MULTISENSOR TRACK TERMINATION FOR TARGETS WITH FLUCTUATING SNR

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ABSTRACT

In active sonar tracking applications, targets frequently undergo fading detection performance in which the target's detection probability can shift suddenly between high and low values. Using a multistatic active sonar problem, we examine the performance of sequential track termination tests where target detections are based on an underlying Hidden Markov Model (HMM) with high and low detection states. We show that the Page test is not optimal in this problem and that a K/N track termination rule yields better performance. Further we show that a Bayesian sequential test (the Shiryaev test) yields dramatic performance improvements over both the K/N rule and the Page test.

Index Terms— Target tracking, multistatic sonar, track termination, Page test, Shiryaev test

1. INTRODUCTION

In undersea surveillance of large areas, multistatic sonar networks show promise in the ability to use many sensors to cover a large area with overlapping detection coverage, the achievement of higher data rates from use of multiple sensors (receivers) to process a single active transmission from a source, and in the geometric diversity that can be achieved by selecting receiver locations. However, it has been observed from at-sea testing that sensor detection performance varies significantly over the sensor network and for a single sensor over time. In particular, due to geometric, environmental, and geographic effects a target may be detected by a given sensor with high probability over a number of scans and then suddenly fade from view as the sensor detection probability decreases drastically. A key tracking issue therefore becomes how to adapt the tracking system to account for this fading detection performance in a multistatic active sonar problem.

In this paper we address the track termination aspect of this problem. We consider a centralized track management model that processes time ordered measurements from all sensors and include sensor origin information. The sensor detection performance is modeled as a two-state Markov chain with high and low detection states. Target-originated measurements (binary detection events) can therefore be described using a Hidden Markov Model (HMM) structure.

There is a well-developed base of literature covering track termination for sensors with a fixed probability of detection (P_d) of the target on a single scan (see e.g., [3]). Examples include K/N tests (a track is terminated if K or fewer detections are received in the last N scans) and track score tests. Track score tests may include the SPRT or Bayesian sequential tests. If the track score (related to the probability that the detection sequence is the result of a true track) falls below a certain value, the track is terminated. However research pertaining to track termination for sensors with P_d based on a Markov model has only recently been considered [6].

In this paper we analyze the performance of K/N-based and sequential track termination tests when target-originated measurements are described by a HMM with high and low P_d Markov states. Using only the binary detection events, it is shown that the K/N test outperforms the Page test over a portion of its operating characteristic region. This result is surprising considering the fact that the Page test is proven to be the optimal sequential test for quickest detection of a change in measurement distribution and we show how when the HMM-based detection statistics are used, a key assumption in the optimality proof for the Page test is no longer satisfied. It is next shown that by using a Bayesian version of a sequential test (the Shiryaev test), significant performance improvement is obtained compared to the K/N test. This work presents significantly updated and improved results from that originally presented in [4].

2. PROBLEM DESCRIPTION

In the multistatic sonar problem one or more transmitters transmit active sonar pings. Receivers are positioned to receive and process acoustic energy in order to detect reflected energy from the target(s). Because each transmitter-receiver combination is unique and provides a set of measurements, a *sensor* is identified as a specific transmitter-receiver combination.

A *scan* of data is defined as the measurement data set from one sensor resulting from one sonar transmission. Although

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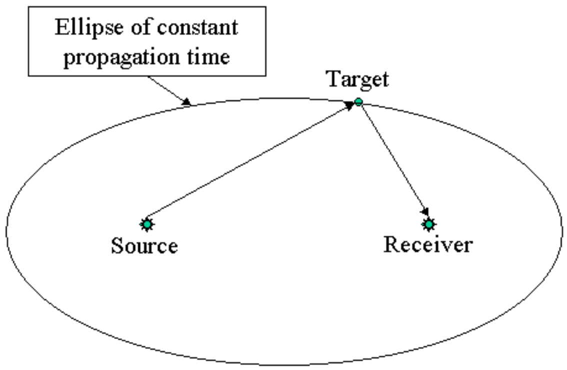

Fig. 1. Multistatic source-target-receiver geometry for a single source-receiver pair.

the results presented in this paper can be adapted to an arbitrary transmit pattern, the mathematics of the problem are made much more transparent when one assumes (as is done in this paper) that each transmitter pings in a regular pattern at the same period (T seconds). One can therefore define a *scan cycle* as the set of scans from all sensors over that period. For example if there are two transmitters transmitting at a 60 s period and two receivers, there are a total of 4 "sensors" and 4 scans of data in one scan cycle.

