USER SELECTION FOR THE MIMO BROADCAST CHANNEL WITH A FAIRNESS CONSTRAINT

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ABSTRACT

In this paper, we present a user selection scheme with a fairness constraint when zero-forcing beamforming (ZFBF) transmission is employed in a multiple-input multiple-output (MIMO) broadcast channel. This problem can be reduced to the maximization of a weighted sum rate with a power constraint. In this paper, we first derive a lower bound on the weighted sum rate. Roughly speaking, this bound is inversely proportional to the Frobenius norm of the inverse of the weighted composite channel matrix. We then present a greedy algorithm to select the users that maximize this lower bound, using the same technique in [1]. Finally, we show that this approach achieves the same scaling law as DPC does as presented in [2]. Simulation suggests that this method achieves comparable or better performance than most existing schemes in the moderate to high SNR realm with comparable complexity.

Index Terms— MIMO systems, zero-forcing beamforming, broadcast channel, user selection

I. INTRODUCTION

For multiple input multiple output (MIMO) broadcast channels, where the access point (AP) is equipped with M transmit antennas and each of the $K \geq M$ users has a single receive antenna, zero-forcing beamforming (ZFBF) proposed in [3] has attracted considerable attention in recent years due to its relative simplicity compared to dirty paper coding (DPC) [4]. By inverting the channel matrix at the transmitter side, a number of orthogonal channels can be created to support independent data streams simultaneously without the interference between different users. Since the number of users ZFBF can optimally support at the same instant is no larger than the number of transmit antennas, user scheduling is always a must when the number of users is large. It has been demonstrated that, coupled with user selection, ZFBF attains the full multiplexing gain M and the multiuser diversity $\log\log K$ asymptotically [5]–[7], which is precisely what DPC can achieve [2].

In practical wireless systems, fairness among all the users is always an important issue that deserves full attention. One criteria of this fairness is the average throughput of each user [8]. On the other hand, in packet-based wireless networks, each user is often associated with a queue where the packets for that user arrive randomly. In [9], the *network capacity region* is defined as the region of stabilizable input data rates. This region can be achieved by the strategy of *maximum-weight matching* where the weights are related to queue sizes. User selection for MIMO broadcast channels with a fairness constraint has been addressed in [6], [10].

Since an exhaustive search among all the channel vectors of all the users is infeasible, in this paper, we present a user selection scheme with a fairness constraint where the ZFBF transmission is assumed. We first derive a lower bound on the weighted sum rate maximization problem. Roughly speaking, this bound is inversely proportional to the Frobenius norm of the inverse of the weighted channel matrix. We then present a greedy algorithm to choose the users that maximize this lower bound, using the same technique in [1]. Finally, we show that this approach achieves the same scaling law as DPC does [2].

The organization of this paper is as follows. Section II gives the system model. Section III provides with some background on ZFBF and user selection with a fairness constraint. Section IV presents the user selection algorithm and its performance analysis. Simulation results are given in Section V. Section VI concludes the paper.

We use lowercase boldface letters to denote vectors and uppercase bold letters to denote matrices. $\|\cdot\|$ denotes the norm of a vector, and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. $(\cdot)^T$ denotes matrix transposition, $(\cdot)^H$ denotes the matrix Hermitian transpose, $\mathbb{E}\{\cdot\}$ denotes expectation, and $|\cdot|$ denotes the absolute value.

II. SYSTEM MODEL

We assume the base station is equipped with M antennas, each user has one antenna, and $K > M$ users are receiving signals from the base station. The received signal y_k at user k is determined by

$$y_k = \boldsymbol{h}_k \boldsymbol{x} + n_k, \qquad (1)$$

for $k = 1, \ldots, K$, where $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal, $\boldsymbol{h}_k \in \mathbb{C}^{1\times M}$ represents the multiple-input-single-output (MISO) channel from the base station to user k, and $\{n_k\}$ are i.i.d complex Gaussian noise terms with unit variance. The power constraint for the input signal is $\mathbb{E}[\boldsymbol{x}^H\boldsymbol{x}] = P$. We assume that the transmit antennas and users are sufficiently spaced apart such that the entries of $\boldsymbol{h}_k$, for $k = 1, \ldots, K$, can be modeled as a set of $i.i.d.$ zero-mean circularly symmetric complex Gaussian random variables. Without loss of generality, we assume that these entries

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have unit variance (i.e., $\boldsymbol{h}_k$ is distributed as $\mathcal{CN}(0, \boldsymbol{I})$) and that the channel is constant for multiple transmission epochs before changing independently. Throughout this paper, we assume that the base station has perfect channel state information (CSI) of all of the downlink channels $\boldsymbol{h}_k$, $k = 1, \ldots, K$. However, each user only has the CSI of its own downlink channel and does not know the CSI of the downlink channel of other users. Furthermore, there is no collaboration between users in decoding the signal.

