Blind Identification and Linear Quadratic Frequency Invariant Beamforming Based Angle of Arrival Estimation

Ananth Subramanian

Abstract— **In this paper, we present wide band frequency invariant beamforming and Angle of Arrival (AoA) estimation techniques. We propose a new Linear Quadratic (LQ) frequency domain frequency invariant beamforming strategy. Based on the proposed beamforming strategy, we give a new AoA estimation technique using blind identification. Simulation results illustrate the performance of the proposed beamforming and AoA estimation strategies.**

Keywords : Beamforming, Angle of Arrival (AoA) estimation.

I. Introduction

Various approaches to choosing the weights in a beamformer have been proposed in the literature (see e.g., [1]–[4]). In one of the earliest works [1], the commonly known Minimum Variance Distortionless Response (MVDR) beamformer was first derived. Subsequently, choosing the weights to minimize the output power or variance subject to linear constraints have been studied (see e.g., [2], [3]). Such beamformers are commonly known as Linear Constrained Minimum Variance (LCMV) or Linear Constrained Minimum Power (LCMP) beamformers depending on whether they minimize the output variance or output power respectively. In [3], quiescent pattern constraints are used to specify the beam pattern so that the weights of the beamformer provide a pattern as close as possible to the quiescent pattern in a mean square sense while minimizing the output power or variance.

However, the above referred works deal with narrow-band beamforming and may not be applicable directly to wide band signals, particularly when the beampattern is required to be approximately the same for all the frequencies in the bandwidth under consideration. Frequency Invariant Beamformers (FIBs) are beamformers which result in the beampatterns being approximately constant with respect to frequency over a design bandwidth. Some of the works on the design of frequency invariant beamformers can be found in [5]–[9]. In this paper, we propose a new linear quadratic frequency domain frequency invariant beamforming strategy. The proposed linear quadratic frequency domain frequency invariant beamformer may be viewed as a wide band beamformer with quiescent pattern constraints. The proposed beamformer seeks to provide a pattern as close as possible to the quiescent pattern in a mean square sense for all frequencies in a design bandwidth.

Now, it is known that the problem of estimating the AoA of signals at an antenna array has many applications in sonar and radar, satellite and cellular networks, and wireless ad-hoc and sensor networks. Some of the works that have been done in the area of narrow band AoA estimation include [10]–[11]. MUSIC [10] and ESPRIT [11] rely on signal and noise subspaces related to a set of generalized eigenvectors of the data covariance matrix pertaining to the array output. Several approaches to wide band AoA estimation have also been investigated (see for e.g., [12]–[16]). In [13] for example, a Coherent Signal Subspace (CSS) approach is introduced, where the wide band data is decomposed in to several non overlapping narrow band frequency bins, and the signal subspaces at different frequencies are combined to form a new signal subspace for which a narrow band scheme like MUSIC can be applied. In [15], [16], wide band AoA estimation approaches based on frequency invariant beamformers are proposed. In this paper, we also propose a new AoA estimation technique based on the proposed linear quadratic frequency domain frequency invariant beamforming strategy and blind identification concepts.

The organization of the paper is as follows. In Sec. II, we describe the system model. Sec. III deals with the design of a new linear quadratic frequency domain frequency invariant beamformer and Sec. IV deals with a new AoA estimation strategy. Sec. V gives the simulation results.

Note that we will use bold lowercase and upper case characters to denote vectors and matrices respectively. We will indicate time series with square brackets (e.g,. $\mathbf{x}[k]$) and frequency responses with round brackets (e.g., $\mathbf{x}(f)$).

II. System Model

Consider an uniformly spaced linear array of N antenna elements with inter element spacing d_o. Assume that there are D broadband sources. Let $\breve{\theta}_d$ denote the direction of arrival of the plane wave from source d measured relative to the line joining the array elements, where $0 \leq d \leq D-1$. The signal received at the i-th antenna element at time k for $0 \leq i \leq N-1$ is given by

$$x_i[k] = \sum_{d=0}^{D-1} s_d[k - \tau_i(\breve{\theta}_d)] + n_i[k] \qquad (1)$$

where $s_d[k]$ is the d-th source signal, $n_i[k]$ is zero mean white noise, and $\tau_i(\breve{\theta}_d) = id_o cos(\breve{\theta}_d)/c$, where c is the velocity of light. The Fourier transform of $x_i[k]$ is given by

$$x_i(f) = \sum_{d=0}^{D-1} \exp(-j2\pi f \tau_i(\breve{\theta}_d)) s_d(f) + n_i(f) \qquad (2)$$