Raw measurements consist of the propagation time from source to receiver and the bearing from receiver to target. For a specific source-receiver combination, the propagation time defines an ellipse of possible target locations. The intersection of the received bearing from the target to this ellipse localizes the measurement to a single point. This geometry is illustrated in Fig. 1. For the target tracking algorithm, these measurements are converted to Cartesian coordinates and the measurement errors quantified using the results from [7] which account for the sources of measurement errors.

Track filtering is performed using a Kalman filter [1]. The nearly constant velocity target motion model is used with the state defined in Cartesian coordinates by $\mathbf{x} = (\, x \quad y \quad \dot{x} \quad \dot{y}\,)'$. The measurements $\mathbf{z}$ are given as positions (x and y).

2.1. Target Detection Model

A two-state Markov chain detection model is used to describe a sensor's probability of detection (P_d) of the target on a single scan, similar to that used in [6] except that state transition probabilities are given in terms of a discrete time model rather than in continuous time. In this model, define $u \in \{0, 1\}$ as the detection state where $u = 0$ represents the low detection probability state (P_d^L), and $u = 1$ represents the high detection probability state (P_d^H). The state transition probabilities are given by p (low to high) and q (high to low).

From this Markov chain detection model, the steady state probability that a sensor is in a particular state is given by

$$\Pr\{u = 0\} = q/(p + q) \tag{1}$$

$$\Pr\{u = 1\} = p/(p + q) \tag{2}$$

2.2. False Alarm Measurements

False alarms (noise-originated measurements) are assumed to be distributed uniformly over the surveillance region of a given sensor with a spatial density of λ, and the number of false alarms is distributed according to a Poisson distribution. The false alarm probability (P_{fa}) for a track at a certain time is given by the probability that at least one false alarm will fall within the validation region of the predicted track estimate $\mathbf{x}_{k+1|k}$ whose volume is given by [3]

$$V(\delta_1^k) = \gamma\pi|\mathbf{S}_{k+1}(\delta_1^k)|^{1/2} \tag{3}$$

where δ_1^k is the detection sequence $\{\delta_1, \ldots, \delta_k\}$ and $\mathbf{S}$ is the innovation covariance. The gate association parameter, γ, is chosen to yield a specified probability (P_G) that a target related measurement will fall within the validation region when a true target is being tracked.

As a result P_{fa} becomes a function of the detection sequence up to the current scan and is given by

$$P_{fa}(\delta_1^k) = 1 - \exp[-\lambda V(\delta_1^k)] \tag{4}$$

3. TRACK TERMINATION

It is well known that the CUSUM (Page) test yields the quickest detection of a change of distribution for the case of i.i.d. observations [2]. In fact, in a (highly) simplified target tracking model where detection of target and false alarms can be described by Bernoulli random processes with fixed parameters (i.e., P_{fa} is independent of the detection sequence), the K/N track termination rule becomes a sufficient statistic for discrete Page test thresholds. The optimality results of the Page test have also been extended to some non-i.i.d distributions (including certain classes of Markov chain structures) [9]. However, there have been no global optimality results proven for the case where the distributions are characterized by HMMs except in the special cases considered by [8] whose conditions are not satisfied in this application.

In addition to examining the Page test, we also implement a stopping rule based on a Bayesian formulation of the problem called the Shiryaev rule [2] which has optimality properties similar to the Page test.

3.1. Page Test for HMMs

In an HMM the formulation for the Page test is given by [5]

$$s_k = \ln \frac{f_0(\delta_k|\delta_1^{k-1})}{f_1(\delta_k|\delta_1^{k-1})} \tag{5}$$

$$c_k = \max(c_{k-1} + s_k, 0) \tag{6}$$

where $c_0 = 0$ and with each scan c_k is compared to a threshold h. The track is terminated if the threshold is exceeded. Note that the hypothesis convention used throughout this paper where H_1 represents target present (pre-change distribution for track termination testing) and H_0 represents target absent (post-change distribution for track termination testing) is opposite of that used in the quickest detection literature.