III. BACKGROUND

III-A. Zero-Forcing Beamforming for the Broadcast Channel

Consider a subset of users $\mathcal{A}$, where $\mathcal{A} \subset \{1, \ldots, K\}$ and $|\mathcal{A}| = n \leq M$. Denote the channel vectors of those users as $\boldsymbol{h}_{i_1}, \ldots, \boldsymbol{h}_{i_n}$. Stacking them on top of each other, we get the $n \times M$ composite channel matrix

$$\boldsymbol{H}_{\mathcal{A}} = \left[\boldsymbol{h}_{i_1}^T, \cdots, \boldsymbol{h}_{i_n}^T\right]^T.$$

Let the columns of the Moore-Penrose inverse of $\boldsymbol{H}_{\mathcal{A}}$ be $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$, i.e.,

$$\boldsymbol{H}_{\mathcal{A}}^{\dagger} = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_n].$$

Denote

$$\tilde{\boldsymbol{v}}_k = \frac{\boldsymbol{v}_k}{\|\boldsymbol{v}_k\|}.$$

We can use $\{\tilde{\boldsymbol{v}}_k\}$ as beamforming vectors for the selected users. Specifically, the transmitted signal $\boldsymbol{x}$ is constructed as

$$\boldsymbol{x} = \sum_{k=1}^{n} \sqrt{P_{i_k}} \tilde{\boldsymbol{v}}_k s_{i_k}. \tag{2}$$

Assuming the data streams for different users are independent to each other, the power constraint $\mathrm{E}\left[\boldsymbol{x}^H \boldsymbol{x}\right] = P$ reduces to $\sum_{k=1}^{n} P_{i_k} = P$. Note that

$$\boldsymbol{h}_{i_k} \tilde{\boldsymbol{v}}_j = \begin{cases} \frac{1}{\|\boldsymbol{v}_k\|} & k = j \\ 0 & k \neq j \end{cases}.$$

In effect, using ZFBF decomposes the MIMO broadcast channel into n subchannels without cross channel interference. Additionally, the kth channel has an equivalent channel gain of $\frac{1}{\|\boldsymbol{v}_k\|}$.

The received signal for user i_k is

$$\begin{aligned} y_{i_k} &= \boldsymbol{h}_{i_k} \boldsymbol{x} + n_{i_k} \\ &= \frac{\sqrt{P_{i_k}}}{\|\boldsymbol{v}_k\|} s_{i_k} + n_{i_k}. \end{aligned} \tag{3}$$

The sum rate for the set $\mathcal{A}$ is

$$\begin{aligned} R^{BF}(\mathcal{A}) &= \max_{\sum_{k=1}^{n} P_{i_k} = P} \sum_{k=1}^{n} \log_2 \left(1 + \frac{P_{i_k}}{\|\boldsymbol{v}_k\|^2}\right) \\ &= \max_{\sum_{k=1}^{n} P_{i_k} = P} \sum_{k=1}^{n} \log_2 \left(1 + P_{i_k} \gamma_{i_k}\right), \end{aligned} \tag{4}$$

where $\gamma_{i_k} = \frac{1}{\|\boldsymbol{v}_k\|^2}$ is the SNR of the kth subchannel with unit power. The optimal power allocation that achieves the maximum sum rate is given by a water-filling scheme,

$$\begin{aligned} P_{i_k} &= \left(\mu - \frac{1}{\gamma_{i_k}}\right)^+ \\ &= \left(\mu - \|\boldsymbol{v}_k\|^2\right)^+ \end{aligned} \tag{5}$$

and

$$\sum_{k=1}^{n} P_{i_k} = P, \tag{6}$$

where $(z)^+$ denotes $\max(z, 0)$ and μ is called *water level*.