Let $\mathbf{x}(f)$ denote the $N \times 1$ vector of frequency responses of signals received by the array and given by

$$\mathbf{x}(f) = [x_0(f),\ x_2(f),\ .....,\ x_{N-1}(f)]^T \qquad (3)$$

The author is with Institute for Infocomm Research.
E-mail: sananth@i2r.a-star.edu.sg
This work was partially supported by A*STAR SERC grant 052-121-0086.

Then

$$\mathbf{x}(f) = \mathbf{A}(\breve{\mathbf{\Theta}}, f)\mathbf{s}(f) + \mathbf{n}(f) \tag{4}$$

where

$$\mathbf{s}(f) = [s_0(f),\ .....,\ s_{D-1}(f)]^T, \quad \breve{\mathbf{\Theta}} = [\breve{\theta}_0,\ ....,\ \breve{\theta}_{D-1}]^T$$

and $\mathbf{n}(f) = [n_0(f),\ .....,n_{N-1}(f)]^T$ is the $N \times 1$ additive noise vector. Moreover

$$\mathbf{A}(\breve{\mathbf{\Theta}}, f) = [\mathbf{a}(\breve{\theta}_0, f),\ ....,\mathbf{a}(\breve{\theta}_{D-1}, f)]$$

is the matrix containing the array manifold vectors, where the $N \times 1$ complex array manifold vector $\mathbf{a}(\theta, f)$ at an angle $\theta \in [0, \pi]$ and frequency f is given by

$$\mathbf{a}(\theta, f) = [1,\ e^{\{-j2\pi f\tau_1(\theta)\}},\ ......, e^{\{-j2\pi f\tau_{N-1}(\theta)\}}]^T \tag{5}$$

Note that $e^{\{-j2\pi f\tau_0(\theta)\}} = 1$ for all $0 \leq \theta \leq \pi$ and all $f > 0$. The beamformer structure adopted in this paper is a frequency domain structure, defined by the quantities $w_i(f)$ for $0 \leq i \leq N-1$ which denote the frequency response of the weight at antenna element i and frequency f. The response of this beamformer for the plane waves arriving at an angle θ at frequency f is

$$q(\theta, f) = \sum_{i=0}^{i=N-1} w_i(f)e^{\{-j2\pi f\tau_i(\theta)\}} \tag{6}$$

This may be rewritten as $q(\theta, f) = \mathbf{w}^H(f)\mathbf{a}(\theta, f)$ where $\mathbf{w}(f) = [w_0(f),\ ...., w_{N-1}(f)]^H$ is the $N \times 1$ filtering vector. The frequency response of the beamformer output is then given by

$$y(f) = \mathbf{w}^H(f)\mathbf{x}(\mathbf{f}) \tag{7}$$

The beamformer output $y[k]$ as a time series is the inverse Fourier transform of $y(f)$.

We will now proceed to give the design methodologies for the frequency invariant beamformer and the AoA estimation strategies.

III. Frequency Invariant Beamformer Design

In this section, we design a frequency invariant beamformer whose response is independent of frequency f, or in other words, $q(\theta, f) \approx q(\theta)$ for all $f \in [f_l, f_h]$, and for all $\theta \in [0, \pi]$.

Firstly, let θ_m for $m = 0, 1, ...M-1$ (M given) be M values equally spaced in $[0, \pi]$. We divide the frequency bandwidth $[f_l, f_h]$ in to Z bins, where the k-th bin for $k = 0, 1, ...Z-1$, starts at $f_k = f_l + k\Delta f$, where $\Delta f = \frac{f_h - f_l}{Z}$. Now, define $w_i(k) = w_i(f_k)$ for $i = 0, 1, ..., N-1$. Also, let

$$\mathbf{w}(k) = [w_0(k),\ ...., w_{N-1}(k)]^H$$

$$\mathbf{q} = [q(\theta_0),\ ...., q(\theta_{M-1})]^H$$

and

$$\mathbf{L}(k) = [\mathbf{a}(\theta_0, f_k),\ ......\mathbf{a}(\theta_{M-1}, f_k)]^H$$

