FAST ALGORITHM FOR JOINT AZIMUTH AND ELEVATION ANGLES, AND FREQUENCY ESTIMATION VIA HIERARCHICAL SPACE-TIME DECOMPOSITION

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ABSTRACT

This paper presents a fast algorithm for joint estimation of the azimuth and elevation angles, and frequencies of the incoming signals using a hierarchical space-time decomposition (HSTD) technique. Based on the HSTD, the proposed algorithm makes use of a sequence of one-dimensional (1-D) Unitary Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithms to estimate these parameters alternatively in a hierarchical tree structure. Also, in between every other 1-D Unitary ESPRIT, a temporal filtering process or a spatial beamforming process is invoked to partition the signals into finer groups to enhance the estimation accuracy and to alleviate the contaminated noise. Furthermore, the pairing of these parameters is automatically determined. Simulation results show that the new algorithm provides satisfactory performance but with drastically reduced computations compared with previous works. Simulation results show that the new algorithm provides satisfactory performance but with drastically reduced computations compared with previous works.

Index Terms— 2-D DOA Estimation, array signal processing, multidimensional signal processing.

1. INTRODUCTION

Joint estimation of azimuth and elevation angles, and carrier frequencies of multiple sources is of importance in wireless communications. For example, these parameters can be applied to locating the mobiles and to allocating pilot tones in space division multiple access systems [1]. Also, a precise estimation of these parameters is helpful in obtaining a better channel estimate and thus enhances the system performance. Therefore, joint estimation of these parameters has received lots of attention recently.

Various higher-dimensional subspace based algorithms such as MUltiple SIgnal Classification (MUSIC) [2] or ESPRIT [3]-based algorithms have been reported to jointly estimate these three parameters. The latter are in particular computationally attractive, as they are free of higher-dimensional search on the azimuth-elevation-frequency plane. For example, Haardt *et al.* [1] addressed a 3-D unitary-ESPRIT to estimate these three parameters by extending the computationally efficient 1-D Unitary ESPRIT to 3-D scenarios. Strobach [4] incorporated a total least squares phased averaging method in the 3-D ESPRIT for signal subspace estimates to increase the estimation accuracy. Despite the effectiveness of [1, 4], they call for high computational overhead due to higher-dimensional data stacking and eigendecomposition. On the other hand, an efficient ESPRIT algorithm was recently considered in [5], which, however, needs a special antenna array geometry.

In order to yield high estimation accuracy with low computational complexity, this paper proposes an HSTD-based algorithm for joint estimation of the azimuth and elevation angles, and frequencies of the incoming signals. The essence of the proposed algorithm lies in a succinct combination of the parameter estimation and temporal filtering/spatial beamforming processes, in which the parameters are estimated alternatively in a hierarchical tree structure. More specifically, the proposed algorithm makes use of a sequence of 1-D Unitary ESPRIT algorithms to estimate these parameters in a coarse-fine manner. In addition, to enhance the estimation accuracy, in between every other 1-D ESPRIT algorithm, a temporal filtering process or a spatial beamforming process is invoked to partition the incoming signals into finer groups. Based on such an HSTD technique, not only the estimation accuracy is enhanced, but the pairing of these parameters is automatically achieved without extra computations. Simulation results show that the new algorithm provides satisfactory performance but with drastically reduced computations compared with previous works.

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2. SIGNAL MODEL

Consider a uniform rectangular array (URA) with $M \times N$ omnidirectional antennas. Assume that there are K uncorrelated narrowband sources $\{s_k(t)\}$, each of which is carried by the frequency f_k, impinging on the URA, where each antenna is followed by a tapped delay line with L time delay elements of delay T_s, as shown in Fig.1.

By sampling the output of each antenna at a rate $f_s = 1/T_s$, the observed signal at time t at the l^{th} delay element output of the $(m,n)^{th}$ antenna element can be expressed as

$$\begin{aligned} x_{mnl}(t) &= \sum_{k=1}^{K} s_k(t)[e^{-j2\pi(\frac{M-1}{2}-m)u_k} e^{-j2\pi(\frac{N-1}{2}-n)v_k} \\ &\quad \cdot e^{-j2\pi(\frac{L-1}{2}-l)f_k T_s}] + n_{mnl}(t), \end{aligned} \tag{1}$$

where $u_k = \frac{d \cdot f_k}{c} \sin\phi_k \cos\theta_k$ and $v_k = \frac{d \cdot f_k}{c} \sin\phi_k \sin\theta_k$, in which ϕ_k and θ_k are the elevation and azimuth angles of the k^{th} signal, respectively, c is the wave propagation speed, and d is the antenna spacing which is equal to half of the wavelength. $n_{mnl}(t)$ denotes the white noise with power σ_n^2 at the l^{th} delay element of the $(m,n)^{th}$ antenna. Note that for the employment of the Unitary ESPRIT, the reference point is set at the center of the URA and, without loss of generality, M N, and L are all assumed to be even.