III-B. User Selection with ZFBF

Most previous approaches to user selection with ZFBF can be formulated as follows. Among the K users, find a subset of users $\mathcal{A} \subset \{1, \ldots, K\}$ such that $|\mathcal{A}| \leq M$ and

$$R^{BF}(\mathcal{A}) = \max_{\mathcal{A}' \subset \{1, \ldots, K\}, |\mathcal{A}'| \leq M} R^{BF}(\mathcal{A}').$$

However, in practical situations, it is more suitable to impose a proportional fairness constraint [8] or queue stability constraint [10]. Note that both proportional fairness and queue stability are defined over a certain time interval. Both of these two problems can be generalized to the weighted sum-rate maximization problem [6]

$$\max_{\mathcal{A}' \subset \{1, \ldots, K\}, |\mathcal{A}'| \leq M} \max_{\sum_{k \in \mathcal{A}'} P_k(t) = P} \sum_{k \in \mathcal{A}'} \alpha_k(t) R_k^{BF}(\mathcal{A}', t) \tag{7}$$

where the $\alpha_k(t)$'s are the weights and

$$R_k^{BF}(\mathcal{A}', t) = \log_2 \left(1 + P_k(t) \gamma_k(t)\right)$$

is the rate for user k at time instant t when the subset $\mathcal{A}'$ is selected. For fairness, $\alpha_k(t)$ can be chosen to be the reciprocal of past throughput of user k [8] or the queue length of user k [10].

For notational simplicity, we can drop the time index, and (7) is reduced to

$$\max_{\mathcal{A}' \subset \{1, \ldots, K\}, |\mathcal{A}'| \leq M} \max_{\sum_{k \in \mathcal{A}'} P_k = P} \sum_{k \in \mathcal{A}'} \alpha_k \log_2 \left(1 + \frac{P_k}{\|\boldsymbol{v}_k\|^2}\right). \tag{8}$$

IV. THE ALGORITHM

IV-A. A Lower Bound on the Sum Rate

The main result in this section is the following theorem.

Theorem 1: *Given a set $\mathcal{A} = \{i_1, \ldots, i_n\} \subset \{1, 2, \ldots, K\}$ of users, where $n \leq M$, the weighted ZFBF sum rate $R^{\mathrm{BF}}(\mathcal{A}; P_{\mathcal{A}})$ with power allocation $P_{\mathcal{A}} = \{P_{i_1}, \ldots, P_{i_n}\}$ and weights $\{\alpha_{i_1}, \ldots, \alpha_{i_n}\}$ has a lower bound, that is,*

$$R^{\mathrm{BF}}(\mathcal{A}; P_{\mathcal{A}}) \geq \alpha \log_2 \left(1 + \frac{\alpha}{\|\widetilde{\boldsymbol{H}}_{\mathcal{A}}^{\dagger}\|_F^2}\right), \tag{9}$$

where $\alpha = \sum_{k=1}^{n} \alpha_{i_k}$ and

$$\widetilde{\boldsymbol{H}}_{\mathcal{A}} = \mathrm{diag}\left\{\sqrt{\frac{P_{i_1}}{\alpha_{i_1}}}, \ldots, \sqrt{\frac{P_{i_n}}{\alpha_{i_n}}}\right\} \boldsymbol{H}_{\mathcal{A}} = \begin{bmatrix} \sqrt{\frac{P_{i_1}}{\alpha_{i_1}}} \boldsymbol{h}_{i_1} \\ \vdots \\ \sqrt{\frac{P_{i_n}}{\alpha_{i_n}}} \boldsymbol{h}_{i_n} \end{bmatrix}$$

This result is immediate from the two lemmas that follow.

Lemma 1: *The weighted arithmetic mean is greater than or equal to the weighted geometric mean and the weighted geometric mean is greater or equal to the weighted harmonic mean, i.e.,*

$$\begin{aligned} A_w(a_1, a_2, \ldots, a_n) &\geq G_w(a_1, a_2, \ldots, a_n) \\ &\geq H_w(a_1, a_2, \ldots, a_n). \end{aligned} \tag{10}$$

Lemma 2: *For the weighted harmonic mean $H_w(a_1, \ldots, a_n)$, we have*

$$H_w(a_1+c, a_2+c, \ldots, c_n+c) \geq c + H_w(a_1, a_2, \ldots, a_n) \quad (11)$$

for any constant positive number c.

Sketch of proof: In fact, let $H(x) = H_w(a_1+x, a_2+x, \ldots, a_n+x)$. We can prove

$$H'(x) = \frac{dH}{dx} \geq 1 \quad (12)$$

for all positive x. Applying the mean value theorem yields the lemma. □

From the proof, we have the following corollaries.