Let $\bar{\mathbf{w}}_Q$ denote a quiescent weight vector whose corresponding quiescent beam pattern $B_Q(\theta) = \bar{\mathbf{w}}_Q^H\mathbf{a}(\theta, \bar{f}_o)$ at center frequency $\bar{f}_o = [(f_l + f_h)/2]$ is $q(\theta)$. Our aim is to design $\mathbf{w}(k)$ for $k = 0, 1, ..., Z-1$ such that

$$\mathbf{w}^H(k)\mathbf{a}(\theta, f_k) \approx \mathbf{w}^H(k_o)\mathbf{a}(\theta, \bar{f}_o) \approx q(\theta)$$

for all $f_k \in [f_l,\ f_h]$ and $f_{k_o} = \bar{f}_o$.

We now design a beamformer that seeks to provide a pattern as close as possible to the quiescent pattern $q(\theta)$ in a mean square sense for all frequencies in a design bandwidth. Our frequency domain frequency invariant beamformer design methodology will be as follows. We initially initialize $\mathbf{w}(0) = \bar{\mathbf{w}}_Q$. We will then recursively determine $\mathbf{w}(k+1)$ from $\mathbf{w}(k)$ using the form

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{u}(k),\ k \geq 0 \tag{8}$$

where $\mathbf{u}(k)$ is a deterministic input called the control vector. At each instant k, we shall determine the control signal $\mathbf{u}(k)$ such that the following cost function $\mathcal{J}$ is minimized.

$$\begin{aligned} \mathcal{J} \quad &= \quad \sum_{k=0}^{Z-1} \left[\ \|\mathbf{L}(k+1)\mathbf{w}(k+1) - \mathbf{q}\|^2_{\tilde{\mathbf{P}}} \ \right] \\ &\quad + \left\{ \sum_{k=0}^{Z-1} \left[\|\bar{\mathbf{w}}_Q - \mathbf{w}(k+1)\|^2_{\tilde{\mathbf{R}}} \ + \ \|\mathbf{u}(k)\|^2 \ \right] \right\} \end{aligned}$$

$\tilde{\mathbf{P}} = \lambda_1 I$ and $\tilde{\mathbf{R}} = \lambda_2 I$ are positive definite weighing matrices. Note that minimizing the above cost function would imply minimizing at every frequency f_k, the following terms - the difference between the quiescent weight vector and the weight vector $\mathbf{w}(k)$, the difference[1] between $\mathbf{w}^H(k)\mathbf{a}(\theta, f_k)$ and $q(\theta)$ for all $f_k \in [f_l, f_h]$ and for all $\theta \in [\theta_0,\ ...., \theta_{M-1}]$, and the energy of the control sequence $\mathbf{u}(k)$ itself. We will rewrite the above cost function in the following form.

$$\mathcal{J} = \left\{ \sum_{k=0}^{Z-1} \ \left[\ \|\bar{\mathbf{L}}(k+1)\mathbf{w}(k+1) - \bar{\mathbf{q}}\|^2_{\bar{P}} \ + \ \|\mathbf{u}(k)\|^2 \ \right] \right\} \tag{9}$$

where

$$\bar{\mathbf{L}}(k) = [\mathbf{L}(k)^T \quad \mathbf{I}_{N \times N}]^T, \quad \bar{\mathbf{q}} = [\mathbf{q}^T \quad \bar{\mathbf{w}}_Q^T]^T$$

$$\bar{\mathbf{P}} = \begin{pmatrix} \tilde{\mathbf{P}} & 0 \\ 0 & \tilde{\mathbf{R}} \end{pmatrix}$$

and $\mathbf{I}_{N \times N}$ represents the Identity matrix. We now wish to choose the sequence of control vectors $\{\mathbf{u}(k)\}$ such that the objective function $\mathcal{J}$ is minimized. It is known that the solution to the above problem is given as follows [17].

