AN EFFICIENT ALGORITHM FOR MUTIPLE DIRECTION OF ARRIVAL TRACKING BASED ON THE CONSTRAINED PROJECTION APPROXIMATION APPROACH AND KALMAN FILTER

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ABSTRACT

In this paper, we present a direction of arrival (DOA) tracking scheme involving a subspace tracking algorithm and a Kalman filter. The proposed subspace tracking algorithm is based on an interpretation of the signal subspace as the solution of a minimization of a constrained projection approximation task. We show that we can apply the matrix inversion lemma to solve this problem recursively. Proposed algorithm avoids orthonormalization process after each update for post-processing algorithms which need orthonormal basis of the signal subspace. The DOA's are obtained via a Newton-type method initialized with the DOA's predicted by the Kalman filter. The tracking capability of the proposed algorithm is verified by computer simulations in a scenario involving targets with crossing trajectories.

Index Terms**—** subspace tracking, Kalman filter, direction of arrival (DOA).

1. INTRODUCTION

Subspace-based methods for estimating the direction of arrival (DOA) of the signal impinging on an array of sensors have drawn considerable interest over recent years. Based on the eigendecomposition of the covariance matrix of the array output, they offer high resolution and give accurate estimates [1]. However, in the applications that the number of sources is very large, the computational burden of these methods may be unacceptably high.

In order to overcome this difficulty, a large number of approaches have been introduced for fast subspace tracking in the context of adaptive signal processing. Most of these techniques can be grouped into three families. In the first one, classical batch methods for EVD/SVD have been modified to fit adaptive processing [2]. In the second family, variations and extensions of Bunch's rank-one updating algorithm [3] have been proposed. The third class of algorithms considers the EVD/SVD as a constrained or unconstrained optimization problem, for which the introduction of a projection approximation hypothesis leads to fast subspace tracking methods (see, e.g., the PAST [4] and NIC [5] algorithms).

Some of these algorithms add orthonormalization step to achieve orthonormal eigenvectors [6]. The necessity of orthonormalization depends on the post-processing method which uses the signal subspace estimate to extract the desired signal information. If we are using MUSIC or minimum-norm method for calculating DOA's from the signal subspace, for which orthonormal basis of the signal subspace is required, orthonormalization step is crucial.

In this paper, we present an algorithm for tracking the signal subspace spanned by the eigenvectors corresponding to the r largest eigenvalues, where r is the dimension of signal subspace. It relies on an interpretation of the signal subspace as the solution of a constrained optimization problem. We call our approach as constrained projection approximation subspace tracking (CPAST). This algorithm avoids orthonormalization step. We show that we can apply the recursive technique to solve this problem.

Here we propose a subspace-based DOA tracking scheme that its implementation involves three functional structures:

1) adaptive subspace estimator;
2) subspace-based DOA estimator;
3) Kalman filter.

From the subspace estimate, obtained through the proposed CPAST algorithm, we attain the DOA's by minimizing a one-dimensional (1-D) cost function via a Newton-type method. These DOA estimates are treated as measurements and smoothed by the Kalman filter.

2. PROBLEM FORMULATION

Consider r incident narrowband point sources which impinge on an array of n sensors ($r<n$). These signal sources are assumed to be emitted by sources moving with arbitrary trajectories. At time t, the sample vector $\mathbf{x}(t) \in C^{n\times 1}$ of the sensor outputs can be written according to the model:

$$\mathbf{x}(t) = \mathbf{A}(\theta(t))\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s} \in C^{r\times 1}$ is the vector of the complex envelopes of the target signals, $\mathbf{n} \in C^{n\times 1}$ is an additive noise vector, $\mathbf{A}(\theta(t)) = [\mathbf{a}(\theta_1(t)),\ldots,\mathbf{a}(\theta_r(t))] \in C^{n\times r}$ is the matrix of the steering vectors $\mathbf{a}(\theta_k(t))$, and $\theta_k(t), k=1,\ldots,r$ is the DOA of the k'th target measured with respect to the broadside of the array. The problem of tracking the DOA's can be formulated as the problem of estimating $\theta_k(t)$'s recursively. In the subspace-based methods, the DOA's are determined as the minimizing arguments of the following cost function:

