \leq N$ is the number of receive antennas selected for the kth user. We note that (9) is similar to (12) except that instead of using a subset of receive antennas we use a subset of eigenmodes.

The kth user's precoder matrix is given by $\mathbf{G}_k = \tilde{\mathbf{V}}_k^0 \mathbf{C}_k$, where $\mathbf{C}_k \in \mathbb{C}^{(M - \sum_{j \in \mathcal{S}, j \neq k} |S_j|) \times |S_k|}$ is determined later. Note that since $\tilde{\mathbf{H}}_k \tilde{\mathbf{V}}_k^{(0)} = \mathbf{0}$ for all $k \in \mathcal{S}$, it follows that $\mathbf{\Gamma}_k \mathbf{G}_j = \mathbf{\Gamma}_k \tilde{\mathbf{V}}_j^{(0)} \mathbf{C}_j = \mathbf{0}$ for $j \neq k$ and any choice of $\mathbf{C}_j$. Therefore from (6), the received signal for the kth user after combining contains no interference:

$$\mathbf{r}_k = \mathbf{\Gamma}_k \mathbf{G}_k \mathbf{d}_k + \mathbf{n}'_k. \tag{13}$$

We perform an SVD

$$\mathbf{\Gamma}_k \tilde{\mathbf{V}}_k^{(0)} = \overline{\mathbf{U}}_k \left[\overline{\mathbf{\Sigma}}_k \; \mathbf{0}\right] \left[\overline{\mathbf{V}}_k^{(1)} \overline{\mathbf{V}}_k^{(0)}\right]^H, \tag{14}$$

where $\overline{\mathbf{\Sigma}}_k$ is the $|S_k| \times |S_k|$ diagonal matrix of eigenvalues, and assign $\mathbf{C}_k = \overline{\mathbf{V}}_k^{(1)}$. From (13), the resulting weighted rate for the kth user is

$$\alpha_k \sum_{j \in S_k} \log\left(1 + \overline{\sigma}_j^{(k)^2} w_j^{(k)}\right), \tag{15}$$

where $\overline{\sigma}_j^{(k)^2}$ is the jth diagonal element of $\overline{\mathbf{\Sigma}}_k^2$ ($j \in S_k$), $\mathbf{W}_k$ is the $|S_k| \times |S_k|$ diagonal matrix of powers allocated to the eigenmodes, and $w_j^{(k)}$ is the jth diagonal element. Therefore the total transmitted power for this user is $tr\left[\mathbf{G}_k \mathbf{W}_k \mathbf{G}_k^H\right] = tr\mathbf{W}_k$, and the mth antenna transmitted power for this user is $\sum_{j=1}^{|S_k|} \left|g_{mj}^{(k)}\right|^2 w_j^{(k)}$ where $g_{mj}^{(k)}$ is the (m,j)th element of $\mathbf{G}_k$. For a given selection of users and eigenmodes $\mathcal{T}$, as determined by $\mathcal{S}$ and S_k for $k \in \mathcal{S}$, the power allocation problem under PAPC can be written as

$$R(\mathcal{T}) \quad = \max_{w_j^{(k)}, k \in \mathcal{S}, j \in S_k} \sum_{k \in \mathcal{S}} \alpha_k \sum_{j \in S_k} \log\left(1 + \overline{\sigma}_j^{(k)^2} w_j^{(k)}\right) \tag{16}$$

$$\text{subject to} \qquad \begin{cases} w_j^{(k)} \geq 0, \quad k \in \mathcal{S}, j \in S_k \\ \sum_{k \in \mathcal{S}} \sum_{j \in S_k} |g_{mj}^{(k)}|^2 w_j^{(k)} \leq P_m, \; m = 1, \ldots, M \end{cases}$$

Problem (16) is a convex optimization problem and can be solved using an interior point method based algorithm. In the SPC case, the M individual power constraints are replaced by a sum-power constraint

$$\sum_{m=1}^{M} \sum_{k \in \mathcal{S}} \sum_{j \in S_k} |g_{mj}^{(k)}|^2 w_j^{(k)} \leq \sum_{m=1}^{M} P_m, \tag{17}$$

and the resulting optimization can be solved using waterfilling. Because the SPC is less restrictive, the weighted sum rate under SPC is equal or better than the PAPC performance for any given channel realization and user/eigenmode assignment.