$$\begin{aligned} \mathbf{u}(k) \quad &= \quad [\mathbf{M}(k+1) + \mathbf{I}]^{-1}[-\mathbf{M}(k+1)\mathbf{w}(k) + \mathbf{\Gamma}(k)] \\ \mathbf{\Gamma}(k) \quad &= \quad [I - \mathbf{M}(k+1)[I + \mathbf{M}(k+1)]^{-1}]\mathbf{\Gamma}(k+1) \\ &\qquad + \bar{\mathbf{L}}(\mathbf{k})^T\bar{\mathbf{P}}\bar{\mathbf{q}} \\ \mathbf{M}(k) \quad &= \quad \mathbf{M}(k+1)[I + \mathbf{M}(k+1)]^{-1} + \bar{\mathbf{L}}(k)^T\bar{\mathbf{P}}\bar{\mathbf{L}}(k) \end{aligned}$$

with initial conditions

$$\mathbf{M}(Z) \quad = \quad \bar{\mathbf{L}}(Z)^T\bar{\mathbf{P}}\bar{\mathbf{L}}(Z), \quad \mathbf{\Gamma}(Z) = \bar{\mathbf{L}}(Z)^T\bar{\mathbf{P}}\bar{\mathbf{q}}$$

[1]Note that we have incorporated a quiescent pattern constraint over $\theta \in [\theta_0,\ ...., \theta_{M-1}]$.

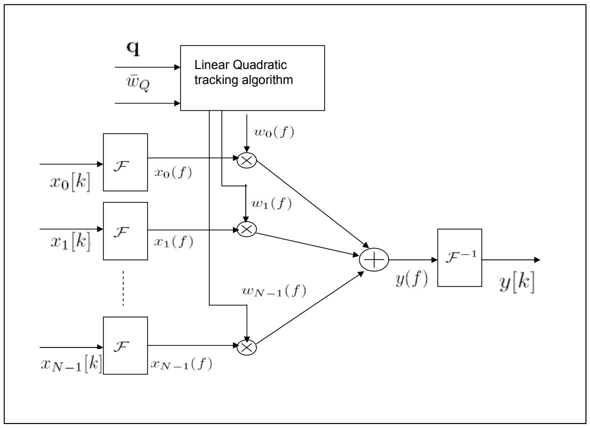

Fig. 1. *Frequency domain frequency invariant beamformer architecture.*

Once the control vectors $\mathbf{u}(k)$ are known, then the frequency domain frequency invariant beamformer weights $\mathbf{w}(f)$ at $f = f_k$ can be calculated [2]. See Fig. 1 for a block diagram of the proposed beamformer.

IV. Angle of Arrival Estimation Algorithm

In this section, we give a new angle of arrival estimation algorithm based on the linear quadratic frequency domain frequency invariant beamforming methodology described in the previous section. Assume that we now form J ($D < J$) frequency invariant beamformers using J different quiescent patterns with the methodology described in the previous section. Let the frequency response of the j-th beamformer be $\mathbf{w}_j(f)$, for $j = 0, 1, ...J-1$. Define

$$r_j(\theta) \triangleq |\mathbf{w}_j^H(\bar{f}_o)\mathbf{a}(\theta, \bar{f}_o)| \approx |\mathbf{w}_j^H(f_k)\mathbf{a}(\theta, f_k)|$$

and

$$\mathbf{r}_j(\breve{\boldsymbol{\Theta}}) = [r_j(\breve{\theta}_0), ..., r_j(\breve{\theta}_{D-1})]$$

for $j = 0, 1, ...J-1$. Let the J beamformers be chosen such that $r_j(\theta) > r_i(\theta)$ for all $j < i$ and for all θ, i.e, the J beampattern gains are ordered for all θ. Define the vector $\mathbf{b}(\theta) = [r_0(\theta) \;\; r_1(\theta) \; ... \; r_{J-1}(\theta)]^T$. Denote the stacked vector of beamformer responses as

$$\mathbf{z}(f) \;\; = \;\; [\mathbf{w}_0^H(f) \; \mathbf{w}_1^H(f) \; ..\mathbf{w}_{J-1}^H(f)]^T \mathbf{x}(f)$$

Let

$$\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}}, f) = [\mathbf{w}_0^H(f) \; \mathbf{w}_1^H(f) \; ..\mathbf{w}_{J-1}^H(f)]^T \mathbf{A}(\breve{\boldsymbol{\Theta}}, f)$$

