ROBUST DISTRIBUTED DETECTION WITH LIMITED RANGE SENSORS

Erhan Baki Ermis, Venkatesh Saligrama

Boston University

ABSTRACT

We consider a multi-target detection problem over a sensor network (SNET) with limited range sensors and communication constraints, which complements the decentralized detection problem where all sensors observe the same target. We consider sensing models where the signal power from targets undergoes a power-law decay. The task is to determine the locations of the targets while minimizing false alarms and communication between sensors. We extend the well-known FDR framework to solve the multi-target detection problem.

***Index Terms*—** Distributed detection, Non-ideal models, Robustness, DTFDR, Multi-object detection

1. INTRODUCTION

Sensor networks (SNET) hold great promise for surveillance and monitoring applications. A wide interest in distributed detection, estimation, and classification in SNETs has emerged in the recent years [1, 2, 3, 4]. A very interesting setup for SNETs is the multi-target detection (MTD) problem with sensors having limited sensing range. In this problem there are an unknown number of targets scattered on a SNET, where the sensors have limited sensing range. For sensor j, H_{1j} is presence of at least one target within its sensing range and H_{0j} is the absence of any. For MTD problems we define the global false alarm event as the event that at least one sensor commits a local false alarm. The general focus is to minimize the misses while suffering minimal false alarms and communication cost, both of which drain the SNET's limited resources.

This problem complements the widely considered decentralized detection problem where all sensors observe the same target and communicate their local observations to the fusion center. While the necessity for collaboration is clear in the latter problem, the benefits of collaboration for sensors with limited sensing range has not been widely explored. In our previous work [5] we showed that collaboration among sensors can be not only beneficial, but also necessary, even with uncorrelated information at the sensors.

Problems involving multiple targets and multiple sensors have an inherent difficulty that prohibits use of global false alarm probability as a performance measure. We refer to this issue as the multiplicity of false alarms and explain below.

This research was supported by ONR Young Investigator Award N00014-02-100362, Presidential Early Career Award (PECASE)

Multiplicity Issue: Suppose $m = 10^4$ sensors scattered on a field. We desire a global false alarm probability, $P_F^G \leq 0.2$. Suppose local false alarm probability, P_F^L is bounded at 0.2 at each sensor. Then as many as 20% of the sensors are expected to commit false alarms with high probability. This implies that P_F^G will be close to one since with high probability at least one sensor will commit a false alarm. Therefore, to reduce P_F^G, P_F^L has to be bounded at much smaller levels, which in turn reduces detection power greatly.

Using global (network-wise) probability of false alarm as the performance measure leads to diminishing detection rates as the number of sensors increase. Using local (sensor-wise) probability of false alarm without global concerns makes it impossible to control the false alarms in the SNET level, wasting network resources. Furthermore, the global false alarm probability and the global miss probability cannot both be less than the local conditional entropy, which can be arbitrarily close to 1/2. For all reasonable setups the local conditional entropy can be high, and the number of errors scale with the number of sensors. This makes using probability of false alarm and probability of miss together as performance measures futile [8]. In view of these issues we previously focused on a novel statistical idea called the false discovery rate (FDR) [9] and communication cost constraints as performance measures. We developed the DTFDR methodology for distributed MTD [6, 7, 8], and showed optimality and scaling properties.

A major assumption in the development of those methodologies was having ideal sensing models (ISM). In the ISM, sensors without a target in their sensing range observe only noise, and sensors with a target in their sensing range observe the signal from that target. Given noise and signal statistics, this setup renders the exact observation models available for all sensors under both H_0 and H_1.

In this work we focus on distributed MTD problems with non-ideal sensing models (NISM). NISM captures a more realistic sensing scenario, in which the the sensors observe target signals attenuated by distance. Therefore, even with known signal statistics at the source and noise statistics at the sensors, the observation models at the sensors cannot be known. For example, the sensors without a target in their sensing range observe noise, distorted by the attenuated signal from far away targets. However the amount distortion is uncertain and can be different for each sensor. Therefore, even if the noise statistics are known, it is impossible to form an exact observation model under H_0. A similar scenario is valid for sensors with a target in their sensing range as well. Thus the observation models under both H_0 and H_1 are uncertain.

