PARTLY-FILLED NONUNIFORM LINEAR ARRAYS FOR DOA ESTIMATION IN MULTIPATH SIGNALS

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ABSTRACT

Partly-filled nonuniform linear arrays (PFNLA) are presented and DOA estimation problem for multipath signals is investigated. A new approach is proposed for DOA estimation in nonuniform linear arrays (NLA) based on array interpolation. A Wiener formulation is used to improve the condition number of the mapping matrix as well as the performance for noisy observations. An initial DOA estimate is obtained by using the uniform part of the PFNLA. This initial estimate is used in array interpolation and a new covariance matrix is found which improves the DOA estimation significantly. Noniterative and iterative algorithms are developed for the DOA estimation in multipath. Proposed approach overcomes some of the limitations of the conventional array interpolation. It is shown that the DOA performance is close to the CRB and the method is robust for a variety of source and DOA scenarios.

***Index Terms*—** Direction of arrival estimation, nonuniformly spaced arrays, music, array signal processing, multipath

1. INTRODUCTION

NLA are shown to be effective in direction of arrival (DOA) estimation [1]. Especially they allow better resolution for the same number of array elements compared to the uniform linear arrays, (ULA). It is well known that when the source signals are coherent, many subspace based DOA estimation methods fail, including the computationally efficient root-MUSIC algorithm. Coherent signals are observed when there are multipath reflections. The solution to the DOA problem in multipath signals is found by forward-backward spatial smoothing [2] in ULA. Forward-backward spatial smoothing requires some kind of shift invariant sub-array structure such as the ULA structure and it cannot be used in arbitrary array geometries such as NLA. A common approach to solve the problem in arbitrary arrays is to use array interpolation to map the real array to a virtual ULA [3].

Array interpolation [4] is an effective technique to map the actual array geometry to a desired virtual array structure. One of the important applications is to convert an array manifold to a uniform linear array form in order to take advantage of the Vandermonde structure, which allows the use of the fast subspace methods [3]. In the conventional array interpolation, mapping matrix requires the solution of a linear equation which may be ill-conditioned in certain cases [5]. A further difficulty is the requirement to define an angular sector where all the sources are assumed to be inside. As the angular sector widens, the mapping accuracy decreases due to the least-squares solution of the mapping equation. In this paper, we overcome the above problems by considering first a Wiener formulation of the mapping relation. This solution improves the condition number of the mapping matrix. In addition, mapping accuracy improves for noisy observations. Furthermore, it allows more sensors in the virtual array than the real array. We also use an initial DOA estimate in order to remove the dependency on an angular sector. The use of initial DOA estimate also improves the mapping accuracy in array interpolation.

In this paper, we define PFNLA structures which form a subset of nonuniform linear arrays. We use PFNLA together with the proposed DOA estimation algorithms when the source signals are coherent. The proposed approach also works for noncoherent sources as well. PFNLA is a NLA which has an ULA and a NLA part. This hybrid form allows one to use forward-backward spatial smoothing [2] to obtain an initial DOA estimate by using the root-MUSIC algorithm. Array interpolation is used together with this initial estimate to obtain a better covariance matrix estimate. This new covariance matrix results significantly better DOA estimate compared to the initial estimate. In addition to this noniterative approach, we also present the iterative method which is simply the repetition of the noniterative algorithm by using the improved DOA estimates. Iterative approach results considerable improvement over the noniterative approach especially at low SNR. It turns out that PFNLA and the proposed DOA estimation algorithms are very effective in case of multipath signals and they perform consistently as the source number and directions change.

2. PROBLEM FORMULATION

We assume that there are n narrowband plane waves impinging on a NLA with DOA's $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_n]$. NLA has M sensors located at integer multiples of unit distance d which is less than half the wavelength, $d < \lambda/2$. Therefore sensor positions, d_i, are integer values, $\mathbf{d} = [0 \ \ d_2 \ \ d_3 \ \ldots \ d_M]$, in units of d.

In this paper, we present the PFNLA to solve the DOA problem in multipath signals.