3. PROPOSED FAST ALGORITHM

The proposed algorithm begins with the estimation of the frequencies of the incoming signals, as in general we have more temporal data to render a precise frequency estimate. To achieve this, we first construct $\mathbf{X}_f(t)$ by stacking $x_{mnl}(t)$ as

$$\mathbf{X}_f(t) = [\mathbf{x}_{11}^{\swarrow}(t), \ldots, \mathbf{x}_{M1}^{\swarrow}(t) \vdots \cdots \vdots \mathbf{x}_{1N}^{\swarrow}(t), \ldots, \mathbf{x}_{MN}^{\swarrow}(t)] \quad (2)$$

where $\mathbf{x}_{mn}^{\swarrow}(t) = [x_{mn1}(t), \cdots, x_{mnL}(t)]^T$, $m = 1, \ldots, M$, $n = 1, \ldots, N$. Based on (1), it can be readily shown that

$$\mathbf{X}_f(t) \quad = \quad \sum_{k=1}^{K} s_k(t)\mathbf{g}_k(\mathbf{v}_k \otimes \mathbf{u}_k)^T + \mathbf{N}_f(t) \quad (3)$$

where $\mathbf{g}_k = [e^{-j2\pi\frac{L-1}{2}f_k T_s}, \cdots, e^{j2\pi\frac{L-1}{2}f_k T_s}]^T$, $\mathbf{u}_k$=$[e^{-j2\pi\frac{M-1}{2}u_k}, \cdots, e^{j2\pi\frac{M-1}{2}u_k}]^T$, $\mathbf{v}_k$=$[e^{-j2\pi\frac{N-1}{2}v_k}, \cdots, e^{j2\pi\frac{N-1}{2}v_k}]^T$, and $\otimes$ denote the Kronecker product [6]. $\mathbf{N}_f(t)$ is constructed by $\{n_{mnl}(t)\}$ in the same way as $\mathbf{X}_f(t)$ from $\{x_{mnl}(t)\}$ given in (2).

We then consider the *frequency covariance matrix* of $\mathbf{X}_f(t)$, $\mathbf{R}_f \triangleq \frac{1}{MN}E[\mathbf{X}_f(t)\mathbf{X}_f^H(t)]$, with $(\cdot)^H$ being the Hermitian operatin. Based on (3), $\mathbf{R}_f$ can be shown as

$$\mathbf{R}_f \quad = \quad \mathbf{G}\mathbf{\Lambda}\mathbf{G}^H + \sigma_n^2\mathbf{I} \quad (4)$$

where we have the fact that $(\mathbf{v}_k \otimes \mathbf{u}_k)^T(\mathbf{v}_k \otimes \mathbf{u}_k)^* = MN$ [6], and that the noise $n_{mnl}(t)$ is white and independent with the imping signals. $\mathbf{G} = [\mathbf{g}_1, ..., \mathbf{g}_K]$ is the frequency signature matrix and $\mathbf{\Lambda} = E[\mathbf{S}(t)\mathbf{S}^H(t)]$, in which $\mathbf{S}(t) = diag\{s_1(t), ..., s_K(t)\}$. Note that $\mathbf{R}_f$ and $\mathbf{G}$ share the same column space and thus the 1-D Unitary ESPRIT can be employed to estimate the frequencies.

