ITERATIVE WATERFILLING FOR WEIGHTED RATE SUM MAXIMIZATION IN MIMO-OFDM BROADCAST CHANNELS

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ABSTRACT

We study the maximization of the weighted sum rate in Gaussian multi-input multi-output OFDM broadcast channel under a total power constraint. This problem is motivated by adaptive resource allocation policies in a multi-carrier wireless system with multiple antennas at the base station. We propose a iterative waterfilling algorithm based on dual decomposition. Two decompositions are considered, one in subcarrier domain and another in both subcarrier and user domain. We show that both decompositions reduce to an identical problem that can be solved by multiuser waterfilling approach simultaneously for all subcarriers. A master problem is solved iteratively to achieve the total power constraint by a simple bisection method. Numerical examples show that our proposed algorithm converges much faster than steepest ascent algorithm and makes convergence almost independent of a number of subcarriers and antennas.

Index Terms— dual decomposition, convex optimization, OFDM, weighted sum rate, iterative algorithms.

1. MOTIVATION

We consider the downlink of a wireless system where the transmitter equipped with M antennas serves K receivers with a single antenna each. Assuming a frequency selective fading channel, we apply orthogonal frequency division multiplexing (OFDM) with N subcarriers to convert the frequency selective channel into N parallel frequency-flat channels. The corresponding system is modeled as the discrete Gaussian multiple input multiple output OFDM broadcast channel (MIMO-OFDM BC), given by

$$y_{k,n} = \mathbf{h}_{k,n}^H \mathbf{x}_n + n_{k,n} \tag{1}$$

where $\mathbf{x}_n \in \mathbb{C}^M$ denotes the transmit signal vector on subcarrier n, $\mathbf{h}_{k,n} \in \mathbb{C}^M$ denotes the channel complex vector of user k on subcarrier n, and $\{n_{k,n}\}$ is an independent identically distributed (i.i.d.) sequence of AWGN $\in \mathcal{N}(0,1)$. The input is subject to the total power constraint $\sum_{n=1}^N \mathbb{E}[||\mathbf{x}_n||^2] \leq P$. The dual uplink channel is the MIMO-OFDM multiple access channel (MAC) where K single-antenna transmitters communicate with a receiver equipped with M antennas. The received signal on subcarrier n is given by

$$\mathbf{r}_n = \sum_{k=1}^K \mathbf{h}_{k,n} s_{k,n} + \mathbf{w}_n \tag{2}$$

where $s_{k,n}$ denotes the symbol of user k on subcarrier n, $\mathbf{w}_n$ is AWGN with i.i.d. components $\sim \mathcal{CN}(0,1)$, and the same total power constraint is imposed, i.e. $\sum_k \sum_n \mathbb{E}[|s_{k,n}|^2] \leq P$. From the standard results of the uplink-downlink duality [1, 2], it is straightforward to show that the capacity regions of the MIMO-OFDM BC (1) and of the MIMO-OFDM MAC (2) coincide and are given by

$$\mathcal{C}(\mathbf{H}; P) = \bigcup_{\sum_k \sum_n p_{k,n} \leq P} \{\mathbf{R} \in \mathbb{R}^K \quad : \forall \mathcal{K} \subseteq \{1, \ldots, K\}$$
$$R_k \leq \frac{1}{N} \sum_{n=1}^N \log \left| \mathbf{I} + \sum_{k \in \mathcal{K}} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^H p_{k,n} \right| \} \tag{3}$$

where $\mathbf{H}$ denotes the sequence of the channel vectors $\{\mathbf{h}_{k,n}\}$ and $p_{k,n}$ denotes the power of user k on subcarrier n. For later use, we let $\mathbf{p}_n = (p_{1,n}, \ldots, p_{K,n})^T$ denote the user power vector on subcarrier n. In this work we assume perfect channel state information at the transmitter (and receivers). Let us consider a set of uplink powers $\{p_{k,n}\}$ satisfying the total power constraint and the decoding order π (where $\pi = (\pi_1, \ldots, \pi_K)$ denotes a permutation of $\{1, \ldots, K\}$ such that user π_K is decoded first and π_1 is decoded last). With this choice, the rate K-tuple given by