$$f(\theta) = \mathbf{a}^H(\theta)(\mathbf{I}_n - \mathbf{W}\mathbf{W}^H)\mathbf{a}(\theta) \tag{2}$$

where $\mathbf{W}$ is an orthogonal basis for the signal subspace and $\mathbf{I}_n$ is the identity matrix of dimension n. Classically, $\mathbf{W}$ is obtained via an eigendecomposition of the covariance matrix of the received sensors' outputs. This involves a prohibitive computational burden, which limits its use for tracking. To overcome this problem, we suggest the CPAST algorithm to estimate $\mathbf{W}$ recursively.

3. THE CPAST METHOD

Let $\mathbf{x} \in C^n$ be a complex valued random vector process with the autocorrelation matrix $\mathbf{C} = E\{\mathbf{x}\mathbf{x}^H\}$ which is assumed to be positive definite. We consider the following minimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J(\mathbf{W}) = E\left\|\mathbf{x} - \mathbf{W}\mathbf{W}^H\mathbf{x}\right\|^2 \tag{3}$$

where $\mathbf{W}$ is an $n\times r$ ($r\leq n$) full rank matrix. It can be shown that $J(\mathbf{W})$ has a global minimum and the columns of the solution of the above problem are orthonormal and span the signal subspace [4]. Thus, the use of an iterative algorithm to minimize $J(\mathbf{W})$ will always converge to an orthonormal basis of the signal subspace without any orthonormalization operation during the iteration. Although the capability of gradient based subspace update approaches is clear to us, it is not the aim of this paper to use these approaches. Instead, we replace the expectation in (3) with an exponentially weighted sum as follows:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{W}^H(t)\mathbf{x}(i)\right\|^2 \tag{4}$$

and we will try to solve this problem recursively. All sample vectors available in the time interval $1\leq i\leq t$ are involved in estimating the signal subspace at the time instant t. The use of the forgetting factor $0<\beta\leq 1$ is intended to ensure that data in the distant past are downweighted. This is useful when the system operates in a nonstationary environment. $J(\mathbf{W}(t))$ is a fourth-order function of elements of $\mathbf{W}(t)$. The key issue of the projection approximation subspace tracking (PAST) approach [4], is to approximate $\mathbf{W}^H(t)\mathbf{x}(i)$ in (4), the unknown projection of $\mathbf{x}$(i) onto the columns of $\mathbf{W}(t)$, by the expression $\mathbf{y}(i)=\mathbf{W}^H(i\text{-}1)\mathbf{x}(i)$, which can be calculated for $1\leq i\leq t$ at the time instant t. This results in a modified cost function:

$$J'(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^2 \tag{5}$$

which is quadratic in the elements of $\mathbf{W}(t)$. This results in the following minimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J'(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i}\left\|x(i) - \mathbf{W}(t)y(i)\right\|^2 \tag{6}$$

The solution to this problem (the PAST solution) is as follows [4]:

$$\mathbf{W}(t) = \mathbf{C}_{\mathbf{xy}}(t)(\mathbf{C}_{\mathbf{yy}}(t))^{-1} \tag{7}$$

where

$$\mathbf{C}_{\mathbf{xy}}(t) = \sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{\mathbf{xy}}(t-1) + \mathbf{x}(t)\mathbf{y}^H(t) \tag{8}$$

$$\mathbf{C}_{\mathbf{yy}}(t) = \sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{\mathbf{yy}}(t-1) + \mathbf{y}(t)\mathbf{y}^H(t) \tag{9}$$

