ACOUSTIC SOURCE DOA ESTIMATION USING THE CROSS-ENTROPY METHOD

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ABSTRACT

This paper deals with the problem of estimating the Directions-of-Arrival (DOA) of multiple wideband sources using an array of sensors. While a variety of estimation techniques have been proposed in the literature, the Maximum-Likelihood (ML) DOA estimator has been shown to have superior performance under many challenging environments. In this paper, we propose a novel implementation for ML DOA estimator based on the Cross-Entropy (CE) method. Simulation results show the CE algorithm converges to the Cramer-Rao Bound (CRB) in all scenarios within several iterations and the convergence speed is insensitive to the coherence of sources.

Index Terms— Cross-Entropy, DOA estimation

1. INTRODUCTION

Over the past thirty years, researchers have been interested in developing array signal processing techniques for DOA estimation and source localization. While many high resolution algorithms have been proposed in the literature, the ML estimator exhibits many advantages over other estimators. For example, TDOA based methods [1] in general assume single source in the data model, and therefore cannot be used to resolve multiple sources. Subspace methods such as MUSIC [2] possess multiple sources DOA estimation capabilities, but require the source waveforms to be incoherent and quasi-static. The ML estimator, on the other hand, gives robust performance under these challenging scenarios and its performance asymptotically achieves the CRB.

The naive implementation of the ML estimator requires multiple dimensional grid search, since the ML metric is a nonlinear function of DOAs. The computational burden of such implementation grows exponentially in the number of sources which is impractical in many applications. Various iterative optimization schemes have been proposed to reduce the complexity. Most of them require a good initial point for the algorithms to converge to the optimal solution, and no global convergence is guaranteed in general.

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In this paper, we propose a novel wideband DOA estimation algorithm based on the CE method [3]. A similar approach has been proposed for narrowband sources [4], while our algorithm is focused on wideband applications. By using the CE method, the ML criterion is translated into a stochastic approximation problem which can be solved efficiently. The performance of the proposed algorithm under both coherent and incoherent scenarios has been studied in this work, and the simulation results show that CE converges to the CRB in both scenarios with a comparable rate.

2. MAXIMUM-LIKELIHOOD CRITERION

In this section, we review the ML criterion for multiple sources DOA estimation [5, 6, 7].

Let there be M wideband sources in the far-field of a P-element randomly distributed array. For simplicity, we assume the sources and the array lie in the same plane, and θ_m denotes the DOA of the mth source with respect to the centroid of the array, where $m = 1, \cdots, M$. Without loss of generality, we set the array centroid to be at the origin, and the position of each sensor is at $\mathbf{r}_p = [r_p \cos(\phi_p), r_p \sin(\phi_p)]^T$, $p = 1, \cdots, P$. With this setting, the time-delay of the mth source to the pth sensor relative to the centroid can be expressed as $t_p^{(m)} = r_p \cos(\theta_m - \phi_p)/v$, where v is the speed of acoustic wave. The received waveform by the pth sensor at time n can then be expressed as

$$x_p(n) = \sum_{m=1}^{M} s^{(m)}(n - t_p^{(m)}) + w_p(n) \tag{1}$$

for $n = 0, \cdots, N-1$. Here N denotes the length of the received waveform, $s^{(m)}(n)$ is the signal, and $w_p(n)$ is modelled as additive white Gaussian noise with variance σ^2.

For the ease of derivation and analysis, the received waveform is transformed into the frequency domain via DFT, where a narrowband model is applied to each frequency bin. This transformation introduces edge effect due to the nature of DFT, but the effect is negligible when N is sufficiently large.