$$R_{\pi_k} = \frac{1}{N} \sum_{n=1}^N \log \frac{\left| \mathbf{I} + \sum_{j=1}^k \mathbf{h}_{\pi_j,n} \mathbf{h}_{\pi_j,n}^H p_{\pi_j,n} \right|}{\left| \mathbf{I} + \sum_{j=1}^{k-1} \mathbf{h}_{\pi_j,n} \mathbf{h}_{\pi_j,n}^H p_{\pi_j,n} \right|} \tag{4}$$

for $k = 1, \ldots, K$ is achievable in the uplink by successive decoding. Then, there exists a set of powers $\{q_{k,n}\}$, also satisfying $\sum_k \sum_n q_{k,n} = P$, such that the same rate K-tuple (4) is achievable in the downlink by successive "Dirty-Paper" encoding, by encoding the users in the reverse order. In the following, we use that duality to cast a downlink problem into a uplink problem, easier to handle.

We address the weighted sum rate maximization in the MIMO-OFDM BC, given by

$$\max \sum_{k=1}^K w_k R_k, \quad \text{subject to } \mathbf{R} \in \mathcal{C}(\mathbf{H}; P) \tag{5}$$

where $\{w_k\}$ denotes the sequence of time-varying positive weights and $\{R_k\}$ denotes the sequence of instantaneous user rates. This problem is motivated by adaptive resource allocation policies in typical downlink scenarios such as a queued downlink with random arrival (see [3] and references therein). We aim to propose an efficient and fast algorithm for the problem (5) via dual decomposition. Dual decomposition is a useful tool that can be applied to a convex problem with a coupled constraint connecting variables. Although any convex optimization tool can be applied to our problem, we propose an iterative algorithm that efficiently exploits the problem structure and enables parallel implementation. Moreover, numerical examples show that the proposed algorithm converges much faster than steepest ascent algorithm.

This work was partially funded by the Generalitat de Catalunya under grant SGR2005-00690.

2. ITERATIVE WATERFILLING ALGORITHM

We apply the results of [4] stating that the solution of (5) is always found in the set of successively decodable rate points if the decoding order sorts the weights in non-increasing order such that $w_{\pi_1} \geq w_{\pi_2} \geq \cdots \geq w_{\pi_K}$. Since the decoding order is fixed by the weights, without loss of generality we can consider $\pi_k = k$, i.e., users are decoded in the order K (first), $K-1, \ldots, 1$ (last). Then the maximization problem (5) reduces to :

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \Delta_k \sum_{n=1}^{N} \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^H p_{j,n} \right| \qquad (6) \\ \text{subject to} \quad & \sum_{k} \sum_{n} p_{k,n} \leq P \\ & p_{k,n} \geq 0, \forall k, n \end{aligned}$$

where we let $\Delta_k = w_k - w_{k+1}$ and we define $w_{K+1} = 0$. Notice that by definition of the decoding order, we have $\Delta_k \geq 0$. We remark that the problem (6) is an extension of the weighted rate sum maximization for the MIMO-BC addressed in [3] to a multi-carrier system. The weighted sum rate maximization in MIMO-OFDM BC is addressed in [5] although only a solution for the sum rate maximization is provided. Due to the total power constraint, a direct solution to (6) seems non-trivial since it requires joint optimization of powers over users and subcarriers. Hence, we follow the approach of [6] based on dual decomposition to break the total power constraint and decompose the original problem (6) into a set of subproblems. On top of subproblems, we solve a so-called master problem to satisfy the total power constraint. The dual method has been applied to wired multicarrier systems (see for example [7]). In the following, we consider two different decompositions and show that the both decompositions turn out to yield the same iterative algorithm.

2.1. Dual decomposition in subcarrier domain

By introducing a set of N auxiliary variables $\{C_1, \ldots, C_N\}$, we rewrite the original problem (6) as follows.

