SPATIALLY DISTRIBUTED SOURCES LOCALIZATION WITH A SUBSPACE BASED ESTIMATOR WITHOUT EIGENDECOMPOSITION

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ABSTRACT

In this paper, a new subspace-based algorithm for parametric estimation of angular parameters of multiple incoherently distributed sources is proposed. This method consists of using the subspace principle without any eigendecomposition of the covariance matrix, so that it does not require the knowledge of the effective dimension of the pseudosignal subspace and then the major difficulty of the existing subspace estimators can be avoided. The proposed idea relies on the use of the property of the inverse of the covariance matrix to exploit approximately the orthogonality property between column vectors of the noise-free covariance matrix and the sample pseudo-noise subspace. Simulation results show that, compared with other known methods, the proposed algorithm exhibits a better estimation performance.

Index Terms — Array signal processing, DOA and angular spread estimation, incoherently distributed sources.

1. INTRODUCTION

In most applications of array processing, source localization techniques are generally based on point source modeling, where the energy of each source is assumed to be concentrated at discrete direction. However, in many environments for modern radio communications, the transmitted signal is often obstructed by buildings, vehicles, *etc*. The multipath propagation in the vicinity of each transmitter may cause angular spreading of its energy. In this complex situation, a *distributed source* model will be more appropriate than the point source one [1].

In the literature, we can find two types of distributed sources, namely, coherently distributed (CD) and incoherently distributed (ID) sources [2]. More precisely, for a CD source the observation period is much smaller than the channel coherency time, in this case the signal components arriving from different directions can be modeled as the delayed and attenuated replicas of the same signal. On the other hand, if all signals coming from different directions are assumed to be uncorrelated, the source is said ID source.

Several methods have been proposed for estimating these two types of distributed sources. Indeed, in CD source case, the rank of the noise-free covariance matrix is equal to the number of sources. Many classical estimation methods can consequently be generalized from the case of point sources to the case of CD sources.

On the other hand, for ID sources, the rank of the noise-free covariance matrix increases as the angular spread of the source increases. Therefore the classical subspace based techniques cannot be easily generalized to the case of ID sources, because the choice of the effective dimension of the pseudo-signal subspace depends on the unknown parameters [3]. Due to the complexity of the subspace based methods in the ID sources case, an interesting alternative is to use of the beamforming methods.

In [4], an interesting approach to generalize the Minimum Variance Distortionless Response (MVDR) beamforming method [5] to multiple ID sources is proposed. The resulting method, called Generalized Capon (GC), maintains a distortionless response to a hypothetical source in mean power sense rather than in the deterministic sense as for the point source case. Another interesting way consists of using the covariance fitting methods. For example, in [6], a new estimator based on an approximation of the array covariance matrix using central and noncentral moments of the source angular power densities has been proposed. This estimator does not require any spectral search and can be used for widely separated multisource scenarios with different angular power densities.

In this paper, we propose a new method that allows the estimation of the angular parameters of multiple ID sources. The proposed approach relies on the use of the subspace principle but it does not require any eigendecomposition of the covariance matrix, so that it does not require the knowledge of the effective dimension of the pseudosignal subspace and then the major difficulty of the existing subspace estimators can be avoided. Our method exploits the orthogonality property between column vectors of the noise-free covariance matrix and the sample pseudo-noise subspace. Numerical examples show that the resulting estimator outperforms several popular estimators (DISPARE [7] and GC [4]).

