POSTERIOR CRLB BASED SENSOR SELECTION FOR TARGET TRACKING IN SENSOR NETWORKS

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ABSTRACT

The objective in sensor collaboration for target tracking is to dynamically select a subset of sensors over time to optimize tracking performance in terms of mean square error (MSE). In this paper, we apply the Monte Carlo method to compute the expected *posterior* Cramér-Rao Lower Bound (CRLB) in a nonlinear, possibly non-Gaussian, dynamic system. The joint recursive one-step-ahead CRLB on the state vector is introduced as the criterion for sensor selection. The proposed approach is validated by simulation results. In the experiments, a particle filter is used to track a single target moving according to a white noise acceleration model through a two-dimensional field where bearing-only sensors are randomly distributed. Simulation results demonstrate the improved tracking performance of the proposed method compared to other existing methods in terms of tracking accuracy.

Index Terms— Target tracking, Particle Filters, Extended Kalman Filter (EKF), *posterior* CRLB, sensor networks

1. INTRODUCTION

For an Ad Hoc sensor network that consists of a large number of spatially distributed sensors, it is desirable not to use all the sensors to track a target at each time, since there always exist constraints on computation, sensing range, communication bandwidth, and energy consumption. Thus a critical task is to select a subset of sensors to optimize system performance under these constraints.

In [1][2], sensor selection is based on an entropy-based information measure, which is calculated by the expected *posterior* distribution of the state to be estimated. Because active sensors are selected before new measurements arrive, expected likelihood function without measurements has to be estimated, which needs considerable extra computation and increases the estimation error. An intuitive method that employs the expected mean squared state estimation error to determine sensor scheduling for multiple steps ahead is presented in [3][4]. An extension of this method for tracking a highly maneuvering target in clutter was proposed in [5]. This approach works only if the sensors to be scheduled have different covariance matrices for the measurement noise. Otherwise the evaluation of the cost function results in equal values. So it is not applicable to a large scale sensor network with homogenous sensors.

The tracking accuracy in terms of mean square error (MSE) is bounded by *posterior* Cramér-Rao lower bound (PCRLB) [6]. This lower bound gives an indication of performance limits, so it can be used as a criterion for sensor selection [7]. The PCRLB depends not only on the sensing accuracy of individual sensors, but also on sensor locations relative to target position and the *posterior* probability density function (PDF) of the target state. Only under very restricted conditions where the target motion model and the sensor measurement model are both linear and the noise for each model is Gaussian, can we get a closed form expression for PCRLB. To overcome the nonlinearity, in this paper we establish our cost function for sensor selection based on the expected PCRLB, which is computed by a particle filter. It will be shown that in our method, even without the new measurements, we still can calculate the PCRLB driven by the particle filter, and estimate of the expected likelihood function is not required. The simulation results show that the particle filter PCRLB driven method outperforms the EKF *posterior* CRLB driven method [7] and the entropy based method [2]. The proposed method can be applied to either homogenous or heterogenous sensor networks with nonlinear models and non-Gaussian noise. .

2. SYSTEM MODELS

2.1. Target Motion Model

In this paper, we consider a single target moving in a 2-D Cartesian coordinate plane according to a dynamic white noise acceleration model [8]:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_k \tag{1}$$

where the constant parameter $\mathbf{F}$ models the state kinematics, the target state at time k is defined as $\mathbf{x}_k = [x_k\, \dot{x}_k\, y_k\, \dot{y}_k]^T$, x_k and y_k denote the target position and $\dot{x}_k$, and $\dot{y}_k$ denote the velocities. $\mathbf{v}_k$ is white Gaussian noise with covariance matrix $\mathbf{Q}$.