After performing N-point unitary DFT transformation to $x_p(n)$, we can obtain the following data model:

$$\mathbf{X}(\omega_k) = \mathbf{D}(\omega_k, \mathbf{\Theta})\mathbf{S}(\omega_k) + \mathbf{W}(\omega_k), \tag{2}$$

for $k = 0, \cdots, N-1$. Here $\mathbf{X}(\omega_k) = [X_1(\omega_k), \cdots, X_P(\omega_k)]^T$ denotes the array data spectrum, and $\boldsymbol{\Theta} = [\theta_1, \cdots, \theta_M]^T$ is the source DOA vector. The steering matrix is a function of both $\boldsymbol{\Theta}$ and ω_k, and can be expressed as $\mathbf{D}(\omega_k, \boldsymbol{\Theta}) = [\mathbf{d}^{(1)}(\omega_k, \theta_1), \cdots, \mathbf{d}^{(M)}(\omega_k, \theta_M)]$, where $\mathbf{d}^{(m)}(\omega_k, \theta_m) = [d_1^{(m)}(\omega_k, \theta_m), \cdots, d_p^{(m)}(\omega_k, \theta_m)]^T$. Assume the response of the pth sensor is $a_p(\omega_k, \theta)$, then $d_p^{(m)}$can be expressed as $d_p^{(m)}(\omega_k, \theta_m) = a_p(\omega_k, \theta_m)e^{-j2\pi k t_p^{(m)}/N}$. We denote $\mathbf{S}(\omega_k) = [S^{(1)}(\omega_k), \cdots, S^{(m)}(\omega_k)]^T$ as the source spectrum and $\mathbf{W}(\omega_k)$ as the noise spectrum. Throughout this paper, we denote the superscript ^{T} as the transpose and ^{H} as the complex conjugate transpose.

Since $\mathbf{W}(\omega_k)$ is also i.i.d. Gaussian, the likelihood function of $\{\boldsymbol{\Theta}, \mathbf{S}(\omega_1), \cdots, \mathbf{S}(\omega_{N/2})\}$ is given by

$$L(\boldsymbol{\Theta}, \mathbf{S}(\omega_1), \cdots, \mathbf{S}(\omega_{N/2})) = \frac{1}{\pi^{PN/2}\sigma^{PN}} \cdot$$
$$\exp\{(-1/\sigma^2)\sum_{k=1}^{N/2} \|\mathbf{X}(\omega_k) - \mathbf{D}(\omega_k, \boldsymbol{\Theta})\mathbf{S}(\omega_k)\|^2\} \qquad (3)$$

Taking logarithm of $L(\boldsymbol{\Theta}, \mathbf{S}(\omega_1), \cdots, \mathbf{S}(\omega_{N/2}))$, replacing $\mathbf{S}(\omega_k)$ by its ML estimate, and omitting the scaling factor independent of $\boldsymbol{\Theta}$, we have the following ML metric expression:

$$J(\boldsymbol{\Theta}) = \sum_{k=1}^{N/2} tr\{\mathbf{P}(\omega_k, \boldsymbol{\Theta})\mathbf{R}(\omega_k)\} \qquad (4)$$

where $\mathbf{P}(\omega_k, \boldsymbol{\Theta}) = \mathbf{D}(\omega_k, \boldsymbol{\Theta})\mathbf{D}^{\dagger}(\omega_k, \boldsymbol{\Theta})$ is the projection matrix that projects to the signal subspace, and $\mathbf{R}(\omega_k) = \mathbf{X}(\omega_k)\mathbf{X}^H(\omega_k)$. Finally, the ML criterion is expressed by: $\hat{\boldsymbol{\Theta}}^{ML} = \arg\max_{\boldsymbol{\Theta}} J(\boldsymbol{\Theta})$.

3. INTRODUCTION TO CROSS-ENTROPY METHOD

The CE method was first proposed by Rubinstein [3] in solving rare event estimation problems, and was soon realized that it can be generalized to solving both combinatorial and continuous optimization problems. In this section, we review the CE method and apply it to our DOA estimation problem.