However, the Unitary ESPRIT can not well resolve closely spaced parameters [7], as the related signature matrix tends to be ill-conditioned. To overcome this setback, we employ the HSTD technique [8, 9] to partition the signals into smaller groups based on the resolvable carrier frequencies estimated above before proceeding to estimate u and v. To achieve this, suppose that after carrying out the 1-D Unitary ESPRIT with respect to $\mathbf{R}_f$, we obtain a set of frequency estimates, say, $\{\hat{f}_1, \ldots, \hat{f}_q\}$, where q is the number of resolvable frequencies, we construct a set of *temporal* projection matrices $\mathbf{P}_{f_i}$ given by

$$\mathbf{P}_{f_i} \quad = \quad \mathbf{I} - \bar{\mathbf{G}}_i(\bar{\mathbf{G}}_i^H\bar{\mathbf{G}}_i)^{-1}\bar{\mathbf{G}}_i^H \quad (5)$$

for $i = 1, \ldots, q$, where $\bar{\mathbf{G}}_i = [\hat{\mathbf{g}}_1, \ldots, \hat{\mathbf{g}}_{i-1}, \hat{\mathbf{g}}_{i+1}, \ldots, \hat{\mathbf{g}}_q]$. We then use these projection matrices to obtain a set of filtered data matrices $\mathbf{X}_{f_i}(t) = \mathbf{P}_{f_i}\mathbf{X}_f(t)$, $i = 1, \cdots, q$, which, based on the data model in (3), can be re-written as

$$\mathbf{X}_{f_i}(t) \cong \sum_{j=1}^{r_i} s_{i,j}(t)\bar{\mathbf{g}}_{i,j}(\mathbf{v}_{i,j} \otimes \mathbf{u}_{i,j})^T + \mathbf{P}_{f_i}\mathbf{N}_f(t) \quad (6)$$

where r_i is the number of signals in the i^{th} group, $\bar{\mathbf{g}}_{i,j} = \mathbf{P}_{f_i}\mathbf{g}_{i,j}$. Note that the incoming sources except those in the i^{th} group will be approximately annihilated by such a temporal filtering process. and that the filtered data matrix only contains signals whose frequencies are close to $\hat{f}_i$ but with diverse (u, v)'s.

Next, in order to estimate u by using the 1-D Unitary ESPRIT with the filtered data matrix, we partition $\mathbf{X}_{f_i}(t)$ into a set of $L \times M$ sub-block matrices and then rebuild them into $\mathbf{X}_{u_i}(t)$ as

$$\mathbf{X}_{u_i}(t) = \Big[\mathbf{X}_{f_i}(t)(:, 1:M)^T \vdots \cdots \vdots \mathbf{X}_{f_i}(t)(:, (N-1)M+1:NM)^T\Big] \quad (7)$$

Based on (7), it can be readily shown that $\mathbf{X}_{u_i}(t)$ renders

$$\mathbf{X}_{u_i}(t) \cong \sum_{j=1}^{r_i} s_{i,j}(t)\mathbf{u}_{i,j}(\mathbf{v}_{i,j} \otimes \bar{\mathbf{g}}_{i,j})^T + \mathbf{N}_{u_i}(t) \quad (8)$$

where $\mathbf{N}_{u_i}(t)$ is constructed by $\mathbf{P}_{f_i}\mathbf{N}_f(t)$ in the same way as $\mathbf{X}_{u_i}(t)$ from $\mathbf{X}_{f_i}(t)$ in (7), whose covariance matrix is given by

$$E[\mathbf{N}_{u_i}(t)\mathbf{N}_{u_i}^H(t)] = \sigma_i^2\mathbf{I} \quad (9)$$

where $\sigma_i^2 = \frac{L-q+1}{L}$, which implies that the noise components in $\mathbf{N}_{u_i}(t)$ remains white and that the noise power is reduced after the temporal filtering process.

Next, we compute the covariance matrices of $\mathbf{X}_{u_i}$, $\mathbf{R}_{u_i} \triangleq \frac{1}{LN} E[\mathbf{X}_{u_i}(t)\mathbf{X}_{u_i}^H(t)]$. Based on (8), it can be shown that

$$\mathbf{R}_{u_i} = \mathbf{U}_i\mathbf{\Lambda}_i\mathbf{U}_i^H + \sigma_{n_i}^2\mathbf{I} \quad (10)$$

where we have used the fact that $(\mathbf{v}_{i,j} \otimes \bar{\mathbf{g}}_{i,j})^T(\mathbf{v}_{i,j} \otimes \bar{\mathbf{g}}_{i,j})^* = LN$. $\mathbf{U}_i = [\mathbf{u}_{i,1}, ..., \mathbf{u}_{i,r_i}]$ is the signature matrix of $\mathbf{u}_{i,j}$ and $\mathbf{\Lambda}_i = E[\mathbf{S}_i\mathbf{S}_i^H]$, in which $\mathbf{S}_i = diag\{s_{i,1}(t), ..., s_{i,r_i}(t)\}$. Carrying out the 1-D unitary ESPRIT, we can get a set of estimates of u, say, $\hat{u}_{i,j}$, $i = 1, 2, ..., q$, $j = 1, 2, ..., \rho_i$, where ρ_i is the number of u's resolvable in the i^{th} group. Thereafter, we use these estimates to construct a set of *spatial* projection matrices given by