Definition: (PFNLA) Partly-filled nonuniform linear array is the combination of two linear arrays, namely a uniform linear array with M_1 sensors and a nonuniform linear array with M_2 sensors.

An example of such an array is $\mathbf{d}_8 = [0 \ 1 \ 2 \ 3 \ 4 \ 8 \ 10 \ 11]$ where $M_1 = 5$, $M_2 = 3$ and $M = M_1 + M_2$. Let $M_\alpha = d_M + 1$ be the number of elements in an equivalent ULA with the same aperture as the PFNLA. Ideally, it is desired to map the covariance matrix of the PFNLA to this ULA and use the advantage of the forward-backward spatial smoothing [2] proposed for the ULA. We assume the narrowband model for the received signal, $\bar{\mathbf{y}}(t) \in \mathcal{C}^{M\times 1}$,

$$\bar{\mathbf{y}}(t) = \bar{\mathbf{A}}\mathbf{s}(t) + \bar{\mathbf{v}}(t) \tag{1}$$

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where $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1) \ \ldots \ \bar{\mathbf{a}}(\theta_n)] \in \mathcal{C}^{M \times n}$ is the manifold matrix for the NLA. Each term in $\bar{\mathbf{A}}$ is given as,

$$\bar{\mathbf{a}}(\theta_i) = \left[1 \ \ \exp(j2\pi\frac{d_2}{\lambda}\sin\theta_i) \ \ldots \ \exp(j2\pi\frac{d_M}{\lambda}\sin\theta_i)\right]^T \quad (2)$$

$\mathbf{s}(t) \in \mathcal{C}^{n\times 1}$ is the signal vector which represents a stationary, zero-mean random process uncorrelated with noise. $\bar{\mathbf{v}}(t) \in \mathcal{C}^{M\times 1}$ is the additive white noise with covariance matrix, $\mathbf{R}_{\bar{v}} = \sigma_v^2\mathbf{I}$. For ULA, the manifold matrix $\mathbf{A} = [\mathbf{a}(\theta_1) \ \ldots \ \mathbf{a}(\theta_n)] \in \mathcal{C}^{M_\alpha \times n}$ has Vandermonde form with,

$$\mathbf{a}(\theta_i) = \left[1 \ \exp(j2\pi\frac{d}{\lambda}\sin\theta_i)\ldots\exp(j2\pi(M_\alpha-1)\frac{d}{\lambda}\sin\theta_i)\right]^T \quad (3)$$

The received signal in this case is, $\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t)$. The covariance matrix for ULA, $\mathbf{R} \in \mathcal{C}^{M_\alpha\times M_\alpha}$, is

$$\mathbf{R} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_v^2\mathbf{I} \quad (4)$$

If we assume that N snapshots are given, the $M \times M$ and $M_\alpha \times M_\alpha$ sample covariance matrices for the NLA and ULA are given respectively as,

$$\bar{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\bar{\mathbf{y}}(t)\bar{\mathbf{y}}^H(t), \qquad \hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{y}(t)\mathbf{y}^H(t) \quad (5)$$

Note that $\bar{\mathbf{y}}(t)$ and $\bar{\mathbf{R}}$ can be obtained from $\mathbf{y}(t)$ and $\hat{\mathbf{R}}$ by selecting the appropriate rows and columns through a binary selection matrix. Let $\mathbf{B}$ be a $M \times M_\alpha$ matrix where the only nonzero element in its j^{th} row is at $d_j, j = 1, \ldots, M$. Then $\bar{\mathbf{A}} = \mathbf{B}\mathbf{A}$ and $\bar{\mathbf{y}}(t) = \mathbf{B}\mathbf{A}\mathbf{s}(t) + \sigma_v^2\mathbf{I}$.