2.2. Sensor Measurement Model

In this paper, we assume that a large number of homogenous bearing only sensors are randomly deployed. There exists a cluster head (CH) that is responsible for collecting information from each sensor and providing the estimate of the target state. The CH has knowledge about the individual sensors, such as their positions and measurement accuracy. At each time, only a small number of sensors are activated to perform the sensing task and providing their observations to the CH. The measurement model is given by

$$\mathbf{z}_k^j = h(\mathbf{x}_k) + \mathbf{w}_k = tan^{-1}\left(\frac{y_k - y^{s_j}}{x_k - x^{s_j}}\right) + \mathbf{w}_k \tag{2}$$

where $\mathbf{z}_k^j$ is the measurement from sensor j, x^{s_j} and y^{s_j} represent the corresponding position of sensor j, and $\mathbf{w}_k$ is the white Gaussian noise with covariance matrix $\mathbf{R}$.

3. POSTERIOR CRAMER-RAO LOWER BOUNDS

Let $\hat{\mathbf{x}}_k$ be an unbiased estimator of the state vector $\mathbf{x}_k$, given $\mathbf{z}_{1:k}$, which denotes all the measurements from time 1 to k. The covariance of the state estimate P_k is bounded below by the recursive PCRLB, which is defined to be the inverse of the Fisher Information Matrix(FIM) J_k

$$P_k = E\{[\hat{\mathbf{x}}_k - \mathbf{x}_k][\hat{\mathbf{x}}_k - \mathbf{x}_k]^T\} \geq J_k^{-1} \tag{3}$$

$$J_k = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \log p(\mathbf{x}_k, \mathbf{z}_k)\} \tag{4}$$

where J_k^{-1} is the *posterior* CRLB and $\Delta_\Psi^\Theta = \nabla_\Psi \nabla_\Theta^T$. ∇ is the first-order partial derivative defined as

$$\nabla_\mathbf{x} = \left[\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_r}\right]^T \tag{5}$$

In [6], Tichavske et al. provide a recursive approach for calculating the sequential FIM J_k

$$J_{k+1} = D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1} D_k^{12} \tag{6}$$

where

$$D_k^{11} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{7}$$

$$D_k^{12} = E\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_{k+1}} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{8}$$

$$D_k^{21} = E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_k} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} = (D_k^{12})^T \tag{9}$$

$$\begin{aligned} D_k^{22} &= E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}} \log p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} + \\ &\quad E\{-\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}} \log p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})\} \\ &= D_k^{22,a} + D_k^{22,b} \end{aligned} \tag{10}$$

Note that all the above expectations are taken with respect to the joint PDF $p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})$, where $\mathbf{X}^{k+1} \triangleq \mathbf{x}_{0:k+1}$ and $\mathbf{Z}^{k+1} \triangleq \mathbf{z}_{1:k+1}$ denote all the states and observations up to time $k+1$.

The recursion of equation(6) starts from an initial FIM J_0, which can be calculated from the *a priori* PDF $p(x_0)$.

$$J_0 = E\{-\Delta_{\mathbf{x}_0}^{\mathbf{x}_0} \log p(\mathbf{x}_0)\} \tag{11}$$

Even though these are elegant expressions to calculate FIM recursively, the expectations in (7)$\sim$(10) generally do not have analytical closed-form results. Here we circumvent the above difficulty by resorting to the Monte Carlo approach by converting the above integrals to summations [9]. Each PDF in the expectations will be represented as a set of samples with associated weights. For the linear motion model and nonlinear measurement model adopted in this paper, the equations (7)$\sim$(10) become

$$D_k^{11} = \mathbf{F}^T \mathbf{Q}^{-1} \mathbf{F} \tag{12}$$

$$D_k^{12} = (D_k^{12})^T = -\mathbf{F}^T \mathbf{Q}^{-1} \tag{13}$$

$$D_k^{22} = \mathbf{Q}^{-1} + D_k^{22,b} \tag{14}$$

It can be proved that if $\mathbf{w}_k$ in Equation (2) is additive white Gaussian noise, $D_k^{22,b}$ can be simplified as

$$\begin{aligned} D_k^{22,b} &= 1/2E\{\Delta_{\mathbf{x}_{k+1}}^{\mathbf{x}_{k+1}}\{(\mathbf{z}_{k+1} - h(\mathbf{x}_{k+1})) \\ &\quad \mathbf{R}^{-1}(\mathbf{z}_{k+1} - h(\mathbf{x}_{k+1}))\}\} \\ &= \frac{1}{R} E_{p(\mathbf{x}_{k+1}|\mathbf{x}_k)}\{\Lambda_k(\mathbf{x}_{k+1}, \mathbf{x}_k)\} \end{aligned} \tag{15}$$