We then have using equation (4) that

$$\mathbf{z}(f) = \mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}}, f)\mathbf{s}(f) + \mathbf{n}_\mathbf{b}(f) \qquad (10)$$

where $\mathbf{n}_\mathbf{b}(f)$ is zero mean noise vector. Now, due to the frequency invariant nature of the beamformers, we have that $\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}}, f) = \mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ for all $f \in [f_l, f_h]$. Hence

$$\mathbf{z}(f) = \mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})\mathbf{s}(f) + \mathbf{n}_\mathbf{b}(f) \qquad (11)$$

[2]Note that, in practice the Fourier transform and the inverse Fourier transform blocks are implemented using FFT and IFFT and hence only $\mathbf{w}(k)$ which are $\mathbf{w}(f)$ at $f = f_k$ are needed.

We assume that the beamformers are designed in such a way that the matrix $\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ is full rank. Note that if $\mathbf{M}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ represents the matrix with components that are magnitudes of corresponding components in $\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$, then it can be seen that the j-th row of $\mathbf{M}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ is $\mathbf{r}_j(\breve{\boldsymbol{\Theta}})$ and the d-th column of $\mathbf{M}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ is $\mathbf{b}(\breve{\theta}_d)$. Since the J beampattern gains are ordered, the largest element of any column in the matrix $\mathbf{M}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ is always in the first row, and each of the columns of matrix $\mathbf{M}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ are ordered. Now using equation (11) and calling upon any blind identification algorithm like AMUSE [18] or SOBI [19], we can estimate the matrix $\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$. We list below the steps to estimate the matrix $\mathbf{A}_\mathbf{b}(\breve{\boldsymbol{\Theta}})$ based on AMUSE [18] and also show how the indeterminacy problem in blind identification due to gain is solvable[3].

1. Estimate the covariance matrix $\mathbf{R}_z = E(\mathbf{z}(f)\mathbf{z}(f)^H)$.
2. Compute an SVD of $\mathbf{R}_z$ as follows.

$$\mathbf{R}_z = [\mathbf{u}_1, ..., \mathbf{u}_N]\mathrm{diag}(\lambda_1^2, \lambda_2^2, ...\lambda_N^2)[\mathbf{u}_1, ..., \mathbf{u}_N]^H$$

3. Estimate the number of sources D from the number of significant singular values. Estimate the noise variance σ^2 from the insignificant singular values.
4. Let $\mu_i = \sqrt{(}\lambda_i^2 - \sigma^2)$ for $i = 1, 2, ...D$. Let $\bar{\mathbf{z}}(f) = \bar{\mathbf{T}}\mathbf{z}(f)$ where

$$\mathbf{U}_s = [\mathbf{u}_1, ..., \mathbf{u}_D], \quad \bar{\mathbf{T}} = diag(\frac{1}{\mu_1}, \frac{1}{\mu_2}, ..., \frac{1}{\mu_D})\mathbf{U}_s^H$$

5. Select a τ such that $(\bar{\mathbf{R}}_z(\tau) + \bar{\mathbf{R}}_z(\tau)^H)/2$ has distinct eigenvalues, where $\bar{\mathbf{R}}_z(\tau) = E(\bar{\mathbf{z}}(f)\bar{\mathbf{z}}(f+\tau)^H)$.
6. Let $\mathbf{V}$ be the eigenmatrix obtained from the eigen decomposition of $(\bar{\mathbf{R}}_z(\tau) + \bar{\mathbf{R}}_z(\tau)^H)/2$. Determine$\hat{\mathbf{A}} = \bar{\mathbf{T}}^H\mathbf{V}$.
7. Form the matrix $\hat{\mathbf{M}}$ whose u-th row and v-th column element is the magnitude of the corresponding u-th row and v-th column element of $\hat{\mathbf{A}}$. Let the d-th column of $\hat{\mathbf{M}}$ be denoted by $\mathbf{c}_d$.
8. The estimate of the d-th angle of arrival is given by $\acute{\breve{\theta}}_d = \arg \min_{\theta, 0<\alpha<\alpha_{\max}} \|\alpha\mathbf{c}_d - \mathbf{b}(\theta)\|$, where $\alpha_{\max}$ is such that $\alpha_{\max}\mathbf{c}_d$ is greater than the maximum element[4] of $\mathbf{b}(\theta)$ for all[5] θ.