We note that the PAST algorithm is derived by minimizing the modified cost function in (5) instead of the original one in (3). Hence, the columns of $\mathbf{W}$(t) are not exactly orthonormal. The deviation from the orthonormality depends on the signal to noise ratio (SNR) and the forgetting factor β. This lack of orthonormality affects seriously the performance of post-processing algorithms which are dependent on orthonormality of the basis. To overcome this problem, we define the following constrained optimization problem:

$$\underset{\mathbf{W}}{\text{minimize}} \quad J'(\mathbf{W}(t)) = \sum_{i=1}^{t} \beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^2 \qquad \textit{subject to} \quad \mathbf{W}^H(t)\mathbf{W}(t) = \mathbf{I}_r \tag{10}$$

where $\mathbf{I}_r$ is the $r\times r$ identity matrix and it is clear that the constraint in (10) guarantees the orthonormality. To solve this constrained problem we use Lagrange multipliers method. So, after expanding the expression for $J'(\mathbf{W}(t))$, we can replace (10) with the following problem:

$$\begin{aligned}\underset{\mathbf{W}}{\text{minimize}} \quad h(\mathbf{W}) = {} & tr(\mathbf{C}) - 2tr\Big(\sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{x}^H(i)\mathbf{W}(t)\Big) + \\ & tr\Big(\sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i)\mathbf{W}^H(t)\mathbf{W}(t)\Big) + \lambda\left\|\mathbf{W}^H(t)\mathbf{W}(t) - \mathbf{I}_r\right\|_F^2\end{aligned} \tag{11}$$

where $tr(\mathbf{C})$ is the trace of the matrix $\mathbf{C}$, $\|.\|_F$ denotes the Frobenius norm, and λ is the Lagrange multiplier. By letting $\nabla h = 0$, where ∇ is the gradient operator with respect to $\mathbf{W}$, we have:

$$\begin{aligned}& -\sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(t) + \sum_{i=1}^{t} \beta^{t-i}\mathbf{W}(\mathbf{t})\mathbf{y}(i)\mathbf{y}^H(t) + \\ & \lambda[-2\mathbf{W}(\mathbf{t}) + 2\mathbf{W}(\mathbf{t})\mathbf{W}^H(\mathbf{t})\mathbf{W}(\mathbf{t})] = 0\end{aligned} \tag{12}$$

If we obtain $\mathbf{W}$ from aforementioned equation and use it in $\mathbf{W}^H\mathbf{W} = \mathbf{I}_r$, after some manipulations we obtain:

$$\begin{aligned}& \sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{y}^H(i) - 2\lambda\mathbf{I}_r + 2\lambda\mathbf{W}^H(t)\mathbf{W}(t) = \\ & \Big[\sum_{i=1}^{t} \beta^{t-i}\mathbf{y}(i)\mathbf{x}^H(i)\sum_{i=1}^{t} \beta^{t-i}\mathbf{x}(i)\mathbf{y}^H(i)\Big]^{\frac{1}{2}}\end{aligned} \tag{13}$$

where $(.)^{1/2}$ denotes the square root of a matrix.

Now, using (13), we can remove λ from equation (12) and abtain the following solution:

$$\mathbf{W}(t) = \mathbf{C}_{\mathbf{xy}}(t)(\mathbf{C}_{\mathbf{xy}}^H(t)\mathbf{C}_{\mathbf{xy}}(t))^{\frac{-1}{2}} \tag{14}$$

This constrained projection approximation subspace tracking (CPAST) algorithm guarantees the orthonormality of the columns of $\mathbf{W}$(t). The general form of solution of CPAST algorithm is similar to PAST except its square root.

4. ADAPTIVE SUBSPACE TRACKING

Subspace tracking methods have applications in numerous domains, including the fields of adaptive filtering, source localization, and parameter estimation. In many of these