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \Delta_k \sum_{n=1}^{N} \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^H p_{j,n} \right| \\ \text{subject to} \quad & \sum_{k=1}^{K} p_{k,n} \leq C_n, \quad n = 1, \ldots, N \\ & \sum_{n=1}^{N} C_n \leq P \\ & p_{k,n} \geq 0, \forall k, n \qquad (7) \end{aligned}$$

We form the Lagrangian of the new problem (7) with respect to the coupled constraint $\sum_{n=1}^{N} C_n \leq P$.

$$\begin{aligned} & L(\mathbf{p}_1, \ldots, \mathbf{p}_N, C_1, \ldots, C_N, \mu) \qquad (8) \\ & = \sum_{k=1}^{K} \Delta_k \sum_{n=1}^{N} \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^H p_{j,n} \right| - \mu \left(\sum_{n=1}^{N} C_n - P \right) \end{aligned}$$

Let us define the master problem as

$$\min_{\mu \geq 0} \mathcal{G}_1(\mu) \qquad (9)$$

where

$$\mathcal{G}_1(\mu) = \max_{\{\mathbf{p}_n, C_n\}_{n=1}^{N}} L(\mathbf{p}_1, \ldots, \mathbf{p}_N, C_1, \ldots, C_N, \mu) \qquad (10)$$

where the constraints are $\sum_k p_{k,n} \leq C_n$ and $p_{k,n} \geq 0, \forall k$ for each n. Notice that the dual function $\mathcal{G}_1(\mu)$ is the pointwise maximum of a family of affine functions of μ, hence it is a convex function (see for example [8]).

Decoupled subproblems First, we have to find $\mathcal{G}_1(\mu)$ for a fixed μ. Due to its separable structure, the dual function (8) can be decomposed into N subproblems and can be computed simultaneously for N subcarriers. This is very appealing because it enables a parallel implementation. We solve the following subproblem for each n.

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^H p_{j,n} \right| - \mu C_n \\ \text{subject to} \quad & \sum_{k=1}^{K} p_{k,n} \leq C_n \\ & p_{k,n} \geq 0, \forall k \qquad (11) \end{aligned}$$

The Lagrangian of the n-th subproblem is given by

$$\begin{aligned} & \mathcal{L}_n(\mathbf{p}_n, C_n, \mu, \nu_n) = \sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n} \mathbf{h}_{j,n}^H p_{j,n} \right| \\ & - \mu C_n - \nu_n \left(\sum_{k=1}^{K} p_{k,n} - C_n \right) \end{aligned}$$

where ν_n denotes a dual variable associated with the constraint $\sum_k p_{k,n} \leq C_n$. Since each subproblem is a convex optimization problem, the KKT conditions are necessary and sufficient for the optimality [8]. By letting $\partial \mathcal{L}_n / \partial p_{k,n} = 0$ and $\partial \mathcal{L}_n / \partial C_n = 0$, we obtain KKT conditions given by

$$\begin{aligned} \frac{\partial \mathcal{L}_n}{\partial p_{k,n}} &= \sum_{j=k}^{K} \Delta_j \frac{\mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}}{1 + p_{k,n} \mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}} - \nu_n = 0, \quad \forall k \\ \frac{\partial \mathcal{L}_n}{\partial C_n} &= \nu_n - \mu = 0 \end{aligned}$$

where we define the covariance matrix for each $k = 1, \ldots, K$, $j \geq k$ and for $n = 1, \ldots, K$

$$\mathbf{\Sigma}_{k,j,n} = \mathbf{I} + \sum_{i=1, i \neq k}^{j} \mathbf{h}_{i,n} \mathbf{h}_{i,n}^H p_{i,n}$$

which is the covariance of the interference plus noise experienced by user k on subcarrier n while users from K to $j+1$ have already been decoded and subtracted from the received signal, in the successive decoder. By combining the both KKT conditions, we obtain

$$\sum_{j=k}^{K} \Delta_j \frac{\mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}}{1 + p_{k,n} \mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}} = \mu \geq 0, \quad \forall k, n \qquad (12)$$