Corollary 1: *Given a set $\mathcal{A} = \{i_1, \ldots, i_n\} \subset \{1, 2, \ldots, K\}$ of users, where $n \leq M$, the weighted ZFBF sum rate, which is maximized over all possible power allocations, has a lower bound for all power allocation schemes given by*

$$R^{BF}(\mathcal{A}) \quad \geq \quad \alpha \log_2 \left(1 + \frac{\alpha P}{n \|\hat{\boldsymbol{H}}_{\mathcal{A}}^{\dagger}\|_F^2}\right) \quad (13)$$

where

$$\hat{\boldsymbol{H}}_{\mathcal{A}} = \mathrm{diag}\left\{\frac{1}{\sqrt{\alpha_1}}, \ldots, \frac{1}{\sqrt{\alpha_n}}\right\} \boldsymbol{H}_{\mathcal{A}}.$$

Note that when $\alpha_i = 1$ for $i = 1, \ldots, n$, we have following corollary.

Corollary 2: Let $\alpha_i = 1$ for $i = 1, \ldots, n$ and the smallest singular values of $\boldsymbol{H}_{\mathcal{A}} \boldsymbol{H}_{\mathcal{A}}^H$ be $\lambda_{\min}$, then

$$R^{\mathrm{BF}}(\mathcal{A}) \quad \geq \quad n \log_2 \left(1 + \frac{P}{\|\boldsymbol{H}_{\mathcal{A}}^{\dagger}\|_F^2}\right) \quad (14)$$

$$\geq \quad n \log_2 \left(1 + \frac{P}{n} \lambda_{\min}\right). \quad (15)$$

IV-B. The MFNPI Algorithm for User Selection

In [11], the authors propose to perform user selection based on maximizing the Frobenius norm of the composite channel matrix. However, Corollary 1 suggests it is more justified to work on the pseudo-inverse of the weighted composite channel matrix $\hat{\boldsymbol{H}}_{\mathcal{A}}$. In other words, it provides us with a way to do user selection, that is, to find the set of users that minimize the Frobenius norm of the weighted composite channel matrix. This method involves the calculation of the inverse of the weighted composite channel matrix. As proposed in [1], there is an efficient way to calculate this in a sequential manner. In the remainder of this section, for simplicity, we only consider the second method under the condition where $\alpha_i = 1$ for $i = 1, \ldots, K$. With this condition, we develop a user selection algorithm based on Minimization of the Frobenius Norm of the Pseudo-Inverse (MFNPI) of the composite channel matrix. We can show that this algorithm achieves the same scaling laws as DPC.

Theorem 2: *For M and P fixed, the sum-rate of the MFNPI algorithm scales as*

$$\lim_{K \to +\infty} \frac{\mathrm{E}\{R_{\mathrm{MFNPI}}\}}{M \log \log K} = 1 \quad (16)$$

Sketch of proof: The proof uses (14) and the fact that the distribution of the smallest eigenvalue of a Wishart matrix is exponentially distributed [12]. For a sequence of exponential random variables $\{X_1, \ldots, X_K\}$ that satisfies certain dependency constraints, $\max_{i=1,\ldots,K} X_i$ still scales like $\log K$ when K is large [13]. On the other hand, the sum rate of any transmission scheme for broadcast channels is bounded by the DPC sum rate which obeys the scaling law of $M \log \log K$. □

IV-C. Procedure

In this section, the procedure of the MFNPI algorithm is included.

- *Step 1:* Initialization: Find the channel vector that has the largest norm, i.e.,

  $$j_1 = \arg \max_{i=1,\ldots,K} \|h_i\|^2.$$

  Perform the initialization as

  $$\begin{aligned} J &\leftarrow j_1, \\ \Omega &\leftarrow \{1, \ldots, j_1 - 1, j_1 + 1, \ldots, K\}. \end{aligned}$$

  where J contains the indices of the selected users and Ω is termed as *candidate set* which contains the indices of the users that have not been selected and are eligible for selection.

At the mth iteration:

- *Step 2:* Find the user $j_m \in \Omega$, such that it minimizes the Frobenius norm of the inverse of the composite channel matrix for the users in $J \cup \{j_m\}$. In the process, we use the technique in [1] to calculate the inverse of the composite channel matrix:

  $$\begin{aligned} J &\leftarrow J \cup \{j_m\}, \\ \Omega &\leftarrow \Omega - \{j_m\}, \\ m &= m + 1. \end{aligned}$$

  if $n < M$, go to Step 2. Otherwise, go to Step 3.
- *Step 3:* Perform water-filling on the set of selected users.

