ASYNCHRONOUS DIFFERENTIAL TDOA FOR SENSOR SELF-LOCALIZATION

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ABSTRACT

Most methods for sensor self-localization require either synchronization or round trip communications among the networked sensors. A differential time-difference-of-arrival (dTDOA) method is proposed in this paper that does not require such time synchronization nor round trip communications, thus greatly reduces demand on sensor communications resources and makes implementation much easier. Theoretical results are presented for the principle of the new method and simulations are also given that confirm the effectiveness of the proposed method.

Index Terms— self-localization, TDOA, sensor networks

1. INTRODUCTION

Many sensor self-localization methods that do not rely on GPS have been explored by researchers [1]. Two ranging methods are commonly used, which are the round-trip TOA [2] and the time difference of arrival (TDOA) [3]. Round-trip TOA requires a sensor to return its received measurement signal with a prescribed delay from its reception and requires dedicated communications bandwidth between the pair of sensors. The TDOA measurements require the receiving sensors' clocks to be synchronized, which in turn requires significant bandwidths and special communication protocols.

A new method using sinusoidal wave radio interferometry has been proposed [4]. This method transmits two high frequency sinusoids at slightly different frequencies from each of two separate sensors at the same time with the same power, and two separate receiving sensors need to be synchronized to measure the phase difference of the received interfering (modulating) waveform of the two sinusoids. This scheme has less bandwidth and synchronization requirements than the TOA and the TDOA methods, but it requires tight coordination of two pairs of sensors in terms of transmitting powers, tuning frequencies, and synchronization of a pair of transmitting sensors and a pair of receiving sensors. Such tight coordination may prove to be quite inconvenient, to say the least, in practical implementation.

In this paper a differential TDOA (dTDOA) method is proposed that does not require tight time synchronization or round-trip TOA measurements, nor does it require tight coordination among the transmitting/receiving sensors. It only requires the sensors to pass their received signals to a location for processing and location computation. This location can be either one of the sensors or a nearby central location. The proposed method imposes no restriction on the transmitted ranging signals, and significantly reduces the required transmission burden and timing constraint. In this paper we also derive the number of independent equations that can be obtained for a network of N sensors under practical (not very high SNR) conditions.

2. DIFFERENTIAL TDOA WITHOUT SENSOR SYNCHRONIZATION

The central idea of the new method is to construct differential TDOA (dTDOA) rather than TDOA. During the ranging process, each sensor in the network takes turns to transmit a ranging signal, and all other sensors listen. When all sensors have finished transmitting the ranging signals, each sensor then takes turns to pass the sampled version (or simply some parameters, c.f., Section 3) of the received ranging signals to a central processing location. No tight time constraints need to be imposed on any of these transmissions.

Denoting the ranging signal transmitted from the m th sensor as $s_m(t)$, the received signal at the i th sensor can be written as $s_m(t-\tau_m-\tau_{mi}-\Delta\tau_i)$, where τ_m is the unknown starting time of transmission by the m th sensor, $\tau_{mi}=d_{mi}/c$ is the unknown propagation delay from the m th sensor to the i th sensor and is determined by the distance d_{mi} between the two sensors and the propagation velocity c, and $\Delta\tau_i$ is the unknown asynchronous sensor clock offset plus the unknown circuit-induced delay of the i th sensor.

The *i*th sensor then samples and packetizes all its received signals, and transmits them to the processing center. It is important that the packet slot sequence in transmitting all received signals be known a priori, so that the *m*th sensor signal received by the *i*th sensor can be reconstructed at the processing center just as how it is received at the *i*th sensor. The knowledge of such sequence can be guaranteed since we can determine *a priori* a packet that contains received signal samples by the *i*th sensor with a pre-determined packet structure, known slot lengths, and a known signal sequence. To avoid too long a packet when many sensors exist in a network, multiple packets each containing samples of a selected subset of sensor signals can be constructed and transmitted at different times, but time re-aligned after the signals are reconstructed at the processing center.