$$\mathbf{P}_{u_{i,j}} = \mathbf{I} - \bar{\mathbf{U}}_{i,j}(\bar{\mathbf{U}}_{i,j}^H\bar{\mathbf{U}}_{i,j})^{-1}\bar{\mathbf{U}}_{i,j}^H \quad (11)$$

for $i = 1, \ldots, q$, $j = 1, \ldots, \rho_i$, and $\bar{\mathbf{U}}_{i,j} = [\hat{\mathbf{u}}_{i,1} \ldots \hat{\mathbf{u}}_{i,j-1} \ \hat{\mathbf{u}}_{i,j+1} \ldots \hat{\mathbf{u}}_{i,\rho_i}]$. Pre-multiplying data $\mathbf{X}_{u_i}(t)$ by $\mathbf{P}_{u_{i,j}}$ yields a set of finer group of data as

$$\mathbf{X}_{u_{i,j}}(t) \cong \sum_{l=1}^{z_{i,j}} s_{i,j}(t)\bar{\mathbf{u}}_{i,j,l}(\mathbf{v}_{i,j,l} \otimes \bar{\mathbf{g}}_{i,j,l})^T + \mathbf{P}_{u_{i,j}}\mathbf{N}_{u_{i,j}}(t) \quad (12)$$

where $z_{i,j}$ is the number of signals in the j^{th} subgroup of the i^{th} group and $\bar{\mathbf{u}}_{i,j,l} = \mathbf{P}_{u_{i,j}}\mathbf{u}_{i,j,l}$. Note that the incoming sources, except those in the $(i,j)^{th}$ groups, will be approximately eliminated by $\mathbf{P}_{u_{i,j}}$.

Note that the incoming signals in the $(i,j)^{th}$ subgroup, which possess close u components (close to $u_{i,j}$), will have diverse v's. As such, the v components for the signals in each subgroup can be well resolved. To estimate v, we partition and re-stack the filtered matrix $\mathbf{X}_{u_{i,j}}(t)$ by

$$\begin{aligned}\mathbf{X}_{v_{i,j}}(t) \quad = \quad \Big[&vec(\mathbf{X}_{u_{i,j}}(t)(:, 1:L)^T) \vdots \cdots \vdots \cdots \vdots \\ &vec(\mathbf{X}_{u_{i,j}}(t)(:, (N-1)L+1:NL)^T \ \Big]^T\end{aligned} \quad (13)$$

where $vec(\cdot)$ denotes vector stacking operation [6]. Based on (13), it can be shown that $\mathbf{X}_{v_{i,j}}(t)$ renders

$$\mathbf{X}_{v_{i,j}}(t) \cong \sum_{l=1}^{z_{i,j}} s_{i,j,l}(t)\mathbf{v}_{i,j,l}(\bar{\mathbf{u}}_{i,j,l} \otimes \bar{\mathbf{g}}_{i,j,l})^T + \mathbf{N}_{v_{i,j}}(t) \quad (14)$$

where $\mathbf{N}_{v_{i,j}}(t)$ is obtained from $\mathbf{P}_{u_{i,j}}\mathbf{N}_{u_{i,j}}(t)$ in the same way as $\mathbf{X}_{v_{i,j}}(t)$ from $\mathbf{X}_{u_{i,j}}(t)$ in (13). Next, we determine the covariance matrix of $\mathbf{X}_{v_{i,j}}$, $\mathbf{R}_{v_{i,j}} \triangleq \frac{1}{LM}E[\mathbf{X}_{v_{i,j}}(t)\mathbf{X}_{u_{i,j}}^H(t)]$. Based on (14), it can be shown that

$$\mathbf{R}_{v_{i,j}} = \mathbf{V}_{i,j}\mathbf{\Lambda}_{i,j}\mathbf{V}_{i,j}^H + \sigma_{i,j}^2\mathbf{I} \quad (15)$$

where $\sigma_{i,j}^2 = \frac{(L-q+1)(M-\rho_i+1)}{LM}\sigma^2$, $\mathbf{V}_{i,j} = [\mathbf{v}_{i,j,1}, ..., \mathbf{v}_{i,j,z_{i,j}}]$ is the signature matrix of $\mathbf{v}_{i,j,l}$, and $\mathbf{\Lambda}_{i,j} = E[\mathbf{S}_{i,j}\mathbf{S}_{i,j}^H]$, in which $\mathbf{S}_{i,j} = diag\{s_{i,j,1}(t), ..., s_{i,j,z_{i,j}}(t)\}$. Along the same line as above, the 1-D Unitary ESPRIT can be applied to estimate v.