2. SETUP AND PROBLEM FORMULATION

Consider a non-Bayesian setting where an unknown number of targets are distributed on a sensor field of m sensors. Targets are observed by a SNET in which the sensor nodes are uniformly distributed. Assume that the noise statistics of sensors and signal statistics of sources are known. We wish to identify the set of sensors that have an object in their limited sensing range. We consider ISM and NISM:

Ideal Sensing Model: Each sensor can observe targets only in its sensing range. This model is valid when each sensor has a limited sensing range and the targets are sparsely distributed. Given noise statistics for n_s and ν_s, and signal statistics at the source for θ, the observation model at sensor s are:

$$H_{0s}: \ Y_s = n_s$$
$$H_{1s}: \ Y_s = \theta + \nu_s$$

Furthermore, observations are independent under H_0 across sensors, i.e. $f\{Y_s = y_s \mid Y_v = y_v, H_{0s}\} = f_{0s}(y), \ \forall v \neq s$.

Non-Ideal Sensing Model: Each sensor observes a signal from each target, and the signal decays as a function of distance. Sensors without a target in their sensing range observe the sum of the decayed signals from far away targets. For simplicity of exposition here we only describe the uncertainty in H_0. See [8] for further details. This model can be viewed as a perturbation of the ISM:

$$H_{0s}: \ X_s = \theta_s + n_s, \ \text{where } \theta_s = \sum_{j=1}^{k} \frac{\theta}{d(s,t_j)}$$
$$H_{1s}: \ X_s = \theta + \nu_s$$

where $t_1 \ldots t_k$ are targets and $d(\cdot,\cdot)$ is a distance function. The uncertainty arises from the unknown θ_s, because both k and $d(s,t_j)$ are unknown. We use this model for the simulations in Section 3.

Below are the variables associated with MTD problem:

	Declared H_0	Declared H_1	Total
True H_0	U	V	m_0
True H_1	T	S	$m - m_0$
Total	$m - R$	R	m

FDR and Problem Formulation: FDR is the expected ratio of the number of false alarms to the total number of observations that are declared significant. In terms of the elements of the above table, $FDR = E(V/R)$.

We now formulate the problem with FDR as a performance constraint. We also add a communication constraint to limit the inter-sensor information exchange. This formulation is valid irrespective of which model is being used:

$$\text{minimize } E(T) \text{ subject to:}$$
$$E(V/R) \leq \gamma \text{ and } \sum_{s,t} C(u_s(t)) \leq \alpha$$

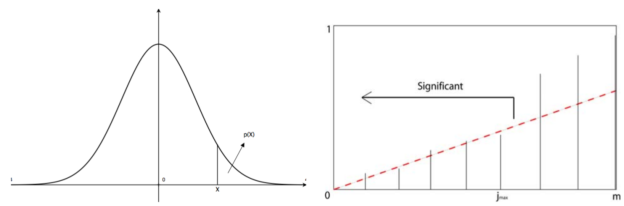
where, $C(u_s(t))$ is the communication cost for sensor s at time t. See [8] for details.

2.1. Solution for the Ideal Sensing Model

Controlling the FDR: The FDR algorithm, introduced in [9], takes the value γ as an input, and in return guarantees a false discovery rate below this input. The FDR procedure is described below and presented in Fig. 1:

1. Calculate the p values for all the observations
2. Order the p values in ascending order
3. Find the largest index, i_{max}, such that $p_i \leq \frac{i}{m}\gamma$
4. Declare p_j significant for $0 \leq j \leq i_{max}$

The p value is defined as $P(X) = \int_X^\infty f_0(t)dt = 1 - F_0(X)$ where f_0 is the pdf of observations under H_0. The p value of the random variable X_0, where X_0 comes from H_0, is defined P_0, and similarly for P_1.



(a) Definition of p value (b) Ordered p values and threshold line

Fig. 1. FDR algorithm

Although it is a very powerful approach, FDR procedure suffers from two significant drawbacks that make it unsuitable in our applications: First, if the realizations P_1 are not clustered around zero, the detection power is very low. Second, the FDR strategy does not lend itself easily to decentralized implementation when F_1 is not concave. Next we present methods to overcome these issues under ISM.

