# NEAR-FIELD SOURCE LOCALIZATION VIA SYMMETRIC SUBARRAYS

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## ABSTRACT

We propose a near-field source localization algorithm with one-dimensional (1-D) search via symmetric subarrays. By dividing the uniform linear array (ULA) into two symmetric subarrays, the steering vectors of the subarrays yield the 1-D (only bearing-related) property of rotational invariance in signal subspace, which allows for the bearing estimation using the generalized far-field ESPRIT. With the estimated bearing, the range estimation of each source is consequently obtained by defining 1-D MUSIC spectrum. This algorithm transforms two-dimensional (2-D) search involved in the parameter estimation to 1-D search, and it does not require high-order statistics computation in contrast with the traditional near-field high-order ESPRIT algorithm.

***Index Terms***— Array signal processing, Direction of arrival estimation, Distance measurement, Position measurement

## 1. INTRODUCTION

Source localization finds its important applications in radar, sonar, seismology and oceanography. Various algorithms have been proposed for bearing estimation of multiple far-field sources, where the propagating waves are considered to be plane waves at the sensor array. However, when the sources are located in the near field of the array, the wavefronts emitted from these sources are spherical rather than planar at each sensor position, and thus far-field bearing estimation algorithms are not applicable. In such situation, more sophisticated algorithms have to be exploited for estimating the azimuth as well as range to localize the sources.

A good approximation of the nonlinear wavefront shape is the Fresnel approximation if the sources are located in the Fresnel region. Some algorithms are recently proposed by applying the Fresnel approximation in the near-field source localization with uniform linear array (ULA). Two-dimensional (2-D) and three-dimensional (3-D) MUSIC have been developed in [1, 2]. A high-order ESPRIT algorithm was studied in [3, 4], where the fourth-order cumulants are computed to formulate the ESPRIT-like model. In contrast to the subspace-based approaches, [5]−[7] presented the maximum likelihood estimation of the near-field parameters, and [8] exploited the cyclic-statistic-based method for the cyclostationary signals.

Considering the computational complexity due to multi-dimensional search or high-order statistics computation involved in most of the above algorithms, more recently, Grosicki presented a method of low computational cost in [9]. It applies the second-order statistics of the array output to transform the near-field parameters into an equivalent number of electrical angles, which are estimated as far-field parameters by a weighted linear prediction algorithm.

In this paper, we propose a second-order-statistics based 1-D algorithm with symmetric-subarray partition to localize multiple near-field sources. It does not require high-order statistics computation or parameter pairing or multi-dimensional search. By dividing a ULA into two symmetric subarrays, the steering vectors of the corresponding subarrays yield the far-field-like rotational invariance property in signal subspace. Based on this property, the algorithm is implemented with the following two steps: 1) Applying the generalized far-field ESPRIT to the symmetric subrrays to estimate the bearing of multiple sources; 2) With the estimated bearing of each source, applying 1-D MUSIC to get its range estimation. Since the noise subspace has been computed in the ESPRIT-based bearing estimation in 1), no additional eigen-decomposition is required in the MUSIC-based range estimation in 2). The symmetric-subarray partition transforms 2-D search for the near-field parameter estimation to $(K + 1)$ times 1-D search, with $K$ being the number of sources.

## 2. NEAR-FIELD SIGNAL MODEL FOR SYMMETRIC SUBARRAYS

### 2.1. Near-Field Signal Model

We consider a near-field scenario of $K$ uncorrelated narrow-band signals impinging to a $(2M + 1)$-element ULA with interelement spacing $d$, which is illustrated in Fig. 1. Let the array center be the phase reference point. The received signal at the $m$th sensor can be modeled as

$$x_m(t) = \sum_{k=1}^{K} e^{j\tau_{mk}} s_k(t) + n_m(t), \; m = -M, \ldots, M, \quad (1)$$

where $s_k(t)$ is the $k$th source signal, $n_m(t)$ is the additive noise, and $\tau_{mk}$ is the phase shift associated with the propagation time delay between sensor 0 and sensor $m$ of the $k$th source signal, which is a function of source signal parameters, range $r_k$, angle $\theta_k$ and wavelength $\lambda$, given by

$$\tau_{mk} = \frac{2\pi}{\lambda} \left( \sqrt{r_k^2 + (md)^2 - 2r_k md \sin\theta_k} - r_k \right). \quad (2)$$