Note that the f's and u's estimated above are rather rough when these parameters are closely spaced. Consequently, we can get a more precise estimate of f's and u's by carrying out the 1-D Unitary ESPRIT again based on the finer groups of data. For this, we construct another set of spatial projection matrices given by

$$\mathbf{P}_{v_{i,j,l}} = \mathbf{I} - \bar{\mathbf{V}}_{i,j,l}(\bar{\mathbf{V}}_{i,j,l}^H\bar{\mathbf{V}}_{i,j,l})^{-1}\bar{\mathbf{V}}_{i,j,l}^H \tag{16}$$

where $\bar{\mathbf{V}}_{i,j,l} = [\hat{\mathbf{v}}_{i,j,1} \ldots \hat{\mathbf{v}}_{i,j,l-1}\ \hat{\mathbf{v}}_{i,j,l+1} \ldots \hat{\mathbf{v}}_{i,j,z_{i,j}}]$, and then pre-multiply $\mathbf{X}_{v_{i,j}}(t)$ by the projection matrix $\mathbf{P}_{v_{i,j,l}}$ to render $\mathbf{X}_{v_{i,j,l}}(t) = \mathbf{P}_{v_{i,j,l}}\mathbf{X}_{v_{i,j}}(t)$, which will annihilate the impinging signals those do not belong to the $(i,j,l)^{th}$ subgroup. Based on (13), the new data matrix can be expressed as

$$\mathbf{X}_{v_{i,j,l}}(t) \cong s_{i,j,l}(t)\bar{\mathbf{v}}_{i,j,l}(\bar{\mathbf{u}}_{i,j,l} \otimes \bar{\mathbf{g}}_{i,j,l})^T + \mathbf{N}_{v_{i,j,l}}(t) \tag{17}$$

where $\bar{\mathbf{v}}_{i,j,l} = \mathbf{P}_{v_{i,j,l}}\mathbf{v}_{i,j,l}$ and $\mathbf{N}_{v_{i,j,l}}(t) = \mathbf{P}_{v_{i,j,l}}\mathbf{N}_{v_{i,j}}(t)$. For a more accurate estimation of f, we partition $\mathbf{X}_{v_{i,j,l}}(t)$ into M $N \times L$ sub-block matrices and stack them as

$$\begin{aligned}\mathbf{X}_{u_{i,j,l}}(t) &= \Big[vec\ \left(\mathbf{X}_{v_{i,j,l}}(t)(:,1:L)\right)^T \vdots \cdots \vdots \cdots \\ &\qquad vec\ \left(\mathbf{X}_{v_{i,j,l}}(t)(:,(M-1)L+1:ML)\right)^T\ \Big]^T \\ &= s_{i,j,l}(t)\bar{\mathbf{u}}_{i,j,l}(\bar{\mathbf{v}}_{i,j,l} \otimes \bar{\mathbf{g}}_{i,j,l})^T + \mathbf{N}_{u_{i,j,l}}(t)\end{aligned} \tag{18}$$

where $\mathbf{N}_{u_{i,j,l}}(t)$ is obtained from $\mathbf{N}_{v_{i,j,l}}(t)$ in the same way as $\mathbf{X}_{u_{i,j,l}}(t)$ from $\mathbf{X}_{v_{i,j,l}}(t)$ in (18). Next, we partition the data matrix $\mathbf{X}_{u_{i,j,l}}(t)$ into N $M \times L$ sub-block matrices and stack them as

$$\begin{aligned}\mathbf{X}_{f_{i,j,l}}(t) &= \Big[\ \mathbf{X}_{u_{i,j,l}}(t)(:,1:L)\ ^T \vdots \cdots \vdots \cdots \\ &\qquad \mathbf{X}_{u_{i,j,l}}(t)(:,(N-1)L+1:NL)\ ^T\Big] \\ &\cong s_{i,j,l}(t)\bar{\mathbf{g}}_{i,j,l}(\bar{\mathbf{v}}_{i,j,l} \otimes \bar{\mathbf{u}}_{i,j,l})^T + \mathbf{N}_{f_{i,j,l}}(t)\end{aligned} \tag{19}$$