2. PROBLEM FORMULATION

Let us consider q electromagnetic scattered waves impinging on the array of m sensors from angular direction θ_i for $i = 1, 2, \ldots, q$. We assume that the delay spread caused by the mutlipath propagation is small compared with the inverse bandwidth of the signals ; therefore, the narrowband assumption continues to be valid, even in the presence of scattering [8]. For simplicity, we assume that sensors and sources are on the same plane. The baseband signals received at the antenna array are collected in the observation vector $\boldsymbol{x}(t) = [x_1(t), \ldots, x_m(t)]^T$, where $(.)^T$ denotes the transpose operator, this vector is modeled as [2]

$$\boldsymbol{x}(t) = \sum_{i=1}^{q} \int_{\phi \in \Theta} \boldsymbol{a}(\phi) s_i(\phi, t; \boldsymbol{\eta}_i) d\phi + \boldsymbol{n}(t) \qquad (1)$$

where $s_i(\phi, t; \boldsymbol{\eta}_i)$ is the angular signal distribution of the i^{th} source, $\boldsymbol{\eta}_i = [\theta_i \ \ \sigma_i]^T$ the corresponding unknown parameter vector, θ_i the nominal DOA of the i^{th} source, σ_i the corresponding angular spread, Θ the angular field of view, $\boldsymbol{n}(t)$ the noise vector and $\boldsymbol{a}(\theta)$ is the steering vector for a point source at DOA θ.

Throughout this paper, we will consider the ID[1] source model presented in [2]. That is, for the i^{th} source, we have :

$$E[s_i(\phi, t; \boldsymbol{\eta}_i) s_i^*(\phi', t; \boldsymbol{\eta}_i)] = \sigma_{s_i}^2 \rho_i(\phi, \boldsymbol{\eta}_i) \delta(\phi - \phi') \qquad (2)$$

where $E[.]$ denotes the expectation, $\delta(\phi - \phi')$ is the Dirac delta-function, $\sigma_{s_i}^2$ is the power of the i^{th} source, and $\rho_i(\phi, \boldsymbol{\eta}_i)$ is the normalized angular power density associated with the i^{th} source : $\int_{\phi \in \Theta} \rho_i(\phi, \boldsymbol{\eta}_i) d\phi = 1$.

The noise $\boldsymbol{n}(t)$ is considered as a Gaussian complex random variable, zero-mean, circular and not necessary spatio-temporally white, the eigenvalues of the covariance matrix may be distinct in this case. The signals and noise are uncorrelated and all distributed sources are mutually uncorrelated. Then, the data model (1) allows us to write the covariance matrix of the array measurements as :

$$\mathbf{R} = \sum_{i=1}^{q} \sigma_{s_i}^2 \boldsymbol{\Psi}_i + \mathbf{R}_n = \mathbf{R}_s + \mathbf{R}_n \qquad (3)$$

where $\mathbf{R}_s = \sum_{i=1}^{q} \sigma_{s_i}^2 \boldsymbol{\Psi}_i$ is the noise-free covariance matrix and $\mathbf{R}_n$ the noise covariance matrix. The $(m \times m)$ normalized noise-free covariance matrix $\boldsymbol{\Psi}_i$ corresponding to i^{th} source is given by

$$\boldsymbol{\Psi}_i = \int_{\phi \in \Theta} \rho_i(\phi, \boldsymbol{\eta}_i) \boldsymbol{a}(\phi) \boldsymbol{a}^H(\phi) d\phi \qquad (4)$$

In this study, we assume that all sources have the same and known shape of the angular distribution of scatterers.

[1] The assumption of ID sources has been theoretically and experimentally shown to be relevant in wireless communications in the case of rural and suburban environments with a high base station [1].

3. SUBSPACE-BASED ESTIMATION

In this section, we shortly describe the problems of some existing subspace-based methods in the case of ID sources and then we present the proposed estimator.

By performing an eigen-decomposition of matrix $\mathbf{R}$ in (3) we get :

$$\mathbf{R} = \mathbf{E}_s \boldsymbol{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \boldsymbol{\Lambda}_n \mathbf{E}_n^H \qquad (5)$$

where $\mathbf{E}_s = [\mathbf{e}_1, \ldots, \mathbf{e}_r]$ contains the signal eigenvectors, $\mathbf{E}_n = [\mathbf{e}_{r+1}, \ldots, \mathbf{e}_m]$ contains the noise eigenvectors, $\boldsymbol{\Lambda}_s = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_r)$ the signal eigenvalues, and $\boldsymbol{\Lambda}_n = \text{diag}(\lambda_{r+1}, \ldots, \lambda_m)$ the noise eigenvalues.