In our case, the problem is to find an estimate of the source DOA's with a fast subspace algorithm such as root-MUSIC given the observed samples, $\bar{\mathbf{y}}$. This problem can be solved satisfactorily only when we have a good estimate of the covariance matrix, $\mathbf{R}$. For nonredundant and partially augmentable arrays, finding an estimate of covariance matrix requires some of the missing lags to be completed. For PFNLA, the same task corresponds to finding a better covariance matrix estimate than $\bar{\mathbf{R}}$. It turns out that the type of processing to obtain a good covariance matrix estimate makes an important difference in DOA estimation performance as it will be shown in the following parts of this paper.

3. ARRAY INTERPOLATION FOR NOISY OBSERVATIONS

Array interpolation is a well known technique in DOA estimation [4], [5], [6]. It is used to map the covariance matrix of a real array to a virtual array. The real array usually has a circular geometry whereas the virtual array is a ULA [3], [6]. This mapping allows one to take advantage of the special Vandermonde structure of the ULA array manifold in order to use the fast subspace algorithms such as root-MUSIC.

Array interpolation is done by considering an interpolation sector $\tilde{\theta} \in [\theta_b \ , \ \theta_f]$ where the source DOA's are all assumed to be inside this sector. If there is a source outside this sector, there is no guarantee that it will be identified correctly. In general, interpolation sector is uniformly divided with $\Delta\theta$ intervals and array manifold is generated by considering $\tilde{\theta}_i = i\Delta\theta$, $i = 1, \ldots, (\theta_f - \theta_b)/\Delta\theta + 1$. Let $\mathbf{A}(\tilde{\theta})$ and $\bar{\mathbf{A}}(\tilde{\theta})$ be the manifold matrices for ULA and NLA respectively. The mapping matrix for the conventional array interpolation [4], [5], $\mathbf{T}$, is given as,

$$\mathbf{T} = \mathbf{A}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H\left(\bar{\mathbf{A}}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H\right)^{-1} \quad (6)$$

$\mathbf{T}$ is found as the least-squares solution (LS) and as the interpolation sector increases the accuracy of the mapping decreases. On the other hand, interpolation sector should be kept as large as possible to cover most of the looking directions. This contradiction is one of the limitations of array interpolation. Multiple sector approach [3] and bias reduction [6] may be used to overcome this problem to some extend. In this paper, we will use initial DOA estimates to remove the sector dependency. Another problem in finding $\mathbf{T}$ is the condition number of $\bar{\mathbf{A}}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H$ [5]. In conventional array interpolation, the number of elements in the virtual array is less than or equal to the real array. Even in this case, $\bar{\mathbf{A}}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H$ may be ill-conditioned for certain angular sectors [5] and different techniques should be used to find an appropriate mapping matrix. In the following part, we will present the Wiener solution for array interpolation which improves the condition number as well as the DOA performance for noisy observations.

Given $\bar{\mathbf{y}} = \bar{\mathbf{A}}\mathbf{s} + \bar{\mathbf{v}}$ for NLA, we need to find $\check{\mathbf{y}} = \mathbf{A}\mathbf{s}$ of the ULA. If we define the error as $\mathbf{e} = \check{\mathbf{y}} - \mathbf{T}\bar{\mathbf{y}}$ and find the MSE optimum solution for $\mathbf{T} \in \mathcal{C}^{M_\alpha\times M}$, we obtain,

$$\mathbf{T} = \mathbf{A}(\tilde{\theta})\mathbf{R}_s\bar{\mathbf{A}}(\tilde{\theta})^H\left(\bar{\mathbf{A}}(\tilde{\theta})\mathbf{R}_s\bar{\mathbf{A}}(\tilde{\theta})^H + \mathbf{R}_{\bar{v}}\right)^{-1} \quad (7)$$

If we assume $\mathbf{R}_{\bar{v}} = \sigma_v^2\mathbf{I}$ and $\mathbf{R}_s = \sigma_s^2\mathbf{I}$ for uncorrelated source signals, we have,

$$\mathbf{T} = \sigma_s^2\mathbf{A}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H\left(\sigma_s^2\bar{\mathbf{A}}(\tilde{\theta})\bar{\mathbf{A}}(\tilde{\theta})^H + \sigma_v^2\mathbf{I}\right)^{-1} \quad (8)$$

Note that the assumption $\mathbf{R}_s = \sigma_s^2\mathbf{I}$ might be seen restrictive since in practice source signals are usually correlated. It turns out that the mapping error due to the violations of this assumption is small compared to the errors due to finite length signals and noise which dominate the MUSIC performance [7]. Furthermore conventional array interpolation does the same assumption implicitly.