Note that $\bar{\mathbf{g}}_{i,j,l}$ in (19) does not possess Vandermonde structure. To overcome this setback, we utilize the fact that $\bar{\mathbf{g}}_{i,j,l}(\bar{\mathbf{u}}_{i,j,l} \otimes \bar{\mathbf{v}}_{i,j,l})^T$ belongs to the subspace spanned by the normalized eigenvector $\mathbf{e}_{f_{i,j,l}}$ associate with the largest eigenvalue of covariance matrix $\mathbf{R}_{f_{i,j,l}}$, where $\mathbf{R}_{f_{i,j,l}} \triangleq \frac{1}{MN}\mathbf{E}[\mathbf{X}_{f_{i,j,l}}(t)\mathbf{X}_{f_{i,j,l}}^H(t)]$. Therefore, $(\mathbf{I} - \mathbf{e}_{f_{i,j,l}}\mathbf{e}_{f_{i,j,l}}^H)\mathbf{P}_{f_i}\mathbf{g}_{i,j,l}(\bar{\mathbf{u}}_{i,j,l} \otimes \bar{\mathbf{v}}_{i,j,l})^T = \mathbf{0}$. $\mathbf{g}_{i,j,l}$ belongs to the subspace spanned by the Denote $\mathbf{\Xi}_{f_{i,j,l}} = \mathbf{I} - (\mathbf{I} - \mathbf{e}_{f_{i,j,l}}\mathbf{e}_{f_{i,j,l}}^H)\mathbf{P}_{f_i}$ and $\mathbf{G}_{i,j,l} = [\bar{\mathbf{G}}_i\ \ \mathbf{g}_{i,j,l}]$, it can be shown that

$$\mathbf{\Xi}_{f_{i,j,l}} = \mathbf{G}_{i,j,l}(\mathbf{G}_{i,j,l}^H\mathbf{G}_{i,j,l})^{-1}\mathbf{G}_{i,j,l}^H \tag{20}$$

Note that $\mathbf{G}_{i,j,l}$ retains the Vandermonde structure and shares the same column space as $\mathbf{\Xi}_{f_{i,j,l}}$, so the 1-D Unitary F-ESPRIT can be utilized and we can get more precise estimates of f's. Similarly, working with $\mathbf{X}_{u_{i,j,l}}$ results in more precise estimates of $u_{i,j,l}$'s.

To sum up, the overall steps of the proposed tree-structured 1-D Unitary ESPRIT based algorithm can be summarized as follows:

Step 1: (**Rough Frequency Estimation**) Estimate the covariance matrix $\hat{\mathbf{R}}_f = \frac{1}{SMN}\sum_{s=1}^{S}\mathbf{X}_f(t_s)\mathbf{X}_f^H(t_s)$, where S is the number of snapshots, and then invoke the 1-D Unitary ESPRIT to yield a set of group frequency estimates $\{\hat{f}_1, ..., \hat{f}_q\}$, where q is the number of resolvable frequencies.

Step 2: (**Temporal Filtering**) Employ $\{\hat{f}_1, ..., \hat{f}_q\}$ to construct the projection matrix $\mathbf{P}_{f_i}$ by (5) and then use $\mathbf{P}_{f_i}$ to obtain the filtered data matrix $\mathbf{X}_{f_i}(t) = \mathbf{P}_{f_i}\mathbf{X}_f(t),\ i = 1, ..., q$.

Step 3: (**Rough Estimation of** u) Stack the data $\mathbf{X}_{f_i}(t),\ i = 1, ..., q$, based on (6) and then estimate the covariance matrix $\hat{\mathbf{R}}_{u_i} = \frac{1}{SLN}\sum_{s=1}^{S}\mathbf{X}_{u_i}(t_s)\mathbf{X}_{u_i}^H(t_s)$. Thereafter, use the 1-D Unitary ESPRIT to estimate the u's, $\{\hat{u}_{1,1}, \ldots, \hat{u}_{1,\rho_1}, \ldots, \hat{u}_{q,1}, \ldots, \hat{u}_{q,\rho_q}\}$, where $\rho_i,\ i = 1, ..., q$, is the number of u's resolvable in the i^{th} group.

