ESTIMATING THE PARAMETERS OF MULTIPLE WIDEBAND POLYNOMIAL-PHASE SIGNALS IN SENSOR ARRAYS USING SPATIAL TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT

A new algorithm for estimating the parameters of multiple wideband polynomial-phase signals (PPSs) in sensor arrays is developed. The spatial high-order instantaneous moments (SHIMs) are first defined using a nonlinear transformation of the array snapshot vectors. Then, the properties of SHIMs of multiple wideband PPSs are employed to obtain recursive estimates of the PPS frequency parameters. The time-frequency properties of SHIMs are then exploited for estimating the source directions-of-arrival (DOAs) using spatial time-frequency distributions (STFDs). The proposed algorithm is shown to have an improved performance compared to the *chirp beamformer* technique. Since our algorithm is based on multiple one-dimensional searches, it also offers a much simpler implementation avoiding multi-dimensional search needed for other algorithms.

Index Terms— Array signal processing, direction-of-arrival estimation, parameter estimation

1. INTRODUCTION

The estimation of the parameters of polynomial-phase signals (PPSs) is an important problem encountered in many practical applications such as radar, sonar and mobile communications. In such applications, the received signal waveforms can often be modeled as PPSs [1], [2]. FM signals can also be intentionally transmitted in synthetic aperture radar (SAR), synthetic aperture sonar (SAS), inverse SAR, inverse SAS, Doppler radar/sonar, and mobile communication systems. There are numerous contributions to the problem of PPS parameter estimation in the single antenna case, see for example [1], [2] and references therein. In particular, high-order instantaneous moments (HIMs) and their Fourier-transformed (referred to as high-order ambiguity functions) have been used in [1] to derive a simple and computationally attractive algorithm for estimating the PPS parameters.

Recently, estimating the parameter of multiple PPSs in sensor arrays has gained a considerable interest [3]-[6]. Several methods that solve this problem using narrowband assumptions have been reported in the literature. For example, a new class of subspace methods that are based on spatial time-frequency distributions (STFDs) has been proposed in [3] to estimate the DOAs of narrowband chirp sources. Several approaches for estimating the parameters of multiple wideband PPSs have also been reported, see [4]-[6]. In particular, an exact ML estimator that takes advantage of the specific PPS structure of the source waveforms has been proposed in [4]. In the same paper, the so-called *chirp beamformer* has been developed that is a suboptimal estimator resulting from the analysis of the log-likelihood function in the single linear FM source case. However, chirp beamformer involves a multi-dimensional search, and hence, it is computationally expensive. Moreover, it suffers from a large bias even at high SNRs.

Another ambiguity function-based approach to DOA estimation of PPS sources has been developed in [6]. However, the method of [6] assumes that the signal initial frequencies and the chirp rates are known or preestimated. Moreover, the latter approach cannot be used for estimating the parameters of wideband PPSs of order higher than two.

In this paper, we introduce a new algorithm for estimating the parameters of multiple wideband PPSs in sensor arrays. We first introduce the so-called spatial high-order instantaneous moments (SHIMs) that are obtained by means of a specific nonlinear transformation of the data snapshots [7], [8]. Then, we employ the SHIM properties to obtain estimates of the frequency parameters in a recursive manner starting with the highest-order frequency coefficients. Also, we exploit SHIMs to estimate the DOAs using a STFD-based approach. Specifically, STFD matrices computed at certain time-frequency points that belong to the SHIM signatures are used to obtain the signal and noise subspaces. Our approach is much more computationally attractive than the chirp beamformer technique because it requires a reasonably small num-

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ber of one-dimensional searches instead of a multi-dimensional search. Moreover, the proposed estimation technique has a substantially improved performance as compared to the chirp beamformer method of [4].

2. ARRAY SIGNAL MODEL

Let us consider L wideband constant-amplitude PPSs impinging on a linear array of M omnidirectional sensors. The vector of array sensor outputs can be modeled as

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t), \quad t = 0, 1, \ldots, N-1 \tag{1}$$

where $\mathbf{A}(t)$ is the $M \times L$ time-varying direction matrix, $\mathbf{s}(t)$ is the $L \times 1$ vector of wideband polynomial-phase source waveforms, N is the number of array snapshots, and t is the discrete-time index. The lth source waveform can be modeled as

$$s_l(t) = \alpha_l \exp\left\{ j \sum_{k=1}^{K} \frac{\omega_{l,k} t^k}{k} \right\}$$

where K is the known PPS order, $\omega_{l,k}$ ($l = 1, \ldots, L; k = 1, \ldots, K$) are the unknown discrete-time frequency parameters, α_l is a complex-valued initial source amplitude, and $j = \sqrt{-1}$. The matrix $\mathbf{A}(t) \triangleq [\mathbf{a}(\tilde{\omega}_1(t), \theta_1), \ldots, \mathbf{a}(\tilde{\omega}_L(t), \theta_L)]$ consists of the time-varying steering vectors