When the source $k$ is in the Fresnel region, which is defined by $r_k$ locating in the range, $0.62\left(D^3/\lambda\right)^{1/2} < r_k < 2D^2/\lambda$, with $D$ representing the aperture of the array [10], $\tau_{mk}$ can be approximated by using the second-order Taylor expansion [9],

$$\tau_{mk}=\left(-\frac{2\pi d}{\lambda}\sin\theta_k\right)m+\left(\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k\right)m^2+O\left(\frac{d^2}{r_k^2}\right), \quad (3)$$

where $O\left(d^2/r_k^2\right)$ denotes terms of order greater than or equal to $d^2/r_k^2$. The second-order Taylor series approximation is found in many references on near-field source localization with ULA [3]−[5], [7], [8]. Using this approximation, the signal in (1) can be reduced to

$$x_m(t)=\sum_{k=1}^{K}e^{j\left(-\frac{2\pi d}{\lambda}\sin\theta_k\right)m+j\left(\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k\right)m^2}s_k(t)+n_m(t). \quad (4)$$

The received signal vector $\mathbf{X}(t)=[x_{-M}(t),\ldots,x_M(t)]^T$, with the superscript $T$ denoting matrix transposition, can be written in

$$\mathbf{X}(t)=\mathbf{A}\mathbf{S}(t)+\mathbf{N}(t), \quad (5)$$

where $\mathbf{S}(t)=[s_1(t),\ldots,s_K(t)]^T$ is the signal vector, $\mathbf{N}(t)=[n_{-M},\ldots,n_M]^T$ is the noise vector and $\mathbf{A}$ is the array manifold matrix given by,

$$\mathbf{A}=\left[\mathbf{a}(r_1,\theta_1),\ldots,\mathbf{a}(r_K,\theta_K)\right], \quad (6)$$

with the steering vector $\mathbf{a}(r_k,\theta_k)$ being expressed as

$$\mathbf{a}(r_k,\theta_k)=\begin{bmatrix} a_{k,-M} \\ \vdots \\ a_{k,M}\end{bmatrix} =\begin{bmatrix} e^{j\left(\frac{2\pi d}{\lambda}\sin\theta_k\right)M+j\left(\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k\right)M^2} \\ \vdots \\ e^{-j\left(\frac{2\pi d}{\lambda}\sin\theta_k\right)M+j\left(\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k\right)M^2}\end{bmatrix}. \quad (7)$$

### 2.2. Signal Model for Symmetric Subarrays

Observing that the elements in (7) are symmetric with respect to the second term, we divide the ULA into two subarrays as shown in Fig. 1. The first subarray is formed with the first $L$ sensors in ascending order (from sensor $-M$ to sensor $-M+L-1$), and the second subarray is formed with the last $L$ sensors in descending order (from sensor $M$ to sensor $M-L+1$). The received signal vectors of the two subarrays can be written as

$$\mathbf{X}_1(t)=[x_{-M}(t),x_{-M+1}(t),\ldots,x_{-M+(L-1)}(t)]^T, \quad (8)$$

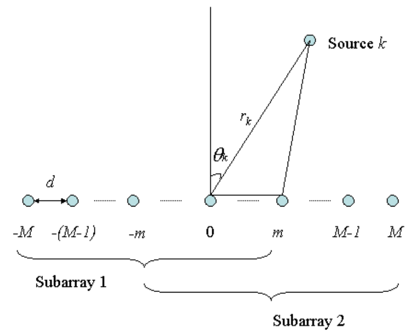


**Fig. 1**. Near field ULA configuration with symmetric partition.

and

$$\mathbf{X}_2(t)=[x_M(t),x_{M-1}(t),\ldots,x_{M-(L-1)}(t)]^T, \quad (9)$$

where $K<L<2M+1$. These two subarray vectors have similar form,

$$\mathbf{X}_1(t)=\mathbf{A}_1\mathbf{S}(t)+\mathbf{N}_1(t), \quad (10)$$

and

$$\mathbf{X}_2(t)=\mathbf{A}_2\mathbf{S}(t)+\mathbf{N}_2(t), \quad (11)$$

where $\mathbf{N}_1(t)=[n_{-M},n_{-M+1}(t)\ldots,n_{-M+(L-1)}]^T$ and $\mathbf{N}_2(t)=[n_M,n_{M-1}(t)\ldots,n_{M-(L-1)}]^T$ are subarray noise vectors. The matrix $\mathbf{A}_1$ is the first $L$ rows of $\mathbf{A}$ and $\mathbf{A}_2$ is constructed with the last $L$ rows of $\mathbf{A}$ in reverse order.