And for the bearing-only measurement model

$$\Lambda_k(\mathbf{x}_{k+1}, \mathbf{x}_k) = \begin{bmatrix} M_{1,1}^k & 0 & M_{1,3}^k & 0 \\ 0 & 0 & 0 & 0 \\ M_{3,1}^k & 0 & M_{3,3}^k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

where

$$M_{1,1}^k = \frac{(y_{k+1} - y^{s_j})^2}{[(x_{k+1} - x^{s_j})^2 + (y_{k+1} - y^{s_j})^2]^2}$$

$$M_{1,3}^k = M_{3,1}^k = -\frac{(x_{k+1} - x^{s_j})(y_{k+1} - y^{s_j})}{[(x_{k+1} - x^{s_j})^2 + (y_{k+1} - y^{s_j})^2]^2}$$

$$M_{3,3}^k = \frac{(x_{k+1} - x^{s_j})^2}{[(x_{k+1} - x^{s_j})^2 + (y_{k+1} - y^{s_j})^2]^2}$$

Note that the expectation of $\Lambda_k(\mathbf{x}_{k+1}, \mathbf{x}_k)$ in (15) is taken with respect to $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$ instead of $p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1})$ because of the following iterative equation.

$$p(\mathbf{X}^{k+1}, \mathbf{Z}^{k+1}) = p(\mathbf{X}^k, \mathbf{Z}^k) \cdot p(\mathbf{x}_{k+1}|\mathbf{x}_k) \cdot p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1}) \tag{17}$$

The likelihood $p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})$ has been integrated out, so the proposed selection strategy based on (15) to be described in Section 4 is computationally much simpler than the information-driven approach [1][2]. The integrals due to expectation can be evaluated approximately by Monte Carlo integrals only if we have a discrete sample-based representation for $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$. In particle filtering, the *posterior* PDF $p(\mathbf{x}_k|\mathbf{Z}^k)$ can be represented approximately by a set of samples with associated weights.

$$p(\mathbf{x}_k|\mathbf{Z}^k) \cong \sum_{i=1}^{N} \omega_k^{(i)} \cdot \delta_{\mathbf{x}_k^{(i)}}(\mathbf{x}_k - \mathbf{x}_k^{(i)}) \quad (18)$$

where N is the number of particles. The PDF $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$ can be represented approximately by propagating the samples $\{\mathbf{x}_k^{(i)}\}$ from time k to $k+1$

$$p(\mathbf{x}_{k+1}|\mathbf{x}_k) \cong \sum_{i=1}^{N} \omega_k^{(i)} \cdot \delta_{\mathbf{x}_{k+1}^{(i)}}(\mathbf{x}_{k+1} - \mathbf{x}_{k+1}^{(i)}) \quad (19)$$

Then Equation (14) can be rewritten as

$$D_k^{22} \cong \mathbf{Q}^{-1} + \frac{1}{R}\sum_{i=1}^{N} \Lambda(\mathbf{x}_{k+1}^{(i)}, \mathbf{x}_k^{(i)}) \quad (20)$$

From the strong law of large numbers, it can be shown that the above sample-based expectation converges almost surely to the true expectation(10). Now plug (12), (13) and (20) into (6), and we can get approximate versions of the recursive FIM $\hat{J}_k$ in terms of samples and associated weights. Note that the cost function (to be described in the next section) established on the approximation of J_k is computed without any future measurements, hence, it can be used as a selection criterion among the sensor candidates for the target tracking problem in sensor networks.