We emphasize that the optimization (16) is performed for a given user and eigenmode allocation. The allocation itself could be performed in a brute-force manner by considering all possible sets of up to M eigenmodes. Due to the high computational complexity of the brute-force case (see [9]) we propose a generalization of the greedy allocation algorithm proposed in [8]. We define $\mathcal{T}_A$ to be the set of all K users' eigenmodes. Assuming $N < M$, each user has at most N eigenmodes, and there are a total of KN eigenmodes in set $\mathcal{T}_A$. On the jth iteration, we let t_j be the candidate eigenmode chosen among any of the available eigenmodes from any user.

initialization. Let $j = 1$, $\mathcal{T}_0 = \emptyset$, $R(\emptyset) = 0$, and $Done = 0$.
while $(j \leq \min(KN, M))$ and (not $Done$)
  find $t_j = \underset{t \in \mathcal{T}_A \setminus \mathcal{T}_{j-1}}{\arg\max} \; R(\mathcal{T}_{j-1} \cup \{t\})$
  if $R(\mathcal{T}_{j-1} \cup \{t_j\}) < R(\mathcal{T}_{j-1})$
    $\mathcal{T}_j = \mathcal{T}_{j-1}$
    $Done = 1$
  else
    $\mathcal{T}_j = \mathcal{T}_{j-1} \cup \{t_j\}$
    $j = j + 1$
  end
end
$\mathcal{T} = \mathcal{T}_j$

On the first iteration, the selected eigenmode t_1 will be the globally dominant eigenmode. In other words, its eigenvalue is the largest among all users' modes. Note however that the chosen set $\mathcal{T}$ will not necessarily contain the dominant eigenmodes of each user. Note also that not all eigenmodes will necessarily be active. Numerical examples in Section 5 show the distribution of allocated eigenmodes. While this greedy algorithm is suboptimum, we feel that it achieves a good balance between performance and complexity. It is also totally flexible in that it can handle any combination of M, K, and N.

4. ASYMPTOTIC OPTIMALITY

In this section we study the asymptotic behavior of the BD scheme [4] with SPC and PAPC in the limit of large K, when a receive antenna selection scheme similar to the one proposed in [6] is used. The main result is given in Theorem 2, where we prove that the BD scheme, with a particular receive antenna selection scheme, is asymptotically optimal in the sense that the ratio of the expected sum-rate capacities between it and DPC approaches one. In Theorem 3 we extend the result to the PAPC case. We recall the results obtained for the case $N = 1$ in [10] for the SPC and in [9] for the PAPC. We let R_{DPC}, R_{ZFSPC} and R_{ZFPAPC} respectively denote the sum rate capacities achieved with DPC (under SPC), ZF under SPC, and ZF under PAPC.

Theorem 1 *In the limit of large K, the zero-forcing beamformer under both a sum power constraint and a per-antenna power constraint can achieve an expected sum-rate equal to that of DPC*[1]

$$\mathcal{E}\{R_{ZFSPC}\} \sim M \log\left(1 + \frac{P}{M}\log K\right) \sim \mathcal{E}\{R_{DPC}\}. \quad (18)$$

As generalization of Theorem 1 for the case $N > 1$, we can give the following result

Theorem 2 *In the limit of large K, the BD scheme [4] under a sum power constraint can achieve an expected sum-rate equal to that of DPC, with a greedy receive antenna selection scheme.*

$$\mathcal{E}\{R_{BD_{RAS}}\} \sim M \log\log NK \sim \mathcal{E}\{R_{DPC}\} \quad (19)$$

Proof We first obtain a lower bound to the expected sum-rate of the BD scheme. Let consider a given set of user $\mathcal{S}$ each one with N_k antennas, and a set $\mathcal{S}'$ of $\sum_{k \in \mathcal{S}} N_k$ "virtual users" obtained by considering not collaborating the receive antennas of each user. The following theorem gives a lower bound for the sum-rate of the BD scheme with the given set of users $\mathcal{S}$.