Unfortunately, it is not possible to solve (12) for a given μ in a closed form. Hence, we resort to an iterative algorithm and in particular apply the algorithm [3] to find the solution of the subproblem. Let $\mathbf{p}_n(\mu)$ denote the solution of the n-th subproblem for a given μ. While in [3] the algorithm determines μ to impose the total power constraint at each iteration, here we find $\mathbf{p}_n(\mu)$ for a fixed μ imposed by the master problem. Consequently, we obtain a set of solutions $\{\mathbf{p}_n(\mu), C_n(\mu)\}_{n=1}^{N}$ where $C_n(\mu) = \sum_k p_{k,n}(\mu)$ is the power on subcarrier n corresponding to a fixed μ. Since C_n is

not a constraint but a optimization variable in the subproblem (11), user powers $p_{1,n}, \ldots, p_{K,n}$ of subcarrier n can be optimized sequentially one user after another. Namely, the objective function is optimized over $p_{k,n}$ while treating $p_{1,n}, \ldots, p_{k-1,n}, p_{k+1,n}, \ldots, p_{K,n}$ fixed. The algorithm updates $\{p_{1,n}\}_{n=1}^N, \ldots, \{p_{K,n}\}_{n=1}^N$ and then $\{p_{1,n}\}_{n=1}^N \ldots$ until the solution of (10) is converged.

Master problem We have to solve the master problem (9) on top of the N subproblems. Since the master problem is always convex, any optimization method (interior-point, subgradient) can be applied. In our case, similar to the dual decomposition for sum rate maximization of MIMO-BC in [9], a simple bisection method is sufficient because the search is one-dimensional. The Lagrangian variable μ can be interpreted as a water level that should be adjusted according to whether the total power constraint is satisfied or not. Let $g_{k,n}(p_{k,n})$ denote the LHS of the KKT conditions (12)

$$g_{k,n}(p_{k,n}) = \sum_{j=k}^{K} \frac{\Delta_j \alpha_{k,j,n}}{1 + p_{k,n}\alpha_{k,j,n}}$$

where we let $\alpha_{k,j,n} = \mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}$. As remarked in [3], the above function is a monotonically decreasing function of $p_{k,n}$ when treating $\{\alpha_{k,j,n}\}$ fixed. By defining the inverse function of $g_{k,n}(p_{k,n}) = \mu$ such that $g_{k,n}^{-1}(\mu) = p_{k,n}$, the total power corresponding to the water level μ is given by the sum over all users and subcarriers, i.e.

$$\sum_{n=1}^{N} C_n(\mu) = \sum_{n=1}^{N}\sum_{k=1}^{K} g_{k,n}^{-1}(\mu) \tag{13}$$

Noticing that this function is a monotonically decreasing function of μ for $0 \le \mu \le \max_{k,n} g_{k,n}(0)$, the water level should be adjusted as follows : increase μ if $\sum_{n=1}^N C_n(\mu) \ge P$ and decrease μ if $\sum_{n=1}^N C_n(\mu) \le P$.

Finally, by taking into account the outer optimization with respect to the water level μ, we propose the following algorithm to maximize the weighted sum rate in the MIMO-OFDM BC.

Iterative waterfilling algorithm for the weighted sum-rate maximization in MIMO-OFDM BC.

1. Initialize $\mathbf{P}^{(0)} = \mathbf{0}$, $\{\alpha_{k,j,n}^{(0)}\}$ and set the initial water interval $[\mu_{\min}^{(0)}, \mu_{\max}^{(0)}] = [0, \max_{k,n} g_{k,n}^{(0)}(0)]$.
2. Water level set : at iteration l, let $\mu^{(l)} = (\mu_{\min}^{(l)} + \mu_{\max}^{(l)})/2$
3. Inner iteration m : repeat until the optimal solution of (10) is converged.
   For $k = 1, \ldots, K$,
   - Compute $\{\alpha_{k,j,n}^{(l,m)}\}$ for $\forall n, j \ge k$
   - Multiuser waterfilling step: for all n find $p_{k,n}^{(l,m)}$ corresponding to a water level $\mu^{(l)}$, solution of

$$\mu^{(l)} = \sum_{j=k}^{K} \Delta_j \frac{\alpha_{k,j,n}^{(l,m)}}{1 + p_{k,n}^{(l,m)} \alpha_{k,j,n}^{(l,m)}} \tag{14}$$

   End
4. Water interval update : If $\sum_n \sum_k p_{k,n}^{(l,\infty)}(\mu^{(l)}) > P$ then set $\mu_{\min}^{(l+1)} = \mu^{(l)}$, else set $\mu_{\max}^{(l+1)} = \mu^{(l)}$
5. Repeat 2-4 until a desired accuracy on $|\mu_{\max}^{(l)} - \mu_{\min}^{(l)}|$ is reached

Notice that this algorithm is a generalization of Yu's algorithm [9] that maximizes the sum rate in MIMO-BC to the weighted sum rate in a multicarrier MIMO-BC, for the case of a single receive antenna.