V. Simulations

In order to illustrate the performance of the proposed frequency domain frequency invariant beamforming and AoA estimation strategies, a three element antenna array is considered with inter element spacing of 0.05m. Figs 2–3 illustrate the performance of the proposed frequency domain frequency invariant beamforming strategy. To illustrate the design methodology (described in Sec. III), we choose a quiescent weight vector as

$$\bar{\mathbf{w}}_\mathbf{Q} = [0.4 + 0.0316i, \; 0.4821 - 0.0013i, \; 0.0718 - 0.0288i]^T$$

The bandwidth under consideration is $[2.0 \text{ GHz}, \; 2.2 \text{ GHz}]$ with center frequency $\bar{f}_o = 2.1$ GHz. Fig. 2 shows the beampatterns for the frequency invariant beamformer designed using $\bar{\mathbf{w}}_\mathbf{Q}$ at 25 different frequencies in the bandwidth under

[3]Indeterminacy due to permutation does not affect the extraction of the D angles of arrival.

[4]Note that $\mathbf{b}(\theta)$ is known for all θ from the design of the beamformers.

[5]Note that, any other blind identification algorithm may also be used in conjunction with equation (11) and steps 7 and 8.

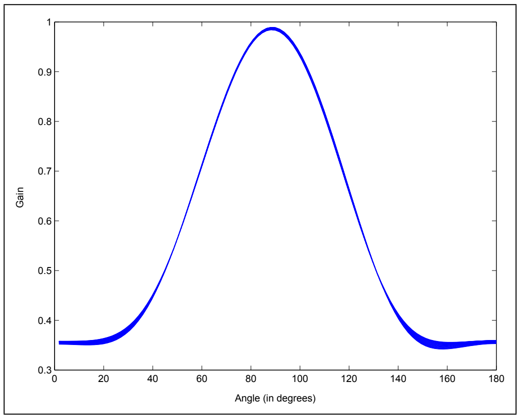

Fig. 2. *Beampatterns (designed from $\bar{\mathbf{w}}_{\mathbf{Q}}$) for twenty five different frequencies in* $[2.0\ \text{GHz},\ 2.2\ \text{GHz}]$.

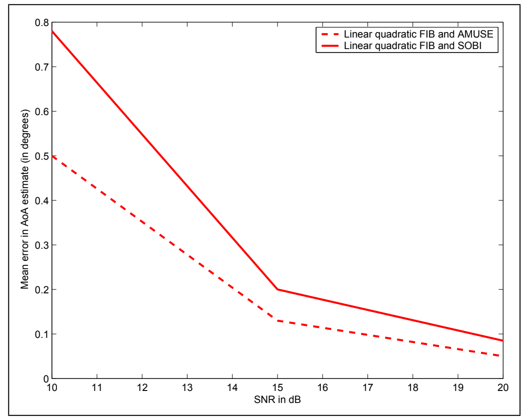

Fig. 4. *Mean error in AoA estimation for angles of arrival as* 40 *degrees.*

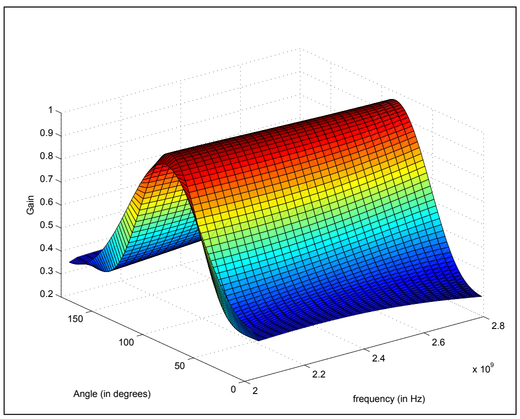

Fig. 3. *Beampatterns (designed from $\bar{\mathbf{w}}_{\mathbf{Q}}$) for* $f \in [2.0\ \text{GHz},\ 2.8\ \text{GHz}]$.

consideration. Fig. 3 shows the three dimensional plot of beampattern gains with respect to frequency and angle. To illustrate the performance of the proposed AoA estimation algorithm, number of sources is assumed 1 whose signal arrives at the antenna array at 40 degrees. Fig. 4 illustrates the performance of the proposed AoA estimation strategy based on AMUSE [18] and SOBI [19].