$$\begin{aligned}\mathbf{a}(\tilde{\omega}_l(t), \theta_l) \ = \ \Bigg[1, \exp\left\{ j \, \frac{\tilde{\omega}_l(t)}{c\,\Delta t} d_1 \sin\theta_l \right\}, \\ \ldots, \exp\left\{ j \, \frac{\tilde{\omega}_l(t)}{c\,\Delta t} d_{M-1} \sin\theta_l \right\}\Bigg]^T\end{aligned} \tag{2}$$

where Δt is the sampling time interval, d_m is the interelement spacing between the 1st and $(m+1)$th sensors, c is the wave propagation speed, θ_l is the DOA of the lth source, $\tilde{\omega}_l(t) \triangleq \sum_{k=1}^{K} \omega_{l,k} t^{k-1}$ is the discrete-time instantaneous frequency of the lth waveform, and $(\cdot)^T$ stands for the transpose. Note that in (2) it is assumed that the instantaneous signal frequencies $\tilde{\omega}_l(t)$ ($l = 1, \ldots, L$) do not change during the time necessary for a wave to travel across the array aperture, i.e., the signals remain narrowband within each snapshot, while being wideband at the observation interval of the N samples [4]. Our objective is to estimate the unknown discrete-time frequency parameters $\omega_{l,k}$ ($k = 1, \ldots, K; l = 1, \ldots, L$) and the unknown DOAs θ_l ($l = 1, \ldots, L$).

3. SHIMS AND THEIR PROPERTIES

Let us define the kth-order SHIM ($k > 1$) of the data vector $\mathbf{x}(t)$ using the following rule

$$\mathbf{x}^{(k)}(t) = \mathbf{x}^{(k-1)}(t+\tau) \odot \left(\mathbf{x}^{(k-1)}(t-\tau) \right)^* \tag{3}$$

where τ is a discrete-time lag (positive number), $\odot$ is the Schur-Hadamard (elementwise) product, $(\cdot)^*$ is the conjugation operator, and $\mathbf{x}^{(1)}(t) \triangleq \mathbf{x}(t)$. The kth-order SHIM can be viewed as a numerical differentiator of the kth order and represents a multi-antenna extension of the corresponding kth-order HIM [1]. Applying (3) to (1), we obtain the following relationship [7]:

$$\begin{aligned}\mathbf{x}^{(K)}(t) \ = \ \sum_{l=1}^{L} s_l^{(K)}(t) \, \mathbf{a}(\Omega_l^{(K)}, \theta_l) \\ + \mathbf{n}^{(K)}(t) + \text{crossterms}\end{aligned} \tag{4}$$

where $s_l^{(K)}(t) \triangleq \alpha_l^{(K)} \exp\{j\Omega_l^{(K)} t\}$ is a harmonic waveform with the complex amplitude $\alpha_l^{(K)}$ (which is not explicitly defined here for the sake of brevity; see [7] for more details), and

$$\Omega_l^{(K)} = (2\tau)^{K-1} (K-1)! \, \omega_{l,K} \, . \tag{5}$$

In (4), $\mathbf{n}^{(K)}(t)$ is the vector that captures all the noise terms contained in the Kth-order SHIM, and the SHIM signature vector associated with $s_l^{(K)}(t)$ is given by

$$\begin{aligned}\mathbf{a}(\Omega_l^{(K)}, \theta_l) = \Bigg[1, \exp\left\{ j \frac{\Omega_l^{(K)}}{c\Delta t} d_1 \sin\theta_l \right\}, \\ \ldots, \exp\left\{ j \frac{\Omega_l^{(K)}}{c\Delta t} d_{M-1} \sin\theta_l \right\}\Bigg]^T .\end{aligned} \tag{6}$$

Note that the frequencies of the sinusoidal components contained in $\mathbf{x}^{(K)}(t)$ are directly related to the highest-order frequency parameters of the original PPSs. Hence, (5) provides a basis for estimating $\omega_{l,K}$ via $\Omega_l^{(K)}$. The simplest way to estimate $\{\Omega_l^{(K)}\}_{l=1}^{L}$ is to search for the L main peaks of the frequency spectrum of each row of a SHIM data matrix composed of the vectors $\mathbf{x}^{(K)}(t)$ taken at different time indices. This means that M estimates can be obtained for each frequency parameter. These multiple estimates can be combined in a certain way (for example, through their averaging, taking their median value, etc.) to obtain a better final estimate of each frequency parameter.

As can be observed from (6), the SHIM signature vectors do not depend on time. This opens an avenue for estimating the source DOAs using subspace-based techniques. The structure of (4) is also suitable for using STFD-based methods [3] for estimating the signal and noise subspaces.