4. DOA ESTIMATION FOR MULTIPATH SIGNALS

One of the most serious problems in DOA estimation is the multipath problem. Multipath occurs when one or more signal reflections are received by the sensors. Therefore a source signal in one direction is a scaled version of another with a different direction, i.e., $s_j(t) = re^{j\phi}s_i(t)$ for some i, j. In this case, we have coherent signals in the sensors and signal covariance matrix $\mathbf{R}_s$ is singular. It turns out that phase term ϕ in the multipath signal is more problematic than the magnitude scaling. Therefore ϕ should be evaluated in 2π range in order to have a good idea of the algorithm resilience to multipath signals.

Most of the DOA estimation algorithms including root-MUSIC completely fail in multipath scenario. Maximum likelihood methods can solve this problem but they are computationally expensive. Fortunately forward-backward spatial smoothing [2] can be used to decorrelate the signals and estimate the true DOA's. However this method requires shift invariant subarray structure as in ULA and therefore cannot be applied to a large number of array geometries including the NLA presented in the previous section. Array interpolation is used to map these geometries to ULA to take advantage of the forward-backward smoothing technique.

It turns out that NLA's are affected more than the other types of array geometries in case of multipath signals. In fact, they fail completely in this case. Some solutions to this problem are proposed [8] but they require large number of array elements and they are computationally expensive. In this respect a simple and robust algorithm for NLA is important. A useful NLA structure and algorithm should perform sufficiently well for a variety of cases and source directions.

4.1. Finding an initial DOA estimate

In order to have a sector independent array interpolation, we need to have an initial estimate of the source angles. In this paper, we propose to find this initial estimate by using a hybrid array structure, PFNLA, as we have described in section II. In this array form, M_1 element ULA is followed by a M_2 element NLA. This type of array structure allows a good initial DOA estimate in multipath case. As the size of the ULA part in PFNLA increases, initial DOA estimates can be found more accurately. Therefore we apply the standard approach in M_1 element ULA for multipath signals and employ forward-backward spatial smoothing [2] to determine the initial DOA's $\hat{\boldsymbol{\theta}}$ for the source signals.

4.2. Construction of the mapping matrix, T

Once we have an initial DOA estimate $\hat{\boldsymbol{\theta}} = [\hat{\theta}_1, \ldots, \hat{\theta}_n]$, we construct $\mathbf{T}$ by considering narrow sectors for each $\theta_i \in [\theta_i - \theta_\epsilon, \theta_i + \theta_\epsilon]$. Each of these small sectors are divided with $\tilde{\theta}_i$ as explained in the previous section and $\mathbf{A}(\tilde{\theta})$ and $\bar{\mathbf{A}}(\tilde{\theta})$ are found. Interpolation matrix is found from (8).

4.3. DOA Estimation

Given $\mathbf{T}$, we can construct the $M_\alpha \times M_\alpha$ covariance matrix as $\mathbf{R}_a = \mathbf{T}\bar{\mathbf{R}}\mathbf{T}^H$. Array interpolation should also be accompanied with noise whitening [3]. In our case, $\mathbf{T}\mathbf{T}^H$ is rank deficient and there is no unique whitening transformation [9]. It turns out that noise whitening does not improve the DOA performance of NLA in this case since the matrix $\mathbf{T}\mathbf{T}^H$ is close to a covariance matrix of a white sequence already. On the contrary, noise whitening as in [9] performs poorly at low SNR since the inverse of some of the close to (or almost) zero eigenvalues are taken as zero for the Moore-Penrose pseudoinverse of the diagonal eigenvalue matrix. Therefore there is no need for noise whitening for our case which is advantageous for computational complexity. The complete procedure for the DOA estimation by array interpolation in NLA is given as follows,

Step 1: Use the output samples of the ULA part of the PFNLA and find $M_1 \times M_1$ covariance matrix. Apply forward-backward spatial smoothing and then root-MUSIC algorithm to find an initial DOA estimate $\hat{\boldsymbol{\theta}}$.