VI. Conclusions

In this paper, we have presented a new linear quadratic frequency domain frequency invariant beamforming strategy and a blind identification algorithm based AoA estimation technique. The proposed frequency domain beamformer is obtained by the solution to a quadratic cost minimization problem. The AoA estimation technique uses the proposed frequency invariant beamforming structure and a blind identification algorithm.

References

[1] J. Capon, High resolution frequency-wavenumber spectral analysis, *Proc. IEEE*, vol. 57, pp. 1408–1418, Aug. 1969.

[2] O. L. Frost III, An algorithm for linearly constrained adaptive array processing, *Proc. IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[3] L. J. Griffiths and K. M. Buckley, Quiescent pattern control in linearly constrained adaptive arrays, *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-35, pp. 917–926, Jul. 1987.

[4] Z. Tian, K. L. Bell, and H. L. Van Trees, A recursive least squares implementation for LCMP beamforming under quadratic constraint, *IEEE Trans. on Signal Processing*, vol. 49, no.6, pp. 1138–1145, Jun. 2001.

[5] M. M. Goodwin and G. W. Elko, Constant beamwidth beamforming, *Proc. ICASSP*, vol. 1, pp. 169-172, 1993.

[6] T. Chou, Frequency-independent beamformer with low response error, *Proc. ICASSP*, pp. 2995-2998, May. 1995.

[7] D. B. Ward, R. A. Kennedy, and R. C. Williamson, FIR filter design for frequency-invariant beamformers, *IEEE Signal Processing Lett.*, vol. 3, pp. 69-71, Mar. 1996.

[8] I. D. Dotlic, Minimax frequency invariant beamforming, *Electronic Letters*, vol. 40, no. 9, Sept. 2004.

[9] W. Liu and S. Weiss, Frequency invariant beamforming in subbands, *Proc. Asilomar Conf. Signals, Syst., and Comp.*, vol. 2, pp. 1968–1972, Nov. 2004.

[10] R. O. Schmidt, Multiple emitter location and signal parameter estimation, *IEEE Trans. Antennas and Propagation*, vol. AP-34, pp. 276–280, Mar. 1986.

[11] R. Roy and T. Kailath, ESPRIT - Estimation of signal parameters via rotational invariance techniques, *IEEE Transactions on Acoustics, speech, and signal processing*, vol. 37, no. 7, pp. 984–994, Jul. 1989.

[12] M. Agrawal and S. Prasad, DOA estimation of wideband sources using a harmonic source model and uniform linear array, *IEEE Transactions on Signal Processing*, vol. 47, no. 3, pp. 619–629, Mar. 1999.

[13] H. Wang and M. Kaveh, Coherant signal-subspace processing for the detection and estimation of angles of arrival of multiple wideband sources, *IEEE Trans. Acoust. Speech and Signal Processing*, vol. 33, pp. 823–831, Aug. 1985.

[14] T. S. Lee, Efficient wideband source localization using beamforming invariance technique, *IEEE Trans. Signal Processing*, vol. 42, pp. 1376–1387, Jun. 1994.

[15] D. B. Ward, Z. Ding, and R. A. Kennedy, Broadband DOA estimation using frequency invariant beamforming, *IEEE Transactions on Signal Processing*, vol. 46, no. 5, pp. 1463–1469, May. 1998.

[16] T. Do-Hong, F. Demmel, and P. Russer, Wideband direction of arrival estimation using frequency-domain frequency invariant beamformers: An analysis of performance, *IEEE Microwave and Wireless Components Letters*, vol. 14, no. 8, pp. 383–385, Aug. 2004.

[17] D. E. Catlin, *Estimation, control, and the discrete Kalman filter*, Sringer-Verlag, NY, 1989.

[18] L. Tong, R. Liu, and et. al., Indeterminacy and identifiability of blind identification, *IEEE Trans. on Circuits and Systems*, vol. 38, no. 5, pp. 409–509, May. 1991.

[19] A. Belouchrani, K. A. Meraim, and et.al., A blind source separation technique using second order statistics, *IEEE Trans. on Signal Processing*, vol. 45, no. 2, pp. 434-444, Feb. 1997.