Table 1. The CPAST algorithm for tracking the signal subspace

Choose $\mathbf{P}(0)$ and $\mathbf{W}(0)$ suitably
FOR $t = 1,2,...$DO
$\mathbf{y}(t) = \mathbf{W}^{\mathbf{H}}(t-1)\mathbf{x}(t)$
$\mathbf{C}_{\mathbf{xy}}(t) = \beta\mathbf{C}_{\mathbf{xy}}(t-1) + \mathbf{x}(t)\mathbf{y}^{\mathbf{H}}(t)$
$\mathbf{\Phi}(t) = \mathbf{C}^{\mathbf{H}}_{\mathbf{xy}}(t)\mathbf{C}_{\mathbf{xy}}(t)$
$\mathbf{a}(t) = \mathbf{y}^{\mathbf{H}}(t)\mathbf{P}(t-1)$
$\mathbf{b}(t) = \mathbf{C}^{\mathbf{H}}_{\mathbf{xy}}(t-1)\mathbf{x}(t)$
$\mathbf{E}(t) = \beta^{-2}\left[\mathbf{I_r} - \mathbf{b}(t)\mathbf{a}(t) / (\mathbf{a}(t)\mathbf{b}(t) + \beta)\right]$
$\mathbf{g}(t) = \mathbf{x}^{\mathbf{H}}(t)\mathbf{C}_{\mathbf{xy}}(t)\mathbf{P}(t-1)\mathbf{E}(t)$
$\mathbf{K}(t) = \mathbf{g}(t) / (\mathbf{g}(t)\mathbf{y}(t) + 1)$
$\mathbf{F}(t) = \mathbf{E}(t)(\mathbf{I_r} - \mathbf{y}(t)\mathbf{K}(t))$
$\mathbf{P}(t) = \mathbf{P}(t-1)\mathbf{F}(t)$
$\mathbf{Q}(t) = \mathbf{x}(t)\left(\mathbf{k}(t) - \beta\,\mathbf{b}^{\mathbf{H}}(t)\mathbf{P}(t)\right) / (\mathbf{x}^{\mathbf{H}}(t)\mathbf{x}(t))$
$\mathbf{W}(t)\mathbf{P}^{\frac{\mathbf{H}}{2}}(t) = \beta\mathbf{W}(t-1)\mathbf{P}^{\frac{\mathbf{H}}{2}}(t-1)\mathbf{F}(t) + \mathbf{Q}(t)$
$\mathbf{W}(t) = [\mathbf{W}(t)\mathbf{P}^{\frac{\mathbf{H}}{2}}(t)]\mathbf{\Phi}^{\frac{\mathbf{H}}{2}}$

applications we have a continuous stream of data. Thus, developing adaptive algorithms is very useful for these applications. An efficient and numerically robust recursive solution for (10) can be obtained by using the matrix inversion lemma to compute the inverse of $\mathbf{C}^{\mathbf{H}}_{\mathbf{xy}}(t)\mathbf{C}_{\mathbf{xy}}(t)$ in (14).

To do so, we apply the inversion lemma to equation (14) after replacing $\mathbf{C}_{\mathbf{xy}}(t)$ from (8) in it. It can be shown that after using inversion lemma for two times and doing some manipulations, we can obtain a recursive solution. Table 1 summarizes this recursive CPAST algorithm for tracking the signal subspace. Because of the limited space, we have omitted the derivation of this recursive algorithm.

Appropriate initial values should be chosen for **P**(0) and **W**(0). **P**(0) must be a Hermitian positive definite matrix and **W**(0) should contain r orthonormal vectors. The choice of these initial values affects the transient behavior but not the steady state behavior of the algorithm. The simplest way is to set **P**(0) to the $r\times r$ identity matrix and the columns of **W**(0) to the first r columns of the $n\times n$ identity matrix.

5. THE DOA TRACKING ALGORITHM

The proposed algorithm utilizes a Newton-type (N-t) method which is presented in [8], to track the minimum of $f(\theta)$ obtained by using CPAST algorithm. This method is an adaptive version of the MUSIC DOA estimation method. This N-t method uses predicted DOA's given by a Kalman filter in order to derive its new DOA estimates.