Step 2: Use $\hat{\boldsymbol{\theta}}$ and construct $\mathbf{T}$ from (8).

Step 3: Given $\mathbf{T}$, find $\mathbf{R}_a = \mathbf{T}\bar{\mathbf{R}}\mathbf{T}^H$. Then use forward-backward spatial smoothing to find the DOA estimate by root-MUSIC algorithm.

Above algorithm will be denoted as MCA-AI. For the iterative improvement, we can add the following additional step.

Step 4: (For iterative improvements) Use estimated angles in Step 1 or 3 and repeat Step 2 and 3 K times to improve the DOA estimates.

Iterative MCA-AI will be denoted as IMCA-AI. In certain cases, IMCA-AI can significantly improve the DOA accuracy with a price paid on the increase in computational complexity.

5. SIMULATION RESULTS

Proposed algorithms are evaluated for a variety of cases in an example of PFNLA, $\mathbf{d}_8 = [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 8\ \ 10\ \ 11]$. DOA performance is compared with the ULA which has the same number of elements, namely, $\mathbf{d}_{ULA} = [0\ \ 1\ \ \ldots\ \ 7]$. CRB for the NLA is also considered as in [3], [10], [11]. There are 1000 trials for each experiment and the number of snapshots is 200. MCA-AI and IMCA-AI are the noniterative and iterative algorithms proposed in this paper. Initial estimate stands for the DOA estimate obtained from the first $M_1 = 5$ elements of the PFNLA. IMCA-AI uses a total of four iterations. $\theta_\epsilon = 0.8$ for the MCA-AI and it is changed at each iteration as $\theta_\epsilon = [1\ \ 0.5\ \ 0.2\ \ 0.1]$ for the the IMCA-AI algorithm. Source angles are considered in degrees where 0 and 180 degrees are endfire and 90 degrees is broadside. When there is multipath, $s_3(t) = re^{j\phi}s_1(t)$ where r is uniformly distributed in $[0.5, 1]$ and ϕ is also uniformly distributed in $[0, 2\pi]$. Figure 1 shows the DOA performance when there is no multipath and two sources are in 78 and 80 degrees respectively. The performance of both MCA-AI and IMCA-AI are good and they approach to the CRB as the SNR is increased. After SNR=10 dB, proposed approach performs better than the same number of element ULA. Figure 2 shows the DOA performance for three sources at 55, 72 and 85 degrees respectively when there is multipath. In this figure, C-AI stands for the conventional array interpolation as in [3] where the interpolation sector is taken as 40 degrees in $[50, 90]$. The performance of the proposed approach is significantly better than the C-AI and ULA. Figure 3 shows the sector independent characteristics of the proposed approach. Three sources are positioned at 55, 85, and 130 degrees and there is multipath. The performance of the proposed methods approaches to the CRB starting from the low SNR values. The robust DOA estimation characteristic of the proposed approach is shown in Figure 4. In this case, SNR=15 dB and there are three sources where two of them are at 55, 85 degrees and the third one is swept from 87 to 149 degrees as shown in the figure. MCA-AI and IMCA-AI show a robust performance for different source directions. As it is shown in Figures 1-4, proposed approach performs significantly better than the ULA with the same number of elements and approaches to the CRB closely. DOA performance is also better than the conventional array interpolation and there is no sector dependence.

6. CONCLUSION

We presented a type of NLA, namely PFNLA, to solve the multipath problem. Iterative and noniterative algorithms are proposed in order to use root-MUSIC algorithm in DOA estimation. These algorithms are based on the array interpolation. A Wiener formulation is presented in order to find the mapping matrix effectively. This formulation allows better mapping error especially at low SNR. An initial DOA estimate is obtained by using a part of the sensors in PFNLA. This initial DOA estimate is used in array interpolation to obtain a better mapping from the NLA to a ULA with the same aperture. It is shown that the proposed approach can effectively solve the multipath problem and it is better than the conventional approaches that can be used in NLA.