Lemma 1 *Let suppose that the conditions to apply the BD scheme [4] on the set $\mathcal{S}$ and the conditions to apply the ZF-SPC scheme on the set $\mathcal{S}'$ are verified. Hence*

$$R_{BD}(\mathcal{S}) \geq R_{ZFSPC}(\mathcal{S}') \quad (20)$$

Proof >From [4] we know that the precoding matrix associated to the kth user has to lie in the null space of $\tilde{\mathbf{H}}_k$, where

$$\tilde{\mathbf{H}}_k = \left[\mathbf{H}_1^H, \ldots, \mathbf{H}_{k-1}^H, \mathbf{H}_{k+1}^H, \ldots, \mathbf{H}_{|\mathcal{S}|}^H\right]^H \quad (21)$$

Let apply the ZF-SPC precoder to the set $\mathcal{S}'$. Let l the index of the virtual user corresponding to the ith receive antenna of the kth user. The lth column of Moore-Penrose pseudoinverse associated to the selected set of user has to lie in the null space of $\tilde{\mathbf{H}}_{(k,i)}$,

$$\tilde{\mathbf{H}}_{(k,i)} = \left[\tilde{\mathbf{H}}_k^H \quad \mathbf{H}_k^H\left([1 : i-1, i+1 : N_k], :\right)\right]^H \quad (22)$$

where the used MATLAB notation. From (21) and (22), we note that

$$\mathcal{N}\left(\tilde{\mathbf{H}}_{(k,i)}\right) \subseteq \mathcal{N}\left(\tilde{\mathbf{H}}_k\right) \quad (23)$$

and therefore the BD algorithm has a number of degrees of freedom for the design of the precoding matrices that is greater with respect to the ZF-SPC scheme. □

Let's consider now the original set of K users each with N antennas, and apply Theorem 1 to the virtual system composed by NK single antenna users. In the limit of large K

$$\mathcal{E}\{R_{ZFSPC}\} \sim M \log\log NK \quad (24)$$

Let S_{opt} the set of virtual users selected with the greedy algorithm proposed in [10]. Hence if the selected virtual users are associated to different users, we apply the BD scheme by considering $|S_{opt}|$ users each one using only one receive antenna, otherwise we permit collaboration between the receive antennas of a given user associated to the selected virtual-users. >From Lemma 1, in the limit of large K

$$\mathcal{E}\{R_{ZFSPC}\} \leq \mathcal{E}\{R_{BD_{RAS}}\} \quad (25)$$

An upper bound to $\mathcal{E}\{R_{BD_{RAS}}\}$ can be obtained by considering that

$$R_{BD_{RAS}} \leq R_{DPC} \quad (26)$$

and in the limit of large K [11]

$$\mathcal{E}\{R_{DPC}\} \sim M \log\log KN \quad (27)$$

From (25) and (27)

$$\mathcal{E}\{R_{BD_{RAS}}\} \sim M \log\log KN. \quad (28)$$

□

Theorem 2 can be extended to the PAPC case as follows

Theorem 3 *In the limit of large K, the BD scheme [4] under a per antenna power constraint can achieve an expected sum-rate equal to that of DPC, with a greedy receive antenna selection scheme.*

$$\mathcal{E}\{R_{BD_{RAS-PAPC}}\} \sim M \log\log NK \sim \mathcal{E}\{R_{DPC}\} \quad (29)$$

Proof *The proof is essentialy the same of the one used for Theorem 2, with the difference that for the lower bound is used the result obtained for the ZF-PAPC in [9].* □

[1] $x \sim y$ indicates that $\lim_{K \to \infty} x(K)/y(K) = 1$.