The processing center then estimates the relative delays of the received ranging signal (from the same source) among two or more sensors. This can be done by, for example, a delay-and-sum beamformer to maximize the beamformer output power on each transmitted ranging signal [5]. For the mth ranging signal, the relative time delay between the received signal time at the ith and the jth sensor can be obtained:

$$\tau_{ij}^{m} = (\tau_{mi} - \tau_{mj}) + (\Delta\tau_i - \Delta\tau_j) \tag{1}$$

Unlike the synchronous TDOA method [3], this TDOA measurement cannot be used directly to solve locations of the sensors i and j, since there are unknown clock offsets without synchronization. However, if we perform the above self-ranging on the ranging signal transmitted from another sensor denoted by n, we can obtain the TDOA between the same pair of receiving sensors as,

$$\tau_{ij}^{n} = (\tau_{ni} - \tau_{nj}) + (\Delta\tau_i - \Delta\tau_j) \tag{2}$$

The differential TDOA (dTDOA) is obtained by subtracting these two TDOA measurements in (1) and (2), thus the term "differential",

$$\begin{aligned} \tau_{ij}^{mn} = \tau_{ij}^{m} - \tau_{ij}^{n} &= (\tau_{mi} - \tau_{mj}) - (\tau_{ni} - \tau_{nj}) \\ &= (d_{mi} - d_{mj} - d_{ni} + d_{nj}) / c. \end{aligned} \tag{3}$$

By constructing TDOA, the unknown start time of transmission is canceled. Furthermore, by constructing differential TDOA, the unknown clock-offsets are also canceled.

Equation (3) was also obtained in [4] by radio interferometry, which requires tight coordination of two pairs of sensors in terms of transmitting powers, tuning frequencies, and synchronization. No such requirements are needed in our proposed dTDOA method. In a way the requirement of simultaneous transmission of two ranging signals from two separate sensors in [4] is changed to sequential transmission, one sensor at a time. While this change considerably eases the implementation, clock drift however becomes an issue. Generally speaking when the clock quality is good or the time span of the ranging process is short, the effect due to clock drifts are negligible. In this paper, due to space limitation, we ignore clock drifts under the assumption that these two conditions are satisfied. A method that overcomes the clock drift problem for inaccurate clocks is delegated to another paper.

The proposed dTDOA method is different from the code ranging method of [6], in that our method does not require transmitting time-stamps as in [6], and our ranging signals do not have to be restricted to wideband codes, c.f. the next section. The dTDOA method is also different from relative positioning with differential GPS that requires simultaneous observation of a group of satellites by a network of ground receivers through different frequency bands or coded channels, which is difficult to achieve in sensor networks that have limited communication resources.

3. RANGING SIGNAL DESIGN

We did not impose any restrictions on ranging signals, which can be, for example, (narrowband) sinusoids or (wideband) pulse trains. To save bandwidth resources, sinusoidal ranging signals are much preferred. When this is the case for a known ranging sinusoidal frequency, all quantities named "time" becomes "phase", "time delay" becomes "phase shift", and "clock offset" becomes "oscillator phase offset" in the previous section. In addition, the receiving sensors do not need to transmit its received signal samples to the processing center. They only need to transmit some parameters such as the measured phase, frequency, and magnitude of the received sinusoid, thus the transmission burden from each sensor to the processing center is much reduced and easy to implement.

However, if we only transmit one sinusoid at a high frequency, then the 2π phase ambiguity becomes a problem. This is because $ft = (fd)/c = d/\lambda$, so the distance ambiguity is an integer multiple of the wavelength. Obviously there is a need to transmit low frequency sinusoids to avoid the distance ambiguity for enough distance coverage. However, transmitting a frequency that is lower than the VHF band on a portable platform is not very practical. To overcome this problem, we can adopt and revise the idea of [4] by transmitting simultaneously two sinusoids with slightly different frequencies, but from *one sensor*, which is much easier to do than from two separate sensors as in [4]. After demodulating at each sensor as in [4] the resulting frequencies contain the difference of those two sinusoids. So the 2π phase ambiguity is now extended to the range of the wavelength of the frequency difference, rather than the carrier frequency. The carrier sinusoid can still be used by itself for refining location accuracy. We may even transmit multiple sinusoids spaced apart strategically to allow self-ranging to cover enough distance range and yet achieve good accuracy.