4. SENSOR SELECTION BASED ON POSTERIOR CRLB

Assume we choose a subset consisting of L_s^k sensors from the total L_s candidates on every tracking snapshot at time k, where L_s^k can change over time. Then the weights can be recursively calculated according to the following equation if the sensor measurements are independent from each other.

$$\omega_k^{(i)} = \omega_{k-1}^{(i)} \frac{p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)})}{\pi(\mathbf{x}_k^{(i)}|\mathbf{x}_{0:k-1}^{i}, \mathbf{Z}_{1:k})} \prod_{j \in L_s^k} p(\mathbf{z}_k^j|\mathbf{x}_k^{(i)}) \quad (21)$$

where $\pi(\mathbf{x}_k|\mathbf{x}_{0:k-1}, \mathbf{Z}_{1:k})$ is the proposal density function, which is used to generate new particles at time k, $\mathbf{Z}_{1:k}$ denotes all the measurements up to time k, $\mathbf{Z}_k \triangleq \{\mathbf{z}_k^j, j \in L_s^k\}$ are the measurements at time k. Now the recursive PCRLB can be evaluated with the help of particle filtering.

The PCRLB gives an indication of performance bounds, and no unbiased estimators can outperform it in terms of MSE for each element of the state. Usually people are more concerned with the target position. So we choose the summation of the position bound along each axis as the cost function for time $k+1$

$$\mathcal{C}_{k+1} = J_{k+1}^{-1}(1,1) + J_{k+1}^{-1}(3,3) \quad (22)$$

where the $J_{k+1}^{-1}(1,1)$ and $J_{k+1}^{-1}(3,3)$ are the bounds on the MSE corresponding to x_{k+1} and y_{k+1} respectively. The cost functions can also be selected as the determinant of J_k^{-1} or trace of J_k^{-1}, but the simulation results did not show much difference. Those sensors that collectively minimize the above cost function will be activated at the next time $k+1$. In this paper, we use the optimal enumerative search method to determine the combination of sensors, which minimizes the cost function.

$$L_s^{k+1,*} \triangleq \underset{L_s^{k+1} \subset \mathcal{S}}{\operatorname{argmin}}\, \mathcal{C}_{k+1}(L_s^{k+1}) \quad (23)$$

where $\mathcal{S}$ denotes the set containing all the sensors.

5. SIMULATION RESULTS

The performance of the proposed sensor selection approach in this paper is evaluated in terms of the MSEs of the state vector. In the simulations, we consider a scenario where 100 homogenous bearing-only sensors are randomly deployed in a 500×500 field. A single target moves in the field for 60 seconds according to the white noise acceleration model (1). At each time, 2 sensors are activated to report the information of the target to CH according to (2). The measurement noise variance is set to $R = 0.025$, and the system noise covariance matrix Q is chosen as

$$Q = \begin{bmatrix} 0.3333 & 0.5000 & 0 & 0 \\ 0.5000 & 1.0000 & 0 & 0 \\ 0 & 0 & 0.3333 & 0.5000 \\ 0 & 0 & 0.5000 & 1.0000 \end{bmatrix}$$

The prior PDF of the target state is assumed Gaussian with mean $[0\ 10\ 0\ 10]^T$ and covariance $P_0 = diag(1, 0.5, 1, 0.5)$. For simplicity and illustrative purposes, the transition PDF $p(\mathbf{x}_{k+1}|\mathbf{x}_k)$ is chosen as the proposal density function $\pi(\mathbf{x}_k|\mathbf{x}_{0:k-1}, \mathbf{z}_{0:k})$. We implement our approach by using $N = 300$ particles, and 100 Monte Carlo repetitions are performed for each experiment.