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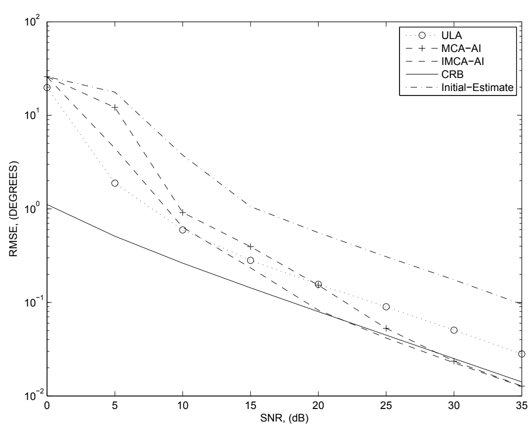

Fig. 1. DOA performance for two uncorrelated sources at 78 and 80 degrees for an eight-element PFNLA. Forward-backward spatial smoothing is applied assuming that there is no information about the source correlation.

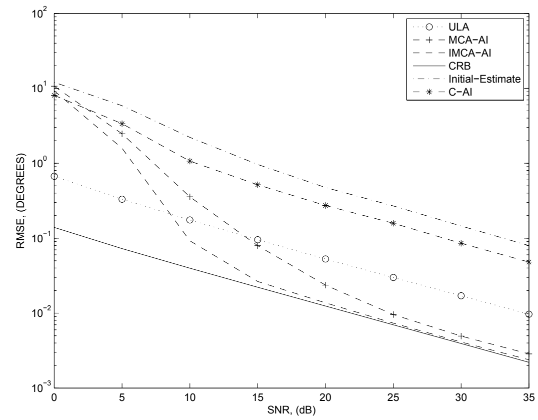

Fig. 2. DOA performance for three sources at 55, 72 and 85 degrees. The first and the third sources are coherent whereas the second one is uncorrelated with the other sources.

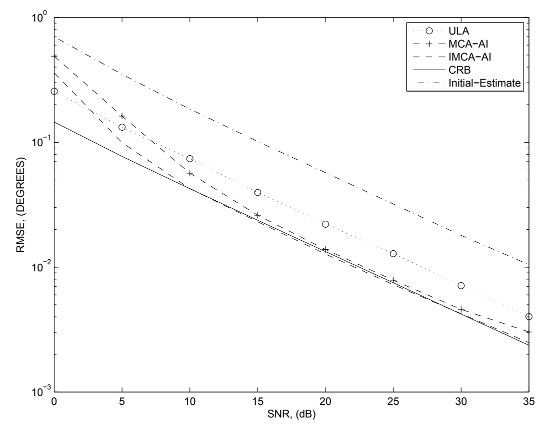

Fig. 3. Performance when the sources are spread over a large sector. Sources are at 55, 85 and 130 degrees. First and third sources are coherent.

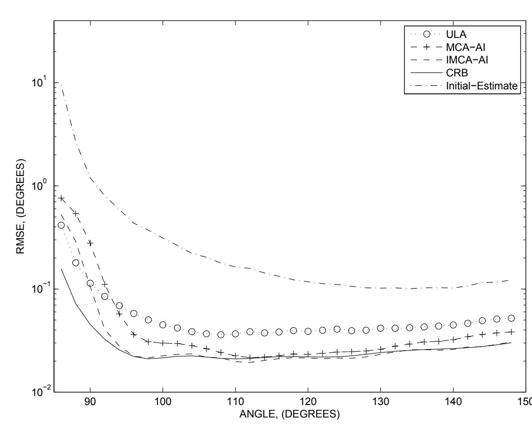

Fig. 4. Two sources at 55 and 85 degrees are fixed whereas the third source is swept between 87 to 149 degrees. The first and third sources are coherent.