Step 4: (**Spatial Beamforming (I)**) Employ $\{\hat{u}_{1,1}\}$ obtained above to construct the projection matrix $\mathbf{P}_{u_{i,j}}$ by (11) and then use $\mathbf{P}_{u_{i,j}}$ to obtain the filtered data matrix $\mathbf{X}_{u_{i,j}}(t) = \mathbf{P}_{u_{i,j}}\mathbf{X}_{u_i}(t),\ i = 1, \ldots, q,\ j = 1, \ldots, \rho_i$.

Step 5: (**Estimation of** v) Re-stack data $\mathbf{X}_{u_{i,j}}(t)$ based on (13) to form $\mathbf{X}_{v_{i,j}}(t)$ and then use $\mathbf{X}_{v_{i,j}}(t)$ to estimate the covariance matrix $\hat{\mathbf{R}}_{v_{i,j}} = \frac{1}{SLM}\sum_{s=1}^{S}\mathbf{X}_{v_{i,j}}(t_s)\mathbf{X}_{v_{i,j}}^H(t_s)$. Thereafter, use the 1-D Unitary ESPRIT to estimate the v's to get $\{\hat{v}_{i,j,l}\}$.

Step 6: (**Spatial Beamforming (II)**) Employ $\{\hat{v}_{i,j,l}\}$ to construct the projection matrices $\mathbf{P}_{v_{i,j,l}}$ as given in (16), then use $\mathbf{P}_{v_{i,j,l}}$ to obtain filtered data matrix $\mathbf{X}_{v_{i,j,l}}(t) = \mathbf{P}_{v_{i,j,l}}\mathbf{X}_{v_{i,j}}(t),\ i = 1, \ldots, q,\ j = 1, \ldots, \rho_i,\ l = 1, \ldots, z_{i,j}$.

Step 7: (**Precise** u **and Frequency Estimation**) Partition and re-stack $\mathbf{X}_{v_{i,j,l}}(t_s)$ as (18) and (19) to obtain $\mathbf{X}_{u_{i,j,l}}(t_s)$ and $\mathbf{X}_{f_{i,j,l}}(t_s)$, respectively. Estimate the covariance matrix $\hat{\mathbf{R}}_{f_{i,j,l}} = \frac{1}{SMN}\sum_{s=1}^{S}\mathbf{X}_{f_{i,j,l}}(t_s)\mathbf{X}_{f_{i,j,l}}^H(t_s)$, and utilize the normalized eigenvector $\hat{\mathbf{e}}_{f_{i,j,l}}$ corresponding to the largest eigenvalue and the projection matrix $\mathbf{P}_{f_i}$ to form $\mathbf{\Xi}_{f_{i,j,l}} = \mathbf{I} - (\mathbf{I} - \mathbf{e}_{f_{i,j,l}}\mathbf{e}_{f_{i,j,l}}^H)\mathbf{P}_{f_i}$. Use the 1-D Unitary ESPRIT to obtain precise frequency estimates from $\mathbf{\Xi}$. Following the above procedures based on $\mathbf{X}_{u_{i,j,l}}(t_s)$ renders precise estimates of u. Note that since every subgroup in this step only contains one signal, the pairing process is automatically achieved. Finally, we can obtain the estimate of elevation and azimuth angles, $\hat{\phi} = \sin^{-1}\frac{c}{fd}\sqrt{\hat{u}^2 + \hat{v}^2}$ and $\hat{\theta} = \tan^{-1}\frac{\hat{v}}{\hat{u}}$.

Note that the number of groups, q, the number of signals in each group, ρ_i, and the number of signals in each subgroup, $z_{i,j}$ are known or have been perfectly estimated, say by the AIC or MDL criterion addressed in [10]. Based on the above steps, the total number of real multiplications required by the proposed algorithm is then about $4LMNSK(L+M+N)$ if we assume that $S \geq M, N, L > K$.

4. SIMULATIONS AND DISCUSSIONS

Some simulations are conducted in this section to assess the proposed approach. Assume that there are $K = 4$ users, and they are received by a 6×6 ($M = N = 6$) element URA which spaced a half wavelength apart, where each antenna is followed by a tapped line with $L = 12$ delay elements and the sampling frequency is $f_s = 400$ MHz. The azimuth and elevation angles of the users are $[63, 29, 75, 63]^o$ and $[23, 44, 46, 13]^o$, respectively, with the center frequencies $[100, 103, 103, 180]$ MHz. $S = 200$ symbols are employed to estimate the temporal and spatial covariances. For each specific SNR, 200 Monte Carlo trials are carried out. The comparison of the root-mean-square-error (*RMSE*) of frequencies, elevation and azimuth angles based on the proposed algorithm and [1] and [4] is shown in Figs. 2, 3 and 4, respectively, where the Cramer-Rao lower bound (CRLB) is also provided for reference.