The relationship between $\mathbf{A}$ and $\mathbf{A}_1$, $\mathbf{A}_2$ is

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_1 \\ \text{last } (M-L) \text{ rows}\end{bmatrix}=\begin{bmatrix}\text{first } (M-L) \text{ rows} \\ \mathbf{J}\mathbf{A}_2\end{bmatrix}, \quad (12)$$

where $\mathbf{J}$ is the anti-identity matrix satisfying $\mathbf{J}^2=\mathbf{I}$.

Define $\mathbf{A}_1$ as

$$\mathbf{A}_1=\left[\mathbf{a}_1(r_1,\theta_1),\ldots,\mathbf{a}_1(r_K,\theta_K)\right], \quad (13)$$

with

$$\mathbf{a}_1(r_k,\theta_k)=\begin{bmatrix} a_{k,-M} \\ \vdots \\ a_{k,-M+L-1}\end{bmatrix}. \quad (14)$$

The symmetric property gives

$$\mathbf{A}_2=[\mathbf{D}(\theta_1)\mathbf{a}_1(r_1,\theta_1),\ldots,\mathbf{D}(\theta_K)\mathbf{a}_1(r_K,\theta_K)], \quad (15)$$

where

$$\mathbf{D}(\theta_k)=\mathrm{diag}\left[e^{-j\left(\frac{4\pi d}{\lambda}\sin\theta_k\right)M},\ldots,e^{-j\left(\frac{4\pi d}{\lambda}\sin\theta_k\right)(M-L+1)}\right], \quad (16)$$

which is only related with the angle $\theta_k$.

It is noteworthy that to avoid the ambiguity of the phase for the element of $\mathbf{D}(\theta_k)$, it is necessary to require $d<\lambda/4$.

## 3. NEAR-FIELD SOURCE LOCALIZATION WITH SYMMETRIC SUBARRAYS

### 3.1. Eigen-decomposition of the Array Output

The eigen-decomposition of the array covariance matrix $\mathbf{R} = \mathrm{E}[\mathbf{X}(t)\mathbf{X}^H(t)]$ yields

$$\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H, \tag{17}$$

where the superscript $H$ denotes the complex conjugate transposition, $\mathbf{U}_s \in C^{(2M+1)\times K}$ contains $K$ eigen vectors spanning the signal subspace of $\mathbf{R}$, and the diagonal matrix $\Lambda_s \in C^{K\times K}$ contains the corresponding eigen values. Similarly, $\mathbf{U}_n \in C^{(2M+1)\times(2M+1-K)}$ contains $2M+1-K$ eigen vectors in the noise subspace of $\mathbf{R}$, whereas the diagonal matrix $\Lambda_n \in C^{(2M+1-K)\times(2M+1-K)}$ is built from the corresponding eigen values.

### 3.2. Generalized ESPRIT for Bearing Estimation

The signal model in (5) implies that there exists a $K \times K$ full-rank matrix $\mathbf{G}$ satisfying $\mathbf{U}_s = \mathbf{AG}$,

$$\mathbf{AG} = \begin{bmatrix} \mathbf{A}_1\mathbf{G} \\ \text{last } (M-L) \text{ rows} \end{bmatrix} = \begin{bmatrix} \text{first } (M-L) \text{ rows} \\ \mathbf{JA}_2\mathbf{G} \end{bmatrix}, \tag{18}$$

(18) means $\mathbf{U}_s$ can be similarly partitioned as

$$\mathbf{U}_s = \begin{bmatrix} \mathbf{U}_{s1} \\ \text{last } (M-L) \text{ rows} \end{bmatrix} = \begin{bmatrix} \text{first } (M-L) \text{ rows} \\ \mathbf{U}_{s2} \end{bmatrix}. \tag{19}$$