• In the point source case with white noise, the eigenvalues of $\mathbf{R}$ can be ordered as $\lambda_1 \geq \cdots \geq \lambda_r \geq \lambda_{r+1} = \cdots = \lambda_m = \sigma_n^2$, where r is equal to the number of point sources ($r = q$). Then the signal subspace is spanned by the steering vectors $\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_q)$ corresponding to q sources.

Many point source DOA estimation techniques rely on the eigendecomposition given in (5). For example, the MUSIC estimator [9] uses the fact that the q steering vectors corresponding to q point sources are orthogonal to the noise subspace $\mathbf{E}_n$. Assuming one point source with corresponding steering vector $\boldsymbol{a}(\theta_0)$, the orthogonality property means $\mathbf{E}_n^H \boldsymbol{a}(\theta_0) = \mathbf{0}$. However, great deterioration in performance of the MUSIC estimator in the case of ID sources has been observed [10]. In fact, in ID case, it is not possible to directly apply the usual subspace based methods because the parameter r in (5) is unknown. Indeed, in a single ID case source the rank of the noise-free covariance matrix increases as the angular spread increases and can reach the number of sensors.

•• In spite of this increase in the rank of the noise-free covariance matrix, many techniques are based on the fact that, most of the signal energy is concentrated within the first few eigenvalues of the noise-free covariance matrix (see [2, 3, 7]). The number of these eigenvalues is referred to as the *effective dimension* of the pseudo-signal subspace. It is generally smaller than the number of sensors.

For example, the *DISpersed Signal PARametric Estimation* (DISPARE) [7] technique has been proposed especially for the case of ID sources. It is based on the fact that the columns vectors of the noise-free covariance matrix $\boldsymbol{\Psi}_i$ corresponding to the i^{th} ID source are orthogonal with those of the pseudo-noise subspace,

$$\mathbf{E}_n^H \boldsymbol{\Psi}_i \approx \mathbf{0} \qquad (6)$$

Note that the major problem with this technique, and with all other existing subspace based techniques, is the choice of the effective dimension of the pseudo-signal subspace, since the optimal choice depends also on the angular spread of the ID source. One indicator of the number of dominant eigenvalues (effective dimension) may be defined as

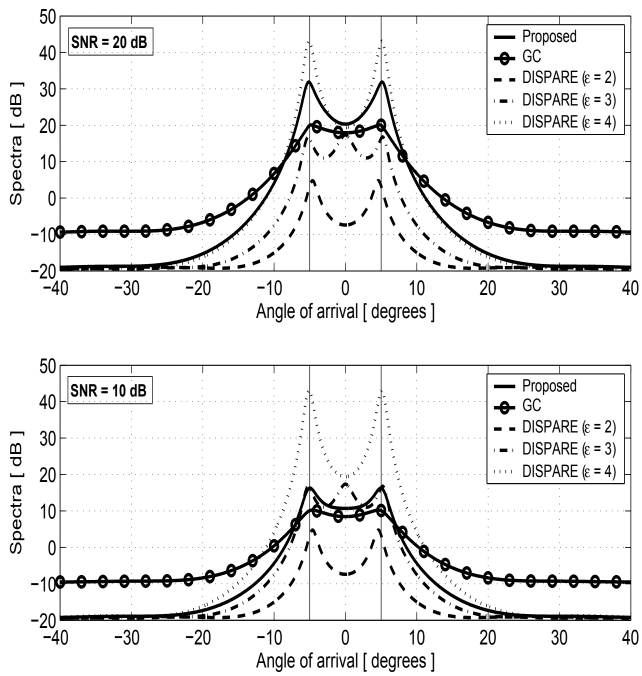

Fig. 1. Theoretical spectra : proposed, GC and DISPARE spectra for different values of $\epsilon = \dim(\mathbf{E}_s)$, $\boldsymbol{\eta}_1 = [-5^\circ\ 2^\circ]^T$, $\boldsymbol{\eta}_2 = [5^\circ\ 2^\circ]^T$, $m = 10$ sensors. White noise with SNR = 20 dB (top) and SNR = 10 dB (bottom) (Gaussian dist.).

the minimum number of eigenvalues whose sum is greater than $\tau \sum_{k=1}^{m} \lambda_k$, where $0 \leq \tau \leq 1$ is close to unity.