For comparison purposes, we also consider three other selection methods. 1) Nearest neighbor: Let $(\hat{x}_{k+1}, \hat{y}_{k+1})$ denote the predicted position based on the current state estimation $p(\mathbf{x}_k|\mathbf{z}_{1:k})$. The sensors that have the closest distance to the predicted position of the target at the next time will be selected. 2) Expected posterior entropy: the sensors that have the largest expected posterior entropies will be selected. 3) PCRLB calculated by EKF [7].

Figure 1 demonstrates the tracking scenario where true target trajectory and estimated trajectories by different sensor selection methods are compared. We can see that the proposed selection method achieves more accurate tracking results. Figure 2 and Figure 3 show the MSEs of target position in x and y coordinates respectively. The proposed sensor selection method by minimizing the PCRLB offers a significant error reduction for most of the tracking time compared to other existing methods. As can also be seen, *posterior* CRLB is also drawn in the figures and demonstrates that it is lower than all the estimation methods employing different sensor selection methods for almost all the points.

6. CONCLUSIONS

In this paper, we considered a sensor selection problem for tracking a single target in sensor networks. The one-step ahead *posterior* CRLB is approximated recursively by using a particle filter without the knowledge of future measurements. Sensors, that collectively minimized the cost function established on PCRLB, are activated, at that time while other sensors are in the idle state. Simulation results are presented to illustrate the improved performance of our proposed sensor selection approach, which outperforms other existing methods.

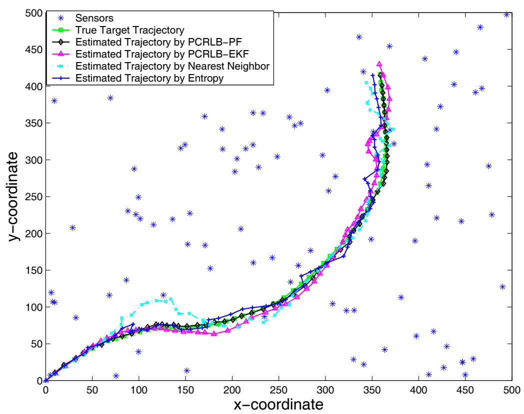

Fig. 1. True and estimated target trajectories using different sensor selection methods.

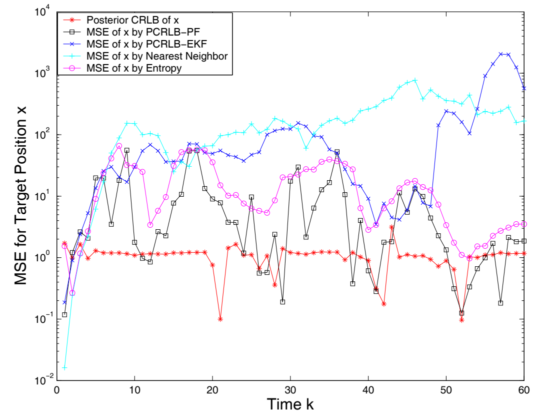

Fig. 2. Comparison of *posterior* CRLB to MSEs for the x-coordinate of target position calculated using different sensor selection methods.

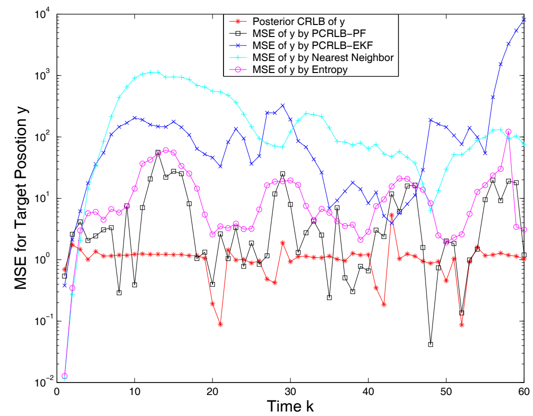

Fig. 3. Comparison of *posterior* CRLB to MSEs for the y-coordinate of target position calculated using different sensor selection methods.

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