5. SIMULATION RESULTS

We assume an independent and identically distributed complex Gaussian channel ($h_{km} \sim \mathcal{CN}(0,1)$) where the channel matrix $\mathbf{H}_k$ is assumed to be perfectly known both at the transmitter and at the kth receiver. For BD we assume that a greedy user selection (GUS) algorithm is used [12]. We consider two types of BD: one where each selected user employs all N antennas (BD-GUS) and another with receive antenna selection (BD-RAS). For BD-RAS we use a modified version of GUS where each candidate user selects the best subset of N receive antennas. For BD-GUS, BD-RAS, and DPC, we assume a sum-power constraint. For MET, we consider both PAPC and SPC. In Figure 1 we compare the average sum-rate versus SNR of the aforementioned structures, for $M = 4$, $N = 4$ and $K = 20$. The MET-SPC gives the best performance among the linear beamformer options. For SNR=10 dB MET-SPC achieves about 90% of

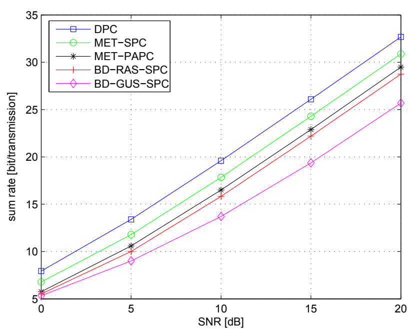

Fig. 1. Average sum rate (bits/transmission) versus SNR, for $N = M = 4$ antennas and $K = 20$ users.

the DPC sum rate. Moreover, MET-PAPC performs better than both BD-GUS and BD-RAS over the range of SNRs. In order to evaluate the effectiveness of the eigenmode selection scheme, in Figure 2 a bar diagram of the average use (in percentages) of the different modes is shown for a single user, with $M = 4$ and $M = 12$, $N = 4$, $K = 20$, and different values of SNR, for respectively the MET-SPC. The modes are ordered according to their powers so that mode

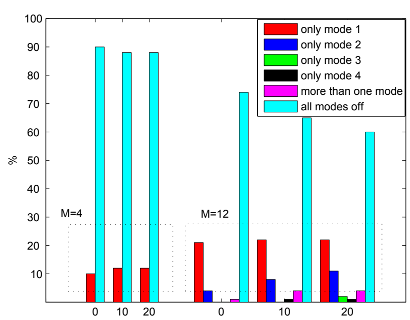

Fig. 2. Average use (in percent) of the different modes for a single user, with $M = 4$ and $M = 12$, $N = 4$, $K = 20$, and different values of SNR, for the MET-SPC.

1 is the largest. For the case $M = 4$, the greedy algorithm almost always chooses mode 1 for each served user. Therefore, even though multiple streams could be sent to a single user (and these streams could be jointly detected), transmitting a single stream to multiple users results in higher throughput. In other words, SDMA using MET is more efficient than time-multiplexing multi-stream transmissions to a single user. This observation holds even for higher SNRs where single-user spatial multiplexing is more efficient. For larger M there may be more eigenmodes to choose from, and in this case, the greedy algorithm sometimes transmits on modes other than the first. For example in Figure 2, we can see that for $M = 12$, the eigenmodes 2, 3 and 4 can be chosen without the allocation of the eigenmode 1. Moreover, when the number of users increases, the probability that more than one mode is used for a given user is small, for both low and high SNR. Therefore in a multiuser scenario, allocating the dominant eigenmodes as done in [4] and [7] or selecting the users without considering the problem of the eigenmode allocation [12] are suboptimum policies when M is large.

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