Let γ^* denote the maximum of our ML metric, $J(\boldsymbol{\Theta})$ over the M dimensional rectangle. First, we randomize our original deterministic problem by defining a family of pdfs $\{f(\cdot, \mathbf{v}), \mathbf{v} \in \mathcal{V}\}$ from which $\boldsymbol{\Theta}$ is drawn; $\mathbf{v}$ is the parameter of the pdf $f(\cdot, \mathbf{v})$ and $\mathcal{V}$ is the set in which $\mathbf{v}$ lies. Then we associate the original problem with a rare-event probability estimation problem:

$$\ell(\gamma) = \mathsf{P}_{\mathbf{v}}(J(\boldsymbol{\Theta}) \geq \gamma) = \mathsf{E}_{\mathbf{v}}\{H(\boldsymbol{\Theta}, \gamma)\}, \qquad (5)$$

where $H(\boldsymbol{\Theta}, \gamma)$ is 1 if $J(\boldsymbol{\Theta}) \geq \gamma$, and 0 otherwise. Here $\ell(\gamma)$ is the rare-event probability for some γ close to γ^*. $\mathsf{P}_{\mathbf{v}}$ and $\mathsf{E}_{\mathbf{v}}$ denote the probability and expectation of the rare event $\{J(\boldsymbol{\Theta}) \geq \gamma\}$. Applying the importance sampling technique to eq. 5, we can approximate $\ell(\gamma)$ by

$$\hat{\ell}(\gamma) = \frac{1}{N_s}\sum_{n=1}^{N_s} H(\boldsymbol{\Theta}^{(n)}, \gamma)\frac{f(\boldsymbol{\Theta}^{(n)}, \mathbf{v})}{g(\boldsymbol{\Theta}^{(n)})}, \qquad (6)$$

where $\boldsymbol{\Theta}^{(n)}$ is the nth sample drawn from some proposal distribution $g(\boldsymbol{\Theta})$, and N_s is the number of samples used in the Monte Carlo approximation. It is easy to see that the optimal proposal distribution $g^*(\boldsymbol{\Theta})$ equals to $H(\boldsymbol{\Theta}, \gamma)f(\boldsymbol{\Theta}, \mathbf{v})/\ell(\gamma)$, since with this choice the variance of $\hat{\ell}(\gamma)$ is zero. However, g^* depends on $\ell(\gamma)$, which is unknown. Therefore the optimal proposal distribution is usually unavailable in practice.

Since g^* is unknown in general, the CE algorithm estimates g^* by constraining the proposal distribution to the same family of $f(., \mathbf{v})$ and seeks the optimal $\mathbf{v}$ such that the Kullback-Leibler distance (cross-entropy) between g^* and $f(., \mathbf{v})$ is minimized.

It can be shown that the optimal parameter of $\mathbf{v}$, denoted by $\mathbf{v}^*$ can be estimated by solving its stochastic counterpart [3]:

$$\mathbf{v}^* = \max_{\mathbf{v}} \frac{1}{N_s}\sum_{n=1}^{N_s} H(\boldsymbol{\Theta}^{(n)}, \gamma)\ln f(\boldsymbol{\Theta}^{(n)}, \mathbf{v}) \qquad (7)$$

Note that when γ is very close to γ^*, most of the $H(\boldsymbol{\Theta}^{(n)}, \gamma)$ values are zero, which makes the estimation in eq. 7 meaningless. To overcome this problem, a two-phased CE procedure has been proposed [3]. In this procedure, the parameter $\mathbf{v}$ and level γ are estimated and updated gradually and converge to $\mathbf{v}^*$ and γ^* that solve eq. 7.