4. CONSTRUCTING DIFFERENTIAL TDOA BASIS

It is easily seen that in a sensor network consisted of N sensors, there are a total of $N(N-2)$ valid TDOA measurements, from which $\binom{N}{2}\binom{N-2}{2}$ dTDOAs can be computed. This is a very large number if N is large. However, not all such dTDOAs are independent. If we use all possible dTDOAs to compute the sensor locations, the computational complexity will likely exceed a practical limit. Thus we would like to eliminate redundant dTDOA measurements. Under ideal conditions (very high SNR) the number of independent dTDOAs that has been derived in [7] and measured in [4] by radio interferometry signals, is $N(N-3)/2$. Here we show that in a practical situation when SNR is not high, this number is $N(N-3)+1$.

The noisy TDOA measurement at the ith and jth sensors for the signal transmitted from the mth sensor is written as

$$t_{ij}^{m} = \tau_{ij}^{m} + n_{ij}^{m} \tag{4}$$

where τ_{ij}^{m} is given in (1), and n_{ij}^{m} is the measurement noise or error. The actual dTDOA (without measurement errors) is given in (3) and the measured dTDOA (with measurement errors) can be written as, due to (4),

$$\begin{aligned} t_{ij}^{mn} &= t_{ij}^{m} - t_{ij}^{n} \\ &= \tau_{ij}^{mn} + (n_{ij}^{m} - n_{ij}^{n}) \end{aligned} \tag{5}$$

From the definition of the measured dTDOAs (5), we obtain linear properties for the measured dTDOAs

$$t_{cd}^{ab} = -t_{cd}^{ba} = -t_{dc}^{ab}, \tag{6}$$

$$t_{cd}^{ab} = t_{cd}^{ax} + t_{cd}^{xb}, \tag{7}$$

and

$$t_{cd}^{ab} = t_{cx}^{ab} + t_{xd}^{ab}. \tag{8}$$

We can obtain similar properties for the actual dTDOAs by replacing t with τ. From (3), we can see that the actual dTDOAs have an additional property

$$\tau_{cd}^{ab} = \tau_{ab}^{cd}. \tag{9}$$

This property is not satisfied for the measured dTDOAs since reversing transmitting and receiving nodes does not produce exactly the same noise and errors. Of course when SNR is very high this equation will also hold approximately for the measured dTDOAs, but not for moderate or low SNR. When we reduce the number of redundant measured dTDOAs, we do not want to incur information loss that will reduce the accuracy of the final location estimation. In other words, we try to keep the resulting statistics sufficient for the location estimates while reducing the number of measured dTDOAs. Under noisy conditions there will be information loss if we reduce the valid dTDOA measurements to $N(N-3)/2$. We now present a theorem that provides the number of independent measured dTDOAs under noisy conditions, i.e. without (9), and a method to select them.

THEOREM 1: The number of linearly independent measured dTDOA in the set $A = \left\{ t_{cd}^{ab} : 1 \le a,b,c,d \le N \right\}$ *is* $N(N-3)+1$ *for* $N \ge 5$, *where* a,b,c,d *are different integers.*

The theorem can be proved by constructing a subset B called a dTDOA basis containing linearly independent measured dTDOAs and showing that every element in A is a linear combination of some elements in B. The details of the proof are omitted here due to space limitation. A possible dTDOA basis $B = B1 \cup B2 \cup B3$ consisting of three sets with no intersections between them is given by

$$\begin{aligned} B1 &= \left\{ t_{3d}^{12} : 4 \le d \le N \right\} \\ B2 &= \left\{ t_{2d}^{1b} : 3 \le b,d \le N, \text{and } b \ne d \right\} \\ B3 &= \left\{ t_{14}^{23}, t_{13}^{2b} : 4 \le b \le N \right\}. \end{aligned} \tag{10}$$

It is worth noting that the dTDOA basis is not unique, but theoretically the choice of the basis does not affect the final localization accuracy of maximum likelihood estimate (MLE). This can be concluded from the invariance of the Cramer-Rao lower bounds (CRLB) [6].