We can note from Figs. 2-4 that the proposed algorithm produces close performance as the algorithms in [1, 4] in all of the estimates. Meanwhile, [1] and [4] need to stack the data to simultaneously estimate these parameters and thus roughly require

$(6S + \frac{2}{3}MNL)M^2N^2L^2$ and $(8S + \frac{8}{3}MNL + 34K)M^2N^2L^2$ real multiplications, respectively, which are far more computationally expensive than the proposed algorithm. In contrast, the proposed algorithm only involves 1-D unitary ESPRIT and thus the computational overhead is substantially reduced.

5. REFERENCES

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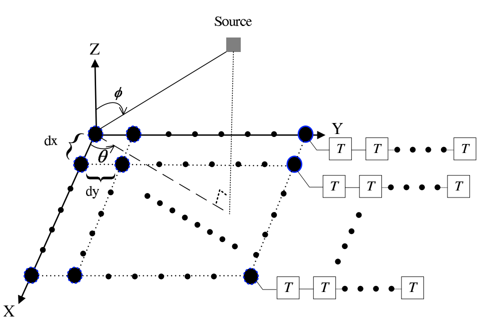

Fig. 1. Uniform rectangular array with tapped delay lines

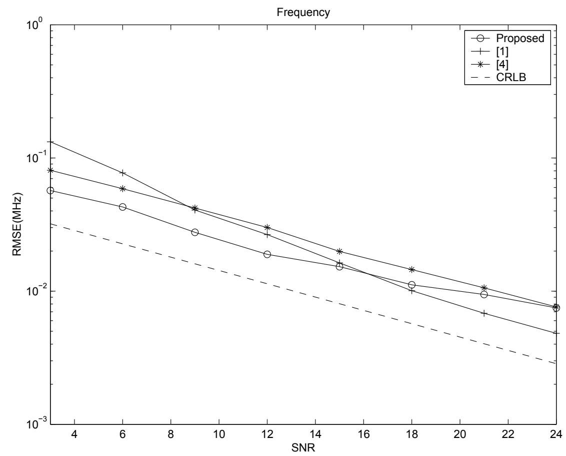

Fig. 2. Comparison of frequency estimates

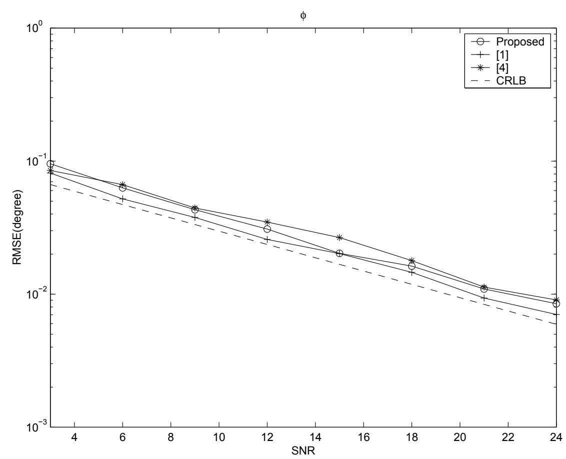

Fig. 3. Comparison of elevation angle estimates.

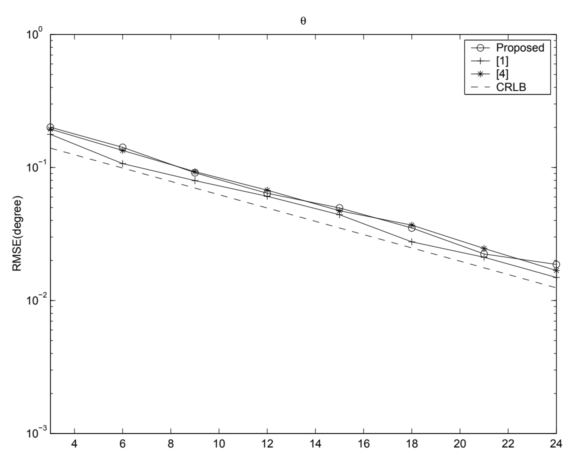

Fig. 4. Comparison of azimuth angle estimates.