4. CE ALGORITHM FOR DOA ESTIMATION

Since the likelihood function eq. 3 is Gaussian, a natural choice of $f(\boldsymbol{\Theta}, \mathbf{v})$ is the Gaussian pdf. For simplicity, we choose $f(\boldsymbol{\Theta}, \mathbf{v})$ to have a decoupled form in each dimension which can expressed by the following equations:

$$f(\theta, \mathbf{v}) = \prod_{m=1}^{M} f_m(\theta_m, \mathbf{v}_m) \qquad (8)$$

$$f_m(\theta_m, \mathbf{v}_m) = \frac{1}{\sqrt{2\pi\sigma_m^2}}e^{-\frac{(\theta_m - \mu_m)^2}{2\sigma_m^2}} \qquad (9)$$

where $\mathbf{v} = [\mathbf{v}_1, \cdots, \mathbf{v}_M]^T$, and $\mathbf{v}_m = [\mu_m, \sigma_m^2]^T$. The CE-procedure contains two phases: Adaptive updating of γ and adaptive updating of $\mathbf{v}$.

The updating procedure for $\gamma(t)$: For a fixed $\hat{\mathbf{v}}(t-1)$, estimate $\gamma(t)$ as the sample $(1-\rho)$-quantile of $J(\boldsymbol{\Theta})$.

The updating procedure for $\mathbf{v}(t)$: For a fixed $\gamma(t)$ and $\hat{\mathbf{v}}(t-1)$, solve eq. 7 using the predefined $f(\boldsymbol{\Theta}, \mathbf{v})$, we then have the following update equations:

$$\hat{\mu}_m = \frac{\sum_{n=1}^{N_s} H(\boldsymbol{\Theta}^{(n)}, \gamma)\theta_m^{(n)}}{\sum_{n=1}^{N_s} H(\boldsymbol{\Theta}^{(n)}, \gamma)} \qquad (10)$$

$$\hat{\sigma}_m^2 = \frac{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \gamma)(\theta_m^{(n)} - \hat{\mu}_m)^2}{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \gamma)} \tag{11}$$

Eq. 10 and 11 suggest the estimated parameters as the sampled mean and variance of those samples that give ML-metric above the threshold, γ. Since the DOA is circular in nature, the above definition needs to be modified to account for the modulus of 2π. As a result, we use the circular sample mean (eq. 12) and circular sample variance (eq. 13) in our proposed algorithm. Note a scaling factor of $\pi^2/3$ is used in eq. 13 such that it gives the same value as the sample variance definition when the DOAs are uniformly distributed within $[0, 2\pi)$.

The proposed CE-procedure for DOA estimation is summarized as follows:

1. Initialize parameters ρ, N_s, α, β, and $\hat{\mathbf{v}}(0)$. $\hat{\mathbf{v}}(0)$ can be chosen to incorporate the *a priori* knowledge of the DOAs. Set $t = 1$.
2. Generate N_s samples $\mathbf{\Theta}^{(1)}, \cdots, \mathbf{\Theta}^{(N_s)}$ from the proposal distribution $f(., \hat{\mathbf{v}}(t-1))$ defined in eq. 8 and 9.
3. Compute $J(\mathbf{\Theta}^{(1)}), \cdots, J(\mathbf{\Theta}^{(N_s)})$. Set $\hat{\gamma}(t)$ as the order statistic $J_{\lceil(1-\rho)N_s\rceil}$.
4. Use the same samples $\mathbf{\Theta}^{(1)}, \cdots, \mathbf{\Theta}^{(N_s)}$ to estimate the parameters $\tilde{\mathbf{v}}(t) = [\tilde{\mu}(t)^T, \tilde{\sigma}^2(t)^T]^T$ through the following update equations.

$$\tilde{\mu}_m(t) = \angle \frac{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \hat{\gamma}(t)) \exp{(j\theta_m^{(n)})}}{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \hat{\gamma}(t))} \tag{12}$$

$$\tilde{\sigma}_m^2(t) = \frac{\pi^2}{3}\{1 - \frac{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \hat{\gamma}(t)) \cos(\theta_m^{(n)} - \tilde{\mu}(t))}{\sum_{n=1}^{N_s} H(\mathbf{\Theta}^{(n)}, \hat{\gamma}(t))}\} \tag{13}$$