Thus $\mathbf{U}_{s1}$ and $\mathbf{U}_{s2}$, corresponding to the first and second subarray, are

$$\mathbf{U}_{s1} = \mathbf{A}_1\mathbf{G}, \tag{20}$$

and

$$\mathbf{U}_{s2} = \mathbf{JA}_2\mathbf{G}, \tag{21}$$

or equivalently

$$\mathbf{JU}_{s2} = \mathbf{A}_2\mathbf{G}. \tag{22}$$

According to the generalized ESPRIT in [11], we introduce a diagonal matrix

$$\Psi(\theta) = \mathrm{diag}[e^{-j\left(\frac{4\pi d}{\lambda}\sin\theta\right)M}, \ldots, e^{-j\left(\frac{4\pi d}{\lambda}\sin\theta\right)(M-L+1)}], \tag{23}$$

and form the matrix $\mathbf{JU}_{s2} - \Psi(\theta)\mathbf{U}_{s1}$,

$$\mathbf{JU}_{s2} - \Psi(\theta)\mathbf{U}_{s1} = [(\mathbf{D}(\theta_1) - \Psi(\theta))\mathbf{a}_1(r_1,\theta_1), \ldots, (\mathbf{D}(\theta_K) - \Psi(\theta))\mathbf{a}_1(r_K,\theta_K)]\mathbf{G}. \tag{24}$$

The $k$th column of the matrix $\mathbf{JU}_{s2} - \Psi(\theta)\mathbf{U}_{s1}$ becomes zero when $\theta = \theta_k$, which implies that the matrix $\mathbf{W}^H\mathbf{JU}_{s2} - \mathbf{W}^H\Psi(\theta)\mathbf{U}_{s1}$ is singular with $\mathbf{W}$ being an arbitrary $M \times K$ full-rank matrix.

The following spectrum function is then used to find the angle,

$$P_{ESPRIT}(\theta) = \frac{1}{\det\left[\mathbf{W}^H\mathbf{JU}_{s2} - \mathbf{W}^H\Psi(\theta)\mathbf{U}_{s1}\right]}. \tag{25}$$

Peaks of the spectrum function $P_{ESPRIT}(\theta)$ indicate the estimated angle $\hat{\theta}_k$, $k = 1, \ldots, K$.

Note that 1-D search can be implemented for bearing estimation of the multiple sources.

### 3.3. MUSIC for Range Estimation

By substituting the estimated angle $\hat{\theta}_k$ back into the steering vectors $\mathbf{a}(r, \theta_k)$ in (7), the problem is reduced to finding the parameter $r$ in $\mathbf{a}(r, \hat{\theta}_k)$ with the received signal $\mathbf{X}(t)$. Since the noise subspace has been obtained in eigen-decomposition in (17), the range spectrum function of the $k$th source can be directly constructed by MUSIC method,

$$P_{MUSIC}^{(k)}(r) = \frac{1}{\mathbf{a}^H(r,\hat{\theta}_k)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(r,\hat{\theta}_k)}, \; k = 1, \ldots, K. \tag{26}$$

The range estimation is obtained by maximizing $P_{MUSIC}^{(k)}(r)$ in the Fresnel region, $\mathbf{r}_F$,

$$\hat{r}_k = \arg\max_{r\in\mathbf{r}_F}\left[P_{MUSIC}^{(k)}(r)\right]. \tag{27}$$

To avoid the parameter pairing, we form the MUSIC spectrum at each estimated bearing. For $K$ sources, $K$ times 1-D search are required to implement the range estimation. Thus, a total of $(K+1)$ times 1-D search is required in this 2-D near-field localization algorithm.

## 4. SIMULATION RESULTS

In the simulations, a ULA with $M = 4$ and $d = \lambda/5$ is employed to localize two uncorrelated narrowband sources in additive white Gaussian noise. The locations of the two sources are set as $(r_1, \theta_1) = (1.8\lambda, -8^o)$ and $(r_2, \theta_2) = (3\lambda, 12^o)$, which are in the Fresnel region of the array $(1.25\lambda < r < 5.12\lambda)$, according to the definition in 2.1.