Under H_0 (target absent), the likelihood function becomes

$$f_0(\delta_k|\delta_1^{k-1}) = [P_{fa}(\delta_1^k)]^{\delta_k}[1 - P_{fa}(\delta_1^k)]^{1-\delta_k} \tag{7}$$

Under H_1 (target present), the likelihood function for the k^{th} measurement from the i^{th} sensor becomes

$$f_1(\delta_k|\delta_1^{k-1}) = \sum_{l=0}^{1} f_1(\delta_k|u_{ik} = l)\Pr\{u_{ik} = l|\delta_1^{k-1}\} \tag{8}$$

where $\Pr\{u_{ik} = l|\delta_1^{k-1}\}$ is obtained using a Bayesian update from the prior measurements.

Let $\Pr\{u_{i1}\}$ (used in (8) for $k = 1$) be given by the steady state probability of the Markov chain from (1) and (2). As a recursive procedure Bayes' rule is applied to obtain the posterior pmf, $\Pr\{u_{ik}|\delta_1^k\}$, that will be used in (10),

$$\Pr\{u_{ik}|\delta_1^k\} = \frac{f_1(\delta_k|\delta_1^{k-1}, u_{ik})\Pr\{u_{ik}|\delta_1^{k-1})}{\sum_{l=0}^{1} f_1(\delta_1^k|\delta_1^{k-1}, u_{ik} = l)\Pr\{u_{ik} = l|\delta_1^{k-1}\}} \tag{9}$$

The prior conditional pmf is updated for the next iteration using the Markov transition matrix $\mathbf{P}$,

$$\begin{pmatrix} \Pr\{u_{ik} = 0|\delta_1^{k-1}\} \\ \Pr\{u_{ik} = 1|\delta_1^{k-1}\} \end{pmatrix} = \mathbf{P} \begin{pmatrix} \Pr\{u_{i(k-1)} = 0|\delta_1^{k-1}\} \\ \Pr\{u_{i(k-1)} = 1|\delta_1^{k-1}\} \end{pmatrix} \tag{10}$$

3.2. Shiryaev Rule

The Shiryaev rule represents the optimal solution (under the i.i.d. assumption) to the quickest detection problem where the problem is formulated using a Bayesian approach.[1] In this approach, there exists *a priori* information regarding the distribution of the change time.

The Shiryaev rule applies the Bayesian concept of declaring that a change in distribution has occured when the *a posteriori* probability of a change exceeds a given threshold. Assume that the *a priori* distribution of the change time k_c is given by a probability that change time is zero, and a geometric distribution of change times greater than zero:

$$\Pr\{k_c = k\} = \begin{cases} \beta_0 & k_c = 0 \\ (1-\beta_0)\rho(1-\rho)^{k-1} & k_c > 0 \end{cases} \tag{11}$$

Applying Bayes rule, the *a posteriori* probability of a change at time $k > 0$ is

$$\beta_k = \frac{[\beta_{k-1} + (1-\beta_{k-1})\rho]f_0}{[\beta_{k-1} + (1-\beta_{k-1})\rho]f_0 + (1-\beta_{k-1})(1-\rho)f_1} \tag{12}$$

[1]The authors thank Dr. Alexander Tartakovsky of the Univ. of Southern California for suggesting this approach.

Table 1. Multistatic Tracking Scenario Parameters

Parameter	Value
Number of sensors, N_s	4
Number of scan cycles, n	4, 5
Scan cycle period, T	60 s
Markov transition probabilities, p, q	0.1/3, 0.1
Detection probabilities, P_d^L, P_d^H	0.1, 0.9
False alarm density, λ	3.14×10^{-8} m^{-2}
Measurement noise covariance	$(243.2\,\text{m})^2$
Kalman Filter process noise	10^{-4} m^2/s^3
Validation gate probability, P_G	99%

The Shiryaev stopping rule becomes [2]

$$g_k = \ln\frac{\beta_k}{1-\beta_k} \tag{13}$$

$$= \ln(\rho + e^{g_{k-1}}) - \ln(1-\rho) + \ln\frac{f_0(\delta_k|\delta_1^{k-1})}{f_1(\delta_k|\delta_1^{k-1})} \tag{14}$$

where with each scan g_k is compared to a threshold h. If the threshold is exceeded then the track is terminated.