4. THE PROPOSED ALGORITHM

In this section, we propose a computationally simple algorithm to estimate the source parameters. Our algorithm can be summarized as follows.

Step 1: *Estimate the highest-order frequency parameters.*

Set $k = K$, choose a positive value τ, compute the vectors $\mathbf{x}^{(k)}(t)$ for different values of t, and form the SHIM data matrix based on these vectors.

Compute the M estimates $\hat{\Omega}_{l,m}^{(k)}$ $(m = 1, \ldots, M)$ by searching for the L highest peaks of the DFT spectrum of each row of the SHIM data matrix, and obtain $\hat{\omega}_{l,k,m} = \frac{\hat{\Omega}_{l,m}^{(k)}}{(k-1)!(2\tau)^{k-1}}$ for all $m = 1, \ldots, M$.

Compute the estimates $\hat{\omega}_{l,k}$ of $\omega_{l,k}$ as the mean (or median) of $\hat{\omega}_{l,k,m}$, $m = 1, \ldots, M$.

Step 2: *Estimate the source DOAs.*

Compute the pseudo Wigner-Ville time-frequency distribution (PWVTFD) in one sensor (or the averaged PWVTFD over all sensors) and detect the points that belong to the time-frequency signatures[1] of the harmonic components in (4). At each such point, compute the spatial pseudo Wigner-Ville distribution (SPWVD) matrices

$$\hat{\mathbf{D}}_{\mathbf{x}^{(K)}}(t,f) = \sum_{T} \mathbf{x}^{(K)}(t+T)\mathbf{x}^{(K)^H}(t-T)e^{-j4\pi fT}$$

where T is the discrete-time SPWVD lag.

Compute the STFD matrix as the average of these SPWVD matrices, and use the eigendecomposition of the latter matrix to estimate the signal and noise subspaces.

Compute the estimates $\{\hat{\theta}_l\}$, $l = 1, \ldots, L$ of the source DOAs using any subspace-based method (e.g., MUSIC).

Step 3: *Estimate the frequency parameters $\omega_{l,k}$ ($l = 1, \ldots, L$; $k = 1, \ldots, K-1$).*

Set $k = k - 1$. Remove the contribution of the already estimated frequency parameters by computing the compensated data snapshots

$$\tilde{\mathbf{x}}_{l,k}(t) = e^{-j\sum_{n=k+1}^{K} \hat{\omega}_{l,n} t^n/n} \mathbf{x}(t) \odot \tilde{\mathbf{a}}_{l,k}$$

where

$$\tilde{\mathbf{a}}_{l,k} \triangleq \left[1, \exp\left\{-j\frac{d_1}{c\Delta t}\sum_{n=k+1}^{K} \hat{\omega}_{l,n} t^{n-1} \sin\hat{\theta}_l\right\}, \right.$$
$$\left. \ldots, \exp\left\{-j\frac{d_{M-1}}{c\Delta t}\sum_{n=k+1}^{K} \hat{\omega}_{l,n} t^{n-1} \sin\hat{\theta}_l\right\}\right]^T.$$

For each source index $l = 1, \ldots, L$, compute the kth-order SHIMs of the compensated data snapshots $\tilde{\mathbf{x}}_{l,k}(t)$ and form the SHIM data matrix. Then obtain the estimates $\hat{\Omega}_{l,m}^{(k)}$ by searching for the highest peak of the DFT spectrum of the mth row of the SHIM data matrix. Find $\hat{\omega}_{l,k,m} = \frac{\hat{\Omega}_{l,m}^{(k)}}{(2\tau)^{k-1}(k-1)!}$ and compute the estimates $\hat{\omega}_{l,k}$ of $\omega_{l,k}$ as the mean (or median) of $\hat{\omega}_{l,k,m}$, $m = 1, \ldots, M$.

Repeat the latter step until $k = 1$.

Step 4: *Refine the frequency parameter estimates using the ML principle.*

Form the matrices $\boldsymbol{G}_m$ $(m = 1, \ldots, M)$, each of size $L \times K$, with the (l,k)th element of $\boldsymbol{G}_m$ being $\hat{\omega}_{l,k,m}$. For each $\boldsymbol{G}_m$, $m = 1, \ldots, M$, evaluate the negative log-likelihood function [4]

$$\sum_{t=0}^{N-1} \|\mathbf{x}(t) - \mathbf{A}(t, \boldsymbol{G}_m, \hat{\boldsymbol{\theta}}_l)\mathbf{s}(t, \boldsymbol{G}_m)\|^2 \qquad (7)$$

where $\hat{\boldsymbol{\theta}}_l \triangleq [\hat{\theta}_1, \ldots, \hat{\theta}_L]^T$. Choose the entries of the particular $\boldsymbol{G}_m$ that minimizes (7) as the final estimate of the frequency parameters.