• • • To avoid the eigendecomposition of the sample covariance matrix into pseudo-signal and pseudo-noise subspaces so that the difficulty of the subspace based estimators for ID case can be avoided, we propose to employ the matrix inversion of (5), given by

$$\mathbf{R}^{-1} = \mathbf{E}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{E}_s^H + \mathbf{E}_n \boldsymbol{\Lambda}_n^{-1} \mathbf{E}_n^H \qquad (7)$$

On one hand, for a high signal to noise ratios, the elements of the matrix $\mathbf{E}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{E}_s^H \boldsymbol{\Psi}(\boldsymbol{\eta})$ are small for all value of $\boldsymbol{\eta}$. On the other hand, the columns vectors of the noise-free covariance matrix $\boldsymbol{\Psi}_i$ corresponding to the i^{th} ID source are orthogonal with those of the pseudo-noise subspace (6).

Consequently, for a high signal to noise ratios, $\left\|\mathbf{R}^{-1}\boldsymbol{\Psi}\right\|_F$ becomes minimal around $\boldsymbol{\eta}_i$, where $\left\|.\right\|_F$ denotes the Frobenius norm.

The main idea of our approach is to exploit the orthogonality property between column vectors of the noise-free covariance matrix and the sample pseudo-noise subspace without really distinguishing the pseudo-signal and the pseudo-noise subspaces.

Considering these results, the parameter vector estimate $\hat{\boldsymbol{\eta}}_i$ ($i = 1, \ldots, q$) can be obtained from the following two-dimensional search problem :

$$\hat{\boldsymbol{\eta}} = \arg\min_{\boldsymbol{\eta}} \left\|\hat{\mathbf{R}}^{-1}\boldsymbol{\Psi}(\boldsymbol{\eta})\right\|_F^2 = \arg\min_{\boldsymbol{\eta}} \mathrm{Tr}\left[\boldsymbol{\Psi}(\boldsymbol{\eta})\hat{\mathbf{R}}^{-2}\boldsymbol{\Psi}(\boldsymbol{\eta})\right] \qquad (8)$$

where $\hat{\mathbf{R}} = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{x}(t)\boldsymbol{x}^H(t)$ is the sample covariance matrix and Tr[.] denotes the trace of a matrix.

To illustrate our approach, figure 1 shows the spectra of the proposed estimator (8) and those of GC [4] and DISPARE [7] estimators in the case of two ID sources. As we can see in this figure, the choice of the pseudosignal dimension is critical for the DISPARE estimator because a bad choice can give erroneous results ($\dim(\mathbf{E}_s) = 3$). We can also observe that the resolution power of the proposed estimator is better than that of the GC estimator presented in [4]. This can be explained by the fact that the GC estimator is based on the beamforming principle, whereas the proposed estimator is based on the subspace principle.

It is worth noting that, in the point source case, the proposed *parametric* estimator reduces to *non-parametric* Pisarenko's extended version of Capon's estimator [11]. Indeed, in this case the vector $\boldsymbol{\eta}$ reduces to the scalar θ and the noise-free covariance matrix $\boldsymbol{\Psi}(\boldsymbol{\eta})$ transforms to $\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)$. Thus, we can readily verify that :

$$\left\|\hat{\mathbf{R}}^{-1}\boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)\right\|_F^2 = m\ \boldsymbol{a}^H(\theta)\hat{\mathbf{R}}^{-2}\boldsymbol{a}(\theta)$$

4. SIMULATION RESULTS

In this section, we present a numerical illustration of the performance of the proposed estimator as well as a comparison with the Generalized Capon (GC) estimator [4] and the DISPARE estimator [7].