5. There are occasions that the algorithm might degenerate too quickly and converge to a suboptimum solution. This problem can be mitigated by smoothing the parameters, where $0 \le \alpha, \beta \le 1$.

$$\hat{\mu}_m(t) = \angle\{\alpha e^{j\tilde{\mu}_m(t)} + (1-\alpha)e^{\hat{\mu}_m(t-1)}\} \tag{14}$$

$$\hat{\sigma}_m^2(t) = \beta\tilde{\sigma}_m^2(t) + (1-\beta)\hat{\sigma}_m^2(t-1) \tag{15}$$

6. Stop the algorithm when it converges; otherwise set $t = t + 1$ and reiterate from step 2.

5. SIMULATION RESULTS

In this section, we present the simulation results of the proposed algorithm. An 8-element Uniform Circular Array (UCA) is assumed in the simulation to estimate the DOAs of three wideband sources which are extracted from real human speech recordings (Fig. 1) sampled at 16kHz. The true DOAs are set to be at DOA1=$\pi/6$, DOA2=$\pi/2$, and DOA3=π, and the radius of the UCA is set to be 0.25m. The effective SNR of all three sources are set at 20dB. We initialize the CE algorithm by setting $\hat{\mu}(0)$ to the value obtained from the same initialization procedure described in [8], and $\hat{\sigma}_m(0)$ is set to be $\pi/6$. The initialization procedure is first proposed for Alternating Projection (AP) algorithm, and global convergence has been observed through extensive simulations [8]. When used as an initialization for CE, it places the generated samples at high likelihood regions and therefore allows a more efficient CE-implementation.

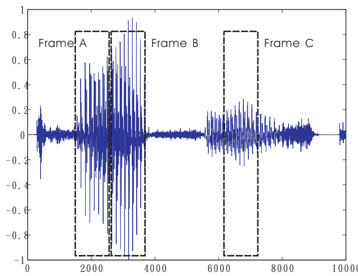


Fig. 1. Acoustic waveform of a human speech.

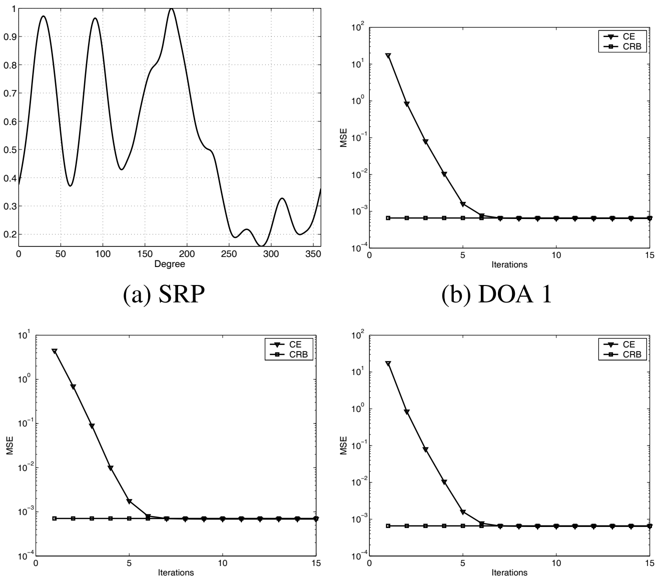
To study the performance of the proposed CE algorithm, the Mean-Square-Errors (MSE) of the estimated DOAs have been computed over 500 Monte Carlo simulations. The MSEs are then compared with their CRBs. Our data model (eq. 2) treated the DOAs and the sources spectra, $S^{(m)}(\omega_k)$ as unknown quantities, and therefore the derived CRBs include penalty factors that depend on the array geometry, the DOA spacings among the sources, and the source spectra. Two scenarios have been investigated throughout the simulation:

Scenario 1 (Incoherent sources): In the first scenario, The source waveforms for source 1, 2, and 3 are extracted from Frame A, B, and C of Fig. 1 respectively and the radius of the UCA is set to 0.15m.