5. ML LOCATION ESTIMATION BASED ON DIFFERENTIAL TDOA BASIS

From (5), the MLE of the sensors' coordinates is derived as

$$\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} [\mathbf{t} - \boldsymbol{\tau}(\boldsymbol{\theta})]^T \mathbf{R}^{-1} [\mathbf{t} - \boldsymbol{\tau}(\boldsymbol{\theta})], \tag{11}$$

where $\boldsymbol{\theta} = [\ldots, x_i, y_i, \ldots]^T \in R^{2N-3}$ consists of the coordinates to be estimated and only relative locations that are subject to rotation, shifting and reflection can be determined. Without loss of generality, the coordinates of node 1 and the x-coordinate of node 2 are assumed as the references for the relative positions of the other sensors. $\mathbf{t} = [\ldots, t_{ij}^{mn}, \ldots, t_{cd}^{ab}, \ldots]^T$ and $\boldsymbol{\tau}(\boldsymbol{\theta}) = [\ldots, \tau_{ij}^{mn}, \ldots, \tau_{cd}^{ab}, \ldots]^T$, in which t_{ij}^{mn}, t_{cd}^{ab} belong to the dTDOA basis B formed in Section 4 and each contains noise terms that satisfy (5) with their covariance $\operatorname{corr}\left(t_{ij}^{mn} - \tau_{ij}^{mn}, t_{cd}^{ab} - \tau_{cd}^{ab}\right)$ forming the covariance matrix $\mathbf{R}$. The dimension of $\mathbf{R}$, $\mathbf{t}$ and $\boldsymbol{\tau}(\boldsymbol{\theta})$ is the same as the number of dTDOAs used and is reduced to $N(N-3)+1$ due to Theorem 1. This MLE can be solved iteratively using Taylor series expansion as

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k + \left(\mathbf{G}^T(\hat{\boldsymbol{\theta}}_k) \mathbf{R}^{-1} \mathbf{G}(\hat{\boldsymbol{\theta}}_k) \right)^{-1} \cdot \\ \mathbf{G}^T(\hat{\boldsymbol{\theta}}_k) \mathbf{R}^{-1} \left(\mathbf{t} - \tau(\hat{\boldsymbol{\theta}}_k) \right), \end{aligned} \tag{12}$$

where $\mathbf{G}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\tau}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ is the gradient of $\boldsymbol{\tau}(\boldsymbol{\theta})$. For certain pathological sensor node deployment or bad initial guess, $\mathbf{G}^T(\hat{\boldsymbol{\theta}}_k) \mathbf{R}^{-1} \mathbf{G}(\hat{\boldsymbol{\theta}}_k)$ may be ill-conditioned, i.e., close to a singular matrix. Then this iterative method would not work properly.

Alternatively, in (11), we can perform Cholesky factorization on $\mathbf{R}^{-1}$ and replace $\mathbf{R}^{-1}$ by an upper triangular matrix $\mathbf{P}$ multiplying its transpose, we have

$$\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \| \mathbf{P}[\mathbf{t} - \boldsymbol{\tau}(\boldsymbol{\theta})] \|^2. \tag{13}$$

This is a non-linear least squares problem that can be solved by Matlab function **lsqnonlin**, which is also an iterative method. Simulations show that this non-linear LS method has better numerical properties than (12).

6. SIMULATION RESULTS

16 sensor nodes are placed uniformly on the cross points of a grid in a square area. Zero-mean Gaussian displacement with standard deviation of one-tenth of the inter-distance between sensor nodes is added to the coordinates. For simplicity the channel model is assumed to be flat fading AWGN channel with line-of-sight. The ranging signal is two sinusoids at 300 MHz and 301 MHz. We initialize the clock-offsets with zero-mean uniformly distributed random variables. A good agreement between the CRLB uncertainty ellipses and simulation results is found in Fig. 1, where no clock drift is added. Since only relative positions are of interest, without loss of generality, we set the position of sensor node 1 and x-coordinate of node 2 to some fixed values as the references, thus the uncertainty ellipse becomes wider when nodes are far away from node 1 (the lower left corner one). Small clock drifts have also been added into the simulations. The result is that for random clock drifts with a maximum of ± 5 ppm, the simulation results of Fig. 1 are unaffected.

Fig. 2 shows how the geometry affects the accuracy, as inter-sensor distances get larger. Note that the performance is not much affected by the inter-sensor spacing, until the 2π ambiguity causes the TDOA estimate to be invalid (80m case). Fig. 3 compares one stage processing using one low frequency sinusoid (301 MHz - 300 MHz = 1 MHz) with two-stage processing where we transmitted three sinusoids once and performed computation twice, first using the 1 MHz low frequency sinusoid and then an intermediate frequency (310 MHz - 300 MHz = 10 MHz) sinusoid as the ranging signals. The intermediate frequency sinusoid refines the low frequency position estimates quite well.