A Bayesian test is appealing in that the *a priori* probability of a distribution change time can be related to the actual tracking problem. One estimates β_0 based on the expected numbers of confirmed true and false tracks using the performance characteristics of the track confirmation module and the expected density of true targets in the surveillance region. Further, the geometric distribution of change time can be used as a model for the probability that a confirmed track on a true target diverges due either to a target maneuver or by being drawn off the target by incorrectly associating noise measurements to the track.

3.3. Model-Based Results

To compare the performance of the Page test, the Shiryaev test and the K/N rule, Monte Carlo simulations were performed in which detection sequences were generated with sensor measurements given by the model from Section 2 and using the parameters from Table 1. For each hypothesis H_0 and H_1, 10^4 simulations were performed. Simulations under H_0 yield the average false track life, called average detection delay (ADD). Simulations under H_1 yield the average true track life, called average run length (ARL)[2]. Results are plotted for each track termination test over a set of threshold values. The Shiryaev test used $\beta_0 = 0.5$ and $\rho = 0.005$. Fig. 2 presents the track termination performance of each test.

Although asymptotically better than the K/N rule, the Page test performed worst over the operating range likely to be used in a track termination module of a track management system (ADD of false tracks 4–15 scan cycles). The Shiryaev test

[2]ADD and ARL are the standard terminology used in change detection literature

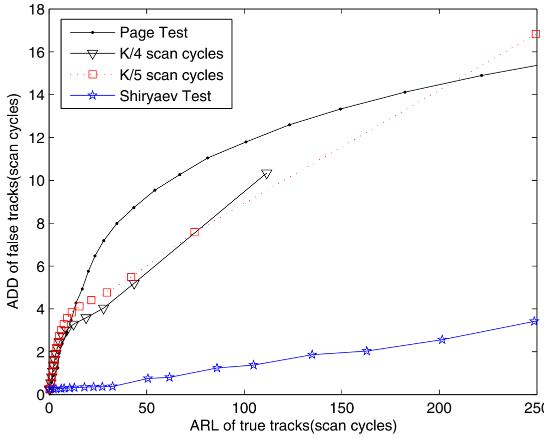

Fig. 2. Comparison of the time to terminate false and true tracks for various track termination tests.

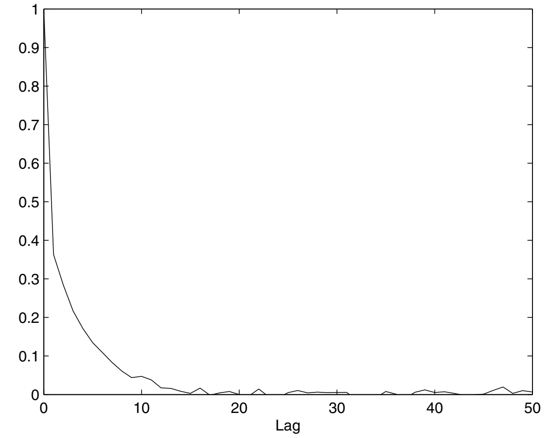

Fig. 3. Autocorrelation of the CUSUM increments, s_k, under H_1 (target present).

performed the best of the three tests considered. This is likely due to the information provided by the prior knowledge of the distribution of change times used in the Shiryaev test. The computational cost of the Shiryaev test is small and easily computable as part of an overall track management system.

The sub-optimality of the Page test was not expected. Examination of the assumptions used in the optimality proofs of the Page test show that the increments of the cumulative sum, s_k, must be i.i.d. [2, 9]. Fig. 3 plots the autocorrelation of the CUSUM increments as a function of lag under H_1 based on data sequences obtained through simulation and shows a significant correlation out to a lag of about 5 measurements. Therefore since the CUSUM increments are not i.i.d., existing optimality proofs of the Page test are not applicable.

4. CONCLUSIONS

In this paper we have examined the performance of track termination tests in a multistatic active sonar problem where the sensor detection performance can be modeled as either high P_d or low P_d with detection statistics based on a hidden Markov model. The Page test is shown to be suboptimal to a K/N test over a portion of its operating range and is due to the HMM structure of the detection sequence under H_1. The Shiryaev test, a Bayesian formulation , performs dramatically better than either the Page test or the K/N rule. The computational complexity of implementing a Shiryaev test for track termination is low.

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