IV-D. Complexity Analysis

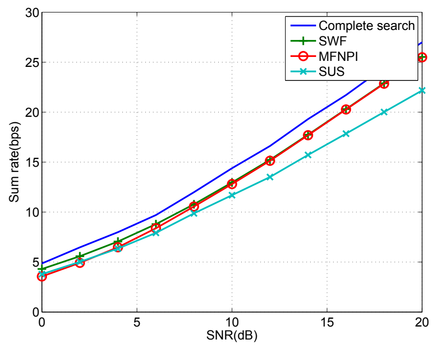
As in [1], the sequential procedure to calculate the Penrose-Moore inverse of a matrix dominates the computation. So the complexity of MFNPI should be also on the order of KM^3. However, for sequential water-filling (SWF), as well as in [14], for each user at iteration n, there are n logarithm operations. In total, if not considering search space pruning, there are roughly $\sum_{n=2}^{M} Kn \approx KM^2$ logarithm operations. On the contrary, for MFNPI, there is no logarithm operation involved. This results in significant complexity reduction.

V. SIMULATION RESULTS

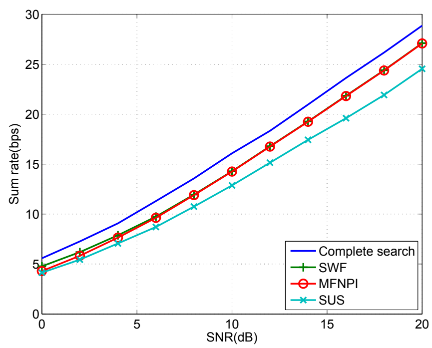
In this section, the performance of the MFNPI algorithm for the equal-weight case is compared with the SWF algorithm [1] and the semiorthogonal user selection (SUS) algorithm [6].

Figure 1 shows that the resultant sum rate of SWF, MFNPI, SUS and complete search ZFBF versus different SNRs. One can see at low SNR, the MFNPI algorithm achieves a lower sum rate than SWF. However, when the SNR is moderately large, e.g., 10dB, the performance of MFNPI and SWF is very close. Moreover, comparing (a) and (b), one can see that at lower SNR, the MFNPI algorithm will outperform the SUS algorithm when the number of users increases.

In Figure 2, the ZFBF sum rate of SWF, MFNPI, SUS and complete search ZFBF are plotted against different number of users. Two transmit SNRs, 2dB and 20dB, are considered in this simulation. One can see that at high transmit SNR (20dB), MFNPI and SWF have close performance and achieve a sum rate around 3bps higher than that of the SUS algorithm. When the transmit SNR is reduced to 2dB and the number of users is small, one can see that the MFNPI algorithm performance worse than the SUS. However, it outperforms SUS and eventually approaches the SWF algorithm as the number of users increases.



(a) Sum rate vs. SNR for 16 users



(b) Sum rate vs. SNR for 32 users

Fig. 1. The sum rate of SWF, MFNPI, SUS, and best sum rate of ZFBF with different transmit SNR. There are 4 transmit antennas.

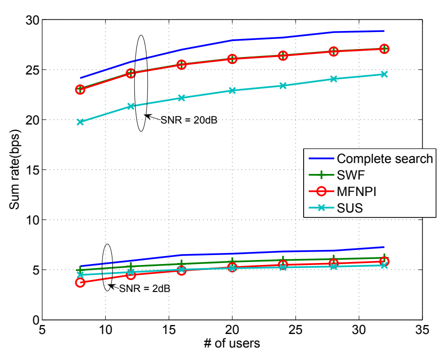


Fig. 2. The ZFBF sum rate of SWF, MFNPI, SUS, and the complete search with different different number of users. There are 4 transmit antennas.

VI. CONCLUSION

In this paper, assuming ZFBF transmission, we proposed a new user selection method to maximize the weighted sum rate based on minimizing the Frobenius norm of the inverse of the composite channel matrix. At moderate to high SNR, the performance of this method is very close to SWF [1] which has the same performance as the greedy method proposed in [14] but with less computation. As the number of users increase, it exhibits the same scaling law of optimal DPC.

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