7. REFERENCES

[1] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero III, R. L. Moses, and N. S. Correal, ‘‘Locating the nodes ’cooperative localization in wireless sensor networks’,” *IEEE Signal Processing Magazine*, pp. 54–69, July 2005.

[2] D.D. McCrady, L. Doyle, H. Forstrom, T. Dempsey, and M. Martorana, ‘‘Mobile ranging using low-accuracy clocks,” *Microwave Theory and Techniques, IEEE Transactions on*, vol. 48, no. 6, pp. 951–958, Jun 2000.

[3] B. Friedlander, ‘‘A passive localization algorithm and its accuracy analysis,” *IEEE Trans. Ocean Engineering*, vol. OE-12, no. 1, pp. 234–245, 1987.

[4] M. Maróti, P. Völgyesi, S. Dóra, B. Kusý, A. Nádas, Á. Lédeczi, G. Balogh, and K. Molnár, ‘‘Radio interferometric geolocation,” in *SenSys ’05*, 2005, pp. 1–12.

[5] J. Chen, R. Hudson, and K. Yao, ‘‘Joint maximum-likelihood source localization and unknown sensor location estimation for near-field wideband signals,” in *Proceedings of the SPIE*, July 2001, vol. 4474.

[6] M. Rydstrom, E.G. Strom, and A. Svensson, ‘‘Clock-offset cancellation methods for positioning in asynchronous sensor networks,” in *Wireless Networks, Comm. and Mobile Computing, 2005 Intl. Conf. on*, 13-16 June 2005, vol. 2, pp. 981– 986.

[7] L. Meertens, ‘‘The dimension of the vector space spanned by sets of radio-interferometric measurements,” Tech. Rep. KES.U.05.02, KestralInstitute, July 2005.

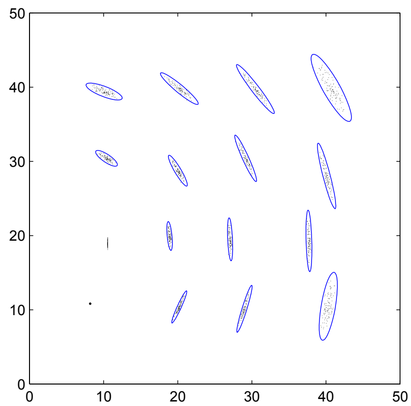

Figure 1. Location results and CRLB uncertainty regions

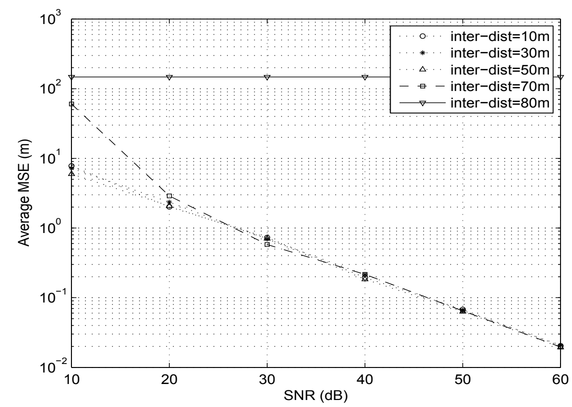

Figure 2. MSE vs SNR for various inter-sensor spacing

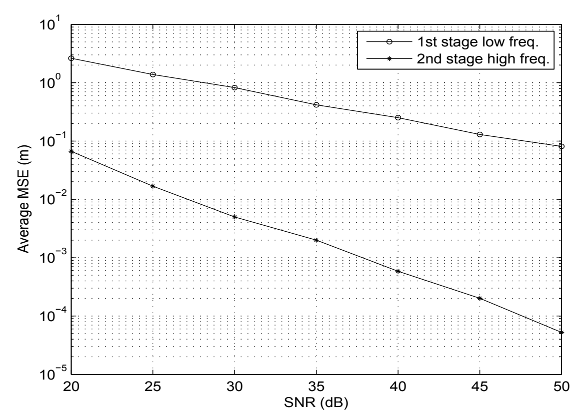

Figure 3. One stage vs. two stage processing