We consider a uniform linear array (ULA) of 8 sensors separated by a half wavelength of incoming uncorrelated signals. The performances of the estimators are obtained by means of 200 Monte Carlo simulations by calculating the root mean square error (RMS Error). The sources emit BPSK modulated signals with a raised cosine pulse shape filter, the roll-off factor is equal to 0.22 and the bit rate is 3.84 Mb/s. The incoming signals are sampled with the frequency 38.4 MHz during approximately $5\mu s$ which gives 200 data samples. The shape of the angular distribution of scatterers is assumed to be Gaussian. For the DISPARE method the effective dimension of the pseudo-signal subspace is chosen such that at least 95% of the signal energy is considered.

In this example, the effect of the signal to noise ratio is examined when two equipower ID sources with the angular parameters $\boldsymbol{\eta}_1 = [-4^\circ\ 2^\circ]^T$ and $\boldsymbol{\eta}_2 = [9^\circ\ 3^\circ]^T$ are simulated. Fig.2 displays the RMS Error of the DOA estimates and Fig.3 considers the estimation of the angular spread. As these two figures show, the proposed estimator has a better estimation performance compared with the other two estimators, especially for low SNRs.

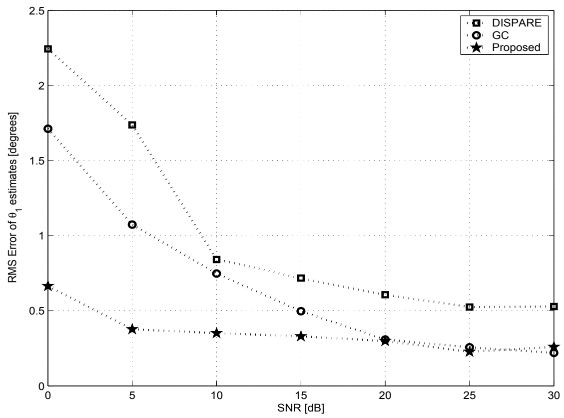

Fig. 2. RMS Errors versus SNR for the DOA θ_1 estimates, $\boldsymbol{\eta}_1 = [-4^\circ\ 2^\circ]^T$, $\boldsymbol{\eta}_2 = [9^\circ\ 3^\circ]^T$.

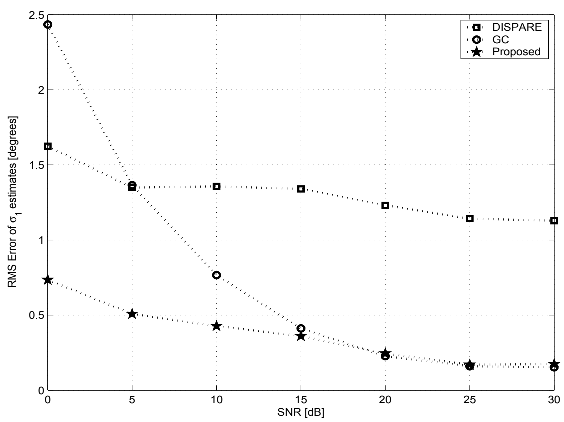

Fig. 3. RMS Errors versus SNR for the spread angle σ_1 estimates, $\boldsymbol{\eta}_1 = [-4^\circ\ 2^\circ]^T$, $\boldsymbol{\eta}_2 = [9^\circ\ 3^\circ]^T$.

5. CONCLUSION

In this paper, we have developed a new technique for estimating the DOAs and angular spreads of incoherently distributed sources. Our approach consists of exploiting the subspace principle without any eigendecomposition of the sample covariance matrix and therefore, it avoids the major difficulty of all subspace estimators in ID case. Indeed, the proposed estimator does not require the knowledge of the effective dimension of the pseudosignal subspace. Numerical examples show that the proposed estimator exhibits better estimation performance compared with the other known estimators.

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