Let us denote $\mathbf{y}_k(t) = [\theta_k(t)\ \dot{\theta}_k(t)\ \ddot{\theta}_k(t)]^T$ as the state vector containing the DOA's $\theta_k(t)$, the bearing velocities $\dot{\theta}_k(t)$, and the acceleration values $\ddot{\theta}_k(t)$ at time t. We model the dynamics and the measurement equations of the kth target by [9]:

$$\mathbf{y}_k(t+1) = \mathbf{F}\mathbf{y}_k(t) + \mathbf{w}_k(t) \tag{15}$$

$$\hat{\theta}_k(t) = \mathbf{h}\mathbf{y}_k(t) + v_k(t), \tag{16}$$

where $\mathbf{F} = \begin{bmatrix} 1 & T & \frac{T}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$, T is sampling period, **h**=[1 0 0] and

$\mathbf{w}_k(t)$ is the noise process assumed to be normally distributed with zero mean and covariance matrix $\mathbf{Q}_k(t)$, and $v_k(t)$ is a zero mean white Gaussian noise with variance $\sigma^2_{vk}(t)$. The proposed tracking algorithm can then be summarized in the following steps:

1) From the existing estimates $\hat{\mathbf{y}}_k(t\mid t)$, obtain a prediction of the state vectors $\hat{\mathbf{y}}_k(t+1\mid t)$ and their covariance matrices $\mathbf{P}_k(t+1\mid t)$ by the equations:

$$\hat{\mathbf{y}}_k(t+1\mid t) = \mathbf{F}\hat{\mathbf{y}}_k(t\mid t) \tag{17}$$

$$\mathbf{P}_k(t+1\mid t) = \mathbf{F}\mathbf{P}_k(t\mid t)\mathbf{F}^T + \mathbf{Q}_k(t) \tag{18}$$

2) Apply the adaptive CPAST algorithm presented in table 1 to update the signal subspace $\mathbf{W}(t+1)$.

3) Using the predicted DOA's and $\mathbf{W}(t+1)$, obtain an estimate of the DOA's via the N-t algorithm:

$$\hat{\theta}_k(t+1) = \hat{\theta}_k(t+1\mid t) - \frac{\mathrm{Re}[d^H(\theta)\mathbf{\Pi}(t+1)a(\theta)]}{d^H(\theta)\mathbf{\Pi}(t+1)d(\theta)} \tag{19}$$

where $\mathbf{\Pi}(t+1) = \mathbf{I}_n - \mathbf{W}(t+1)\mathbf{W}^H(t+1)$, $\theta = \hat{\theta}_k(t+1\mid t)$, $d(\theta) = da(\theta)/d\theta$, and Re[.] stands for the real part.

4) Estimate the innovation errors as in the following form:

$$\delta\theta_k(t+1) = \hat{\theta}_k(t+1) - \hat{\theta}_k(t+1\mid t) \tag{20}$$

From the dynamic model and the measurement equations (17) and (18) and from the innovation errors (20), we have:

$$\begin{aligned} \hat{y}_k(t+1\mid t+1) &= \hat{y}_k(t+1\mid t) + \mathbf{G}_k(t+1)\delta\theta_k(t+1) \\ \mathbf{P}_k(t+1\mid t+1) &= [\mathbf{I}_3 - \mathbf{G}_k(t+1)h]\mathbf{P}_k(t+1\mid t) \end{aligned} \tag{21}$$

where $\mathbf{I_3}$ is the identity matrix of dimension 3 and the matrix $\mathbf{G}_k(t+1)$ is the Kalman gain given by:

$$\mathbf{G}_k(t+1) = \mathbf{P}_k(t+1\mid t)h^T[h\mathbf{P}_k(t+1\mid t)h^T + \sigma^2_{vk}(t+1)]^{-1} \tag{22}$$

Note that $\hat{\theta}_k(t+1\mid t+1)$ is the final output.