We divide the array into two subarrays, and each subarray consists of $L = 8$ elements. To test the performance of the algorithm, 500 Monte Carlo simulations are performed at different SNRs (from 0 dB to 30 dB) and with different snapshots (from 200 to 6000). The results are compared with the Cramer-Rao Bound (CRB) in [9].

Figs. 2 and 3 illustrate the standard deviation of the estimated bearing and range versus SNR, respectively. The solid lines in these figures indicate the estimated parameters, whereas the dash-dot lines indicate the corresponding CRBs. 1024 snapshots are used in this simulation.

Similarly, Figs. 4 and 5 display the bearing and range estimation versus snapshots, respectively. In this simulation, SNR = 10 dB.

For two sources localization, this algorithm only needs 3 times 1-D search, and it does not require parameter pairing or high-order statistics computation.

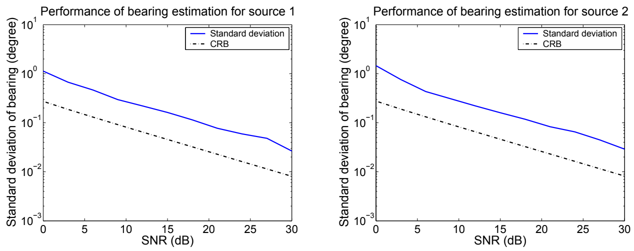


**Fig. 2**. Standard deviation and CRB versus SNR of the bearing estimation.

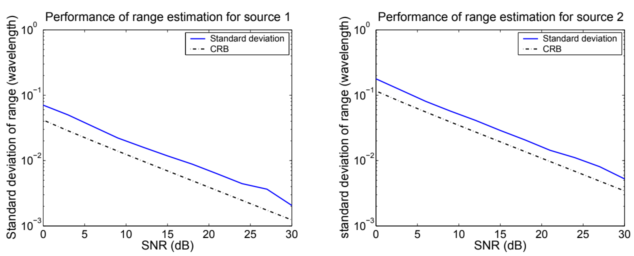


**Fig. 3**. Standard deviation and CRB versus SNR of the range estimation.

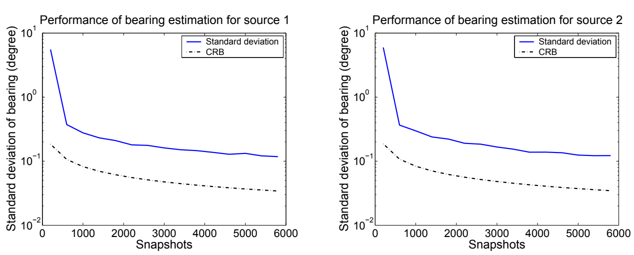


**Fig. 4**. Standard deviation and CRB versus snapshots of the bearing estimation.

## 5. CONCLUSION

Based on the second-order-statistics, this paper proposes a 1-D algorithm via symmetric subarrays for 2-D near-field multiple sources localization. By exploiting the far-field-like rotational invariance property in signal subspace of the symmetric subarrays, 2-D parameter estimation is transformed to 1-D estimation. 1-D generalized ESPRIT is applied to estimate the bearings, and 1-D MUSIC is applied to estimate the range of each source with its estimated bearing. By implementing $(K+1)$ times 1-D search, the algorithm can localize $K$ sources in the near field without parameter pairing. Unlike the traditional near-field high-order ESPRIT, this algorithm does not require high-order statistics computation.

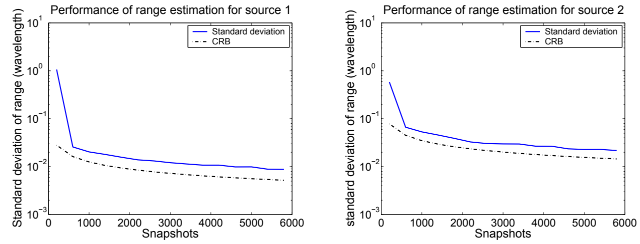


**Fig. 5**. Standard deviation and CRB versus snapshots of the range estimation.

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