2.2. Dual decomposition in subcarrier/user domain

Here we consider the dual decomposition not only in the subcarrier but also in the user domain. Let us introduce KN new variables $C_{k,n}$ for $k = 1, \ldots, K, n = 1, \ldots, N$. Then, the original problem (6) can be expressed as

$$\begin{aligned} \text{maximize} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n}\mathbf{h}_{j,n}^H p_{j,n} \right| \\ \text{subject to} \quad & 0 \le p_{k,n} \le C_{k,n}, \forall n, k \\ & \sum_{n=1}^{N}\sum_{k=1}^{K} C_{k,n} \le P \end{aligned} \tag{15}$$

We form the Lagrangian with respect to the coupled constraint such that

$$\begin{aligned} & L(\{p_{k,n}\}, \{C_{k,n}\}, \mu) \\ & = \sum_{n=1}^{N}\sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n}\mathbf{h}_{j,n}^H p_{j,n} \right| - \mu\left(\sum_{n=1}^{N}\sum_{k=1}^{K} C_{k,n} - P\right) \end{aligned} \tag{16}$$

Let us define the dual objective as

$$\mathcal{G}_2(\mu) = \max_{\{p_{k,n}, C_{k,n}\}, \forall n,k} L(\{p_{k,n}\}, \{C_{k,n}\}, \mu) \tag{17}$$

where the constraints in the maximization are $0 \le p_{k,n} \le C_{k,n}$ for all k and n. The master problem is given by

$$\min_{\mu \ge 0} \mathcal{G}_2(\mu) \tag{18}$$

Again noticing that the dual objective can be decoupled into N subproblems, the evaluation of $\mathcal{G}_2(\mu)$ for a given μ reduces to solving the following subproblem for each n

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n}\mathbf{h}_{j,n}^H p_{j,n} \right| - \mu \sum_{k=1}^{K} C_{k,n} \\ \text{subject to} \quad & 0 \le p_{k,n} \le C_{k,n}, \forall k \end{aligned}$$

The Lagrangian corresponding to the n-th subproblem is given by

$$\begin{aligned} \tilde{\mathcal{L}}_n(\mathbf{p}_n, \{C_{k,n}\}, \{\beta_{k,n}\}) &= \sum_{k=1}^{K} \Delta_k \log \left| \mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_{j,n}\mathbf{h}_{j,n}^H p_{j,n} \right| \\ &- \mu \sum_{k=1}^{K} C_{k,n} - \sum_{k=1}^{K} \beta_{k,n} \left(p_{k,n} - C_{k,n}\right) \end{aligned}$$

where $\beta_{k,n}$ is a dual variable associated with the constraint $p_{k,n} \le C_{k,n}$. By letting $\partial\tilde{\mathcal{L}}_n/\partial p_{k,n} = 0$ and $\partial\tilde{\mathcal{L}}_n/\partial C_{k,n} = 0$, we obtain for all k, n

$$\begin{aligned} \frac{\partial \mathcal{L}_n}{\partial p_{k,n}} &= \sum_{j=k}^{K} \Delta_j \frac{\mathbf{h}_{k,1}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}}{1 + p_{k,n}\mathbf{h}_{k,n}^H \mathbf{\Sigma}_{k,j,n}^{-1} \mathbf{h}_{k,n}} - \beta_{k,n} = 0 \\ \frac{\partial \mathcal{L}_n}{\partial C_{k,n}} &= -\mu + \beta_{k,n} = 0 \end{aligned}$$

Combining both equations, we obtain exactly the same KKT conditions (12) as for the decomposition in subcarrier domain. This means that the two decompositions reduce to the identical problem that can be solved by the same algorithm provided before.