Note that, in contrast to chirp beamformer, the proposed algorithm does not require any multi-dimensional search over the DOA and frequency parameters. It uses multiple one-dimensional searches instead, and this greatly reduces its computational complexity as compared to chirp beamformer.

5. SIMULATION RESULTS

In our simulations, we assume a ULA of $M = 10$ omnidirectional sensors. Following [4], a sonar-type scenario is considered with the sound propagation speed $c = 1500$ m/s and the interelement spacing $d = 1.5$ m. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally white process that has identical variances in each array sensor. We assume that the ULA receives two equipowered chirp signals impinging on the array from the directions $\theta_1 = 10°$ and $\theta_2 = 20°$ relative to the broadside and having the initial continuous-time frequencies 408 Hz and 401 Hz, respectively, and the continuous-time chirp rates -50 Hz/s and 60 Hz/s, respectively. The signals are sampled with $\Delta t = 0.0039$ s and the observation interval of $N = 256$ snapshots is taken. Prior to the processing, the received signals are downconverted to the baseband frequency interval. The second-order SHIM $\mathbf{x}^{(2)}(t)$ of the received data is computed using the time lag of $\tau = 25\Delta t$. For each time-frequency signature that corresponds to harmonic waveforms, the SPWVD matrices are computed at 200 most significant time-frequency points that belong to that signature. The root-MUSIC algorithm is used to obtain the DOA estimates using the averaged STFD matrix. A total of 500 independent Monte-Carlo simulation runs have been used to obtain each point in simulations. The experimental RMSEs of the initial frequency, chirp rate, and DOA estimates are shown in

[1]Techniques to detect such signature points are available in the literature [9].

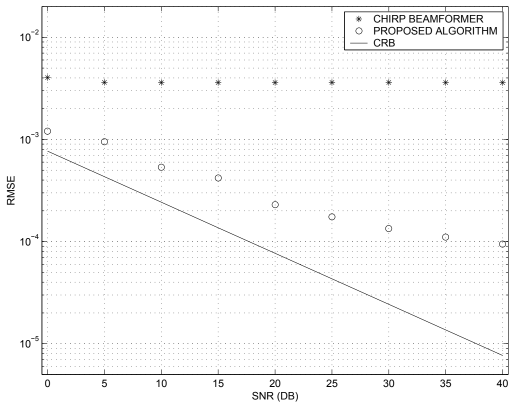

Fig. 1. Initial frequency estimation RMSEs versus SNR.

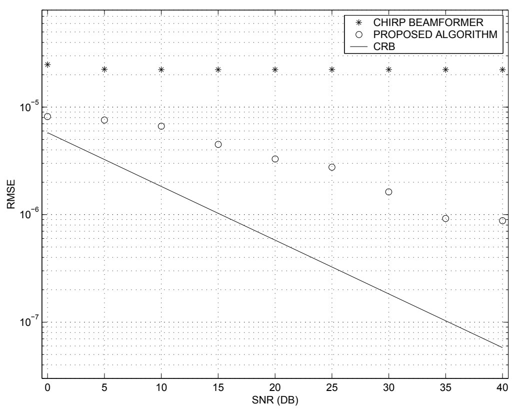

Fig. 2. Chirp rate estimation RMSEs versus SNR.

Figs. 1, 2, and 3, respectively, along with the PPS CRB of [4]. From these figures it can be observed that, although the proposed method does not achieve the corresponding CRB, it has a much better performance than the chirp beamformer technique.

6. CONCLUSIONS

A new algorithm to estimate the parameters of multiple wideband polynomial-phase signals in sensor arrays is proposed. The properties of spatial higher-order instantaneous moments are employed to recursively estimate the signal frequency parameters and to find the estimates of the source DOAs using the spatial time-frequency distributions approach. Simulation results illustrate substantial performance improvements achieved by the proposed approach relative to the earlier chirp beamforming technique.

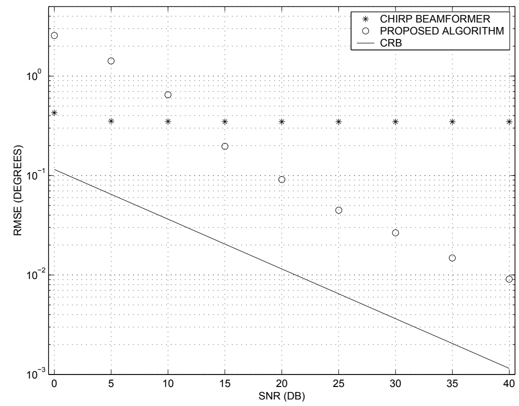

Fig. 3. DOA estimation RMSEs versus SNR.

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