Fig. 2a shows the steered-response-power (SRP) of a conventional beamformer. Three local maximums centered around the true DOAs can be observed. In this scenario, the steered beamformer gives reasonable results, although the estimated DOAs are still biased by the other interfering sources. The proposed CE-implementation of a ML estimator, on the contrary, achieves the CRBs of all three DOAs within 7 iterations (Fig. 2b, 2c, 2d).

Scenario 2 (Coherent sources): In real-applications, multipath effect is one of the biggest issues that degrades the performance. In our data model, the multipath effect can be considered as the presence of multiple coherent sources. In scenario 2, we simulate such condition by choosing the source waveforms all extracted from Frame A of Fig. 1.

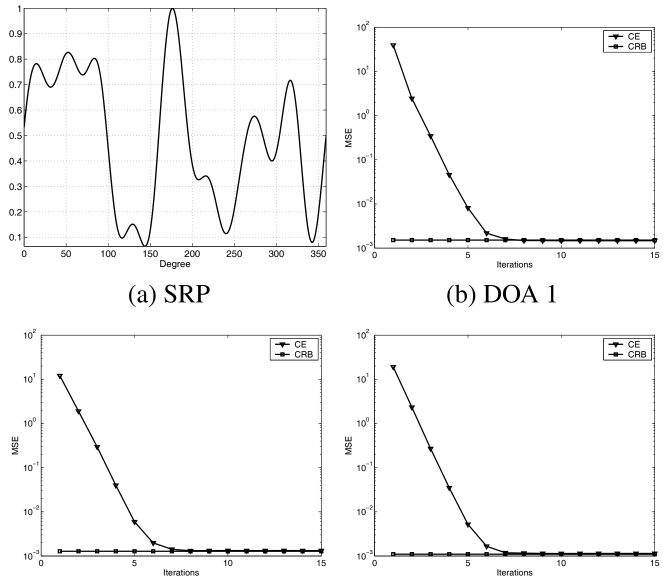
Again, Fig. 3a shows the SRP of a conventional beamformer. Three local maximums centered around 15, 52, 84 degrees can be observed. Clearly DOA1 and DOA2 can not be estimated accurately by a conventional beamformer under this scenario while our proposed algorithm again converges to the CRBs of all DOAs within 7 iterations (Fig. 3b, 3c, 3d).



(a) SRP (b) DOA 1

(c) DOA 2 (d) DOA 3

Fig. 2. Simulation results under scenario 1 ($N_s = 200, \rho = 0.05, \alpha = 1, \beta = 1$): a) SRP of a conventional beamformer. b) MSE of DOA1. c) MSE of DOA2. d) MSE of DOA3



(a) SRP (b) DOA 1

(c) DOA 2 (d) DOA 3

Fig. 3. Simulation results under scenario 2 ($N_s = 200, \rho = 0.05, \alpha = 1, \beta = 1$): a) SRP of a conventional beamformer. b) MSE of DOA1. c) MSE of DOA2. d) MSE of DOA3

Comparing to subspace based approaches, the proposed algorithm is much robust. Fast convergence is observed in both scenarios using a small number of samples, and the complexity of CE is comparable to AP according to our empirical experiences. A more detailed comparison among CE and other deterministic implementations of MLE will be left to our future work.

6. CONCLUSIONS

In this paper, we propose a CE-based implementation for wideband sources DOA ML estimator. The performance of the CE-algorithm has been studied through computer simulations. While the performance of many subspace-based methods degrades significantly when sources are coherent, simulation results show that the CE-algorithm converges to the CRB for both coherent and incoherent sources scenarios at a comparable rate. These numerical studies demonstrate the efficiency and robustness of the proposed algorithm, and suggest the practicality for applications in strong multipath environments.

7. REFERENCES

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