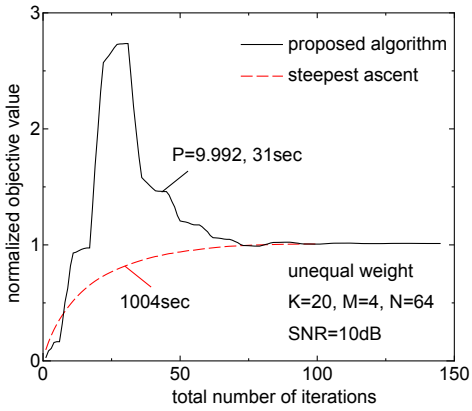

Fig.1 Convergence behavior for $M = 4, N = 64$

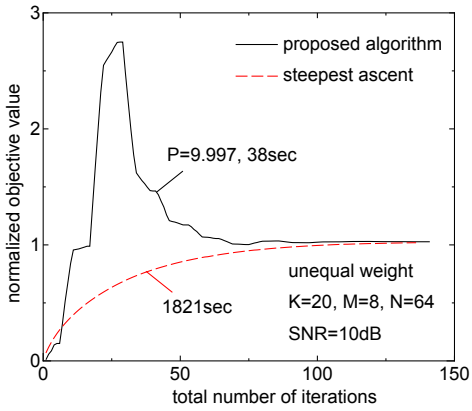

Fig.2 Convergence behavior for $M = 8, N = 64$

3. NUMERICAL EXAMPLES

We evaluate the convergence behavior of the proposed algorithm. For the sake of comparison, steepest ascent algorithm of [10] is also considered. We consider four classes of weights such that $w_4/w_1 = 32$, $w_3/w_1 = 16$, $w_2/w_1 = 8$ and let $K = 20$. Fig. 1 shows the convergence evolution for $M = 4, N = 64$ as a function of the total number of iterations including the inner iterations for our proposed algorithm. Fig. 2 and 3 show the same convergence evolution for $M = 8, N = 64$ and for $M = 4, N = 128$ respectively. The objective value is normalized so that the final value should be one. The channel is i.i.d. over antenna, user, subcarrier dimensions and randomly generated (one realization). The CPU time in second as well as the final total power (only for our proposed algorithm) is provided for a reference.

As observed, the both algorithms converge to the optimum with a power accuracy smaller than 10^{-3} with respect to $P = 10$. While the objective increases monotonically with steepest ascent algorithm, the proposed algorithm requires the water level adjustment which results in a jump in the objective between two consecutive outer iterations. It is found that our proposed algorithm converges much faster than steepest ascent algorithm in terms of required CPU time. Steepest ascent algorithm does not make use of a structure of the convex problem and requires a line search of roughly 200 samples at each iteration, which is computationally high. Comparing the three figures, we remark that the convergence of our proposed algorithm is almost insensitive to the number of antennas and of subcarriers because it updates the powers of all subcarriers simultaneously for a given user. On the other hand, steepest ascent algorithm updates the best user/subcarrier combination at each iteration. As a result, its convergence depends on the total number of active users with positive power over all subcarriers, which in turn depends on the number of antennas and that of subcarriers.

In conclusions, our proposed algorithm yields extremely fast convergence compared to steepest ascent algorithm and moreover makes the convergence almost independent of the number of antennas and the number of subcarriers.

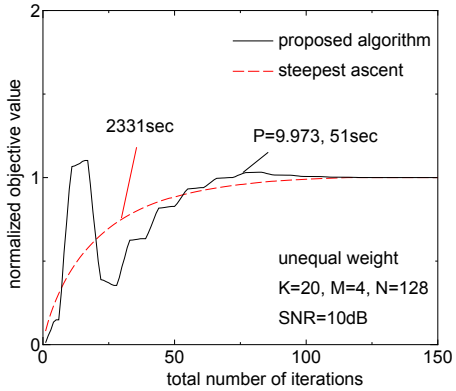

Fig.3 Convergence behavior for $M = 4, N = 128$

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