6. SIMULATION RESULTS

We consider three targets emitting uncorrelated narrow-band

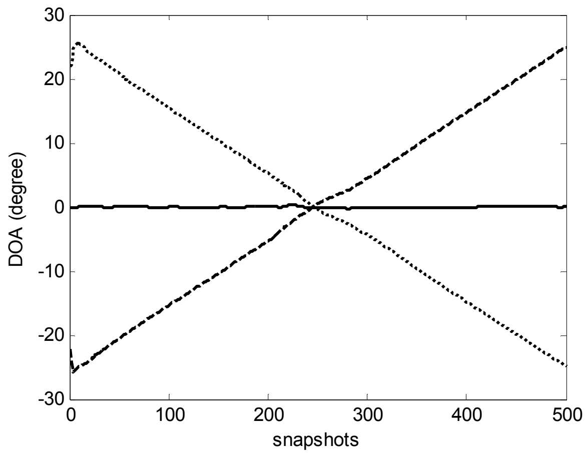

Figure 1. Result of DOA tracking with three crossing targets

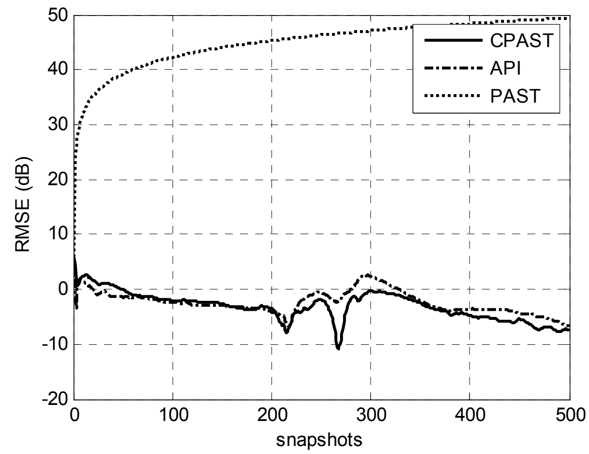

Figure 2. Root mean square error

signals that impinge on a uniform linear array of 17 sensors separated by half wavelength. The targets have been tracked over an interval of 100 seconds with T=0.2 seconds. The targets have individual SNRs of 10 dB. To initialize the Kalman filter, we obtained DOA estimate $\hat{\theta}_k(0)$ for each target from an initial block of data. Then, with these estimates, we initialized the state vector by $\theta_k(0\,|\,0)=\hat{\theta}_k(0)$, $\dot{\theta}_k(0\,|\,0)=0$ and $\ddot{\theta}_k(0\,|\,0)=0$. The corresponding covariance matrix $\mathbf{P}_k(0\,|\,0)$ and the covariance matrix of the noise process $\mathbf{Q}_k$ are, respectively, given by:

$$\mathbf{P}_k(0|0)=\begin{bmatrix} 1 & \frac{1}{T} & 0 \\ \frac{1}{T} & \frac{2}{T^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}\sigma_{vk}^2(0) \tag{23}$$

$$\mathbf{Q}_k=\begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} & \frac{T^2}{2} \\ \frac{T^3}{2} & T^2 & T \\ \frac{T^2}{2} & T & 1 \end{bmatrix}\sigma_{wk}^2 \tag{24}$$

with $\sigma_{vk}^2(0)=0.1$ and $\sigma_{wk}^2=0.001$.

To demonstrate the capability of proposed algorithm in tracking targets in non-stationary environment, we assume two sources that change their locations uniformly from (-25°, 25°) to (25°, -25°), and one source is constantly in 0°. Figure 1 shows the result of applying the DOA tracking algorithm to the data of the mentioned scenario. From the figure, it turns out that the proposed algorithm can track the trajectories properly.

To evaluate the performance of the subspace tracking part of this algorithm, we compare the root mean square error (RMSE) of the proposed CPAST algorithm with two subspace-based tracking algorithms PAST [4] and approximated power iteration (API) [7]. This performance comparison is obtained using 100 simulation runs of the aforementioned scenario. The results are shown in figure 2.

7. CONCLUSION

In this paper, we introduced an interpretation of the signal subspace as the solution of a constrained minimization problem. We derived a recursive solution for adaptive implementation of this solution. Then we used a DOA tracking algorithm based on the subspace tracking algorithm, called CPAST, Kalman filter and an N-t method. This algorithm overcomes the difficulty of crossing over sources. Simulations show excellent performance of this algorithm in a non-stationary environment.

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