Domain Transformation ($\mathcal{T}$): Define the following tuple, for any measure μ, function ϕ, and uniform measure U:

$$(\alpha_\mu(y), \beta_\mu(y)) = (E_U[I_{\{\phi(x)\geq y\}}(x)], E_\mu[I_{\{\phi(x)\geq y\}}(x)])$$

Define the new measure: $\hat{\mu}(0, \alpha_\mu(y)) = \beta_\mu(y)$. If $\alpha_\mu(y)$ has a jump at $y = y_0$ from a to b, then set

$$\hat{\mu}(0,z) = \frac{\beta_\mu(y_0^-) - \beta_\mu(y_0^+)}{b-a}(z-a) + \beta_\mu(y_0^+) \text{ for } z \in (a,b)$$

which is a conditionally uniform distribution in (a,b).

Now let $g_1(\cdot)$ be the pdf of P_1. Define $\mathcal{T}$ as follows:

1. Let $y_{max} = \max_x\{g_1(x)\}$
2. Define $(\alpha_\mu(y), \beta_\mu(y))$ by setting $\phi(\cdot) = g_1(\cdot)$ for $y \in (0, y_{max})$ and $\mu(B) = \int_B g_1(x)dx \ \forall B \in (0,1)$
3. Construct $\hat{\mu}(0, \alpha_\mu(y)) = \beta_\mu(y) \ \ \forall y \in (0, y_{max})$
4. Let $\hat{g}_1(\cdot)$ be the corresponding density of $\hat{\mu}$. Generate $\hat{P} = \mathcal{T}[P]$ as follows:
   (a) For $P \in (0,1)$ find $Y = g_1(P)$
   (b) Find $S = \{x : \hat{g}_1(x) = Y\}$ and draw $\hat{P} \sim U(S)$

Fig. 2 illustrates the nature of transformation.

The following procedure is referred to as the Domain Transformed FDR (DTFDR) procedure. An efficient distributed DTFDR algorithm can be found in [8]:

1) Apply $\mathcal{T}$ to P_0 and P_1, **2)** Follow the FDR procedure.

Fig. 3 shows ROC-like curves for FDR and DTFDR. The DTFDR is uniformly stronger.

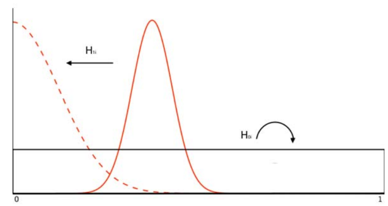
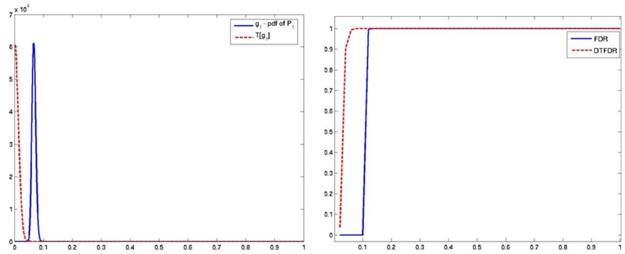


Fig. 2. $\mathcal{T}$ generates a monotonically decreasing pdf.



(a) Original and transformed pdf of P_1 (not normalized)

(b) Detection rate vs γ

Fig. 3. ROC-like curves for FDR and DTFDR procedures.

2.2. Extension to the Non-Ideal Sensing Model

In [8] we have shown the optimality and scaling results of the DTFDR procedure, and we have established that the distributed algorithm is robust with respect to observation noise. Note that this robustness is not with respect to the perturbation of the observation model. We now present the robustness of $\mathcal{T}$ with respect to perturbation of the observation model. Here we state the theorems, the proofs can be found in [8].

There are two parts of the robustness property: The first part establishes that the miss rate of the DTFDR increases gracefully with the perturbation of the ISM.

The second part establishes that a family of distributions that are close in terms of a total variation metric remain close upon application of $\mathcal{T}$. Combining this with the first part of the robustness property allows us to address the distributed MTD problem with NISM within the framework that has been described in this paper. See [8] for more details.

Before we proceed to presenting the results, we formally define the false non-discovery rate (FNR) [10], the natural counterpart of FDR. In terms of the variables presented in the table, $FNR = E(T/(m-R))$. Let F be the concave distribution of observations under H_1 such that $F'(0) > \beta$ where $\beta = (\frac{1}{\gamma} - \frac{m_0}{m})/(\frac{m_1}{m})$. Let $\theta_0 = \frac{m_0}{m}$ and $\theta_1 = \frac{m_1}{m}$. In [10] it is shown that if c is the solution to $F(x) = \beta x$, asymptotically the following holds true:

$$FNR = \frac{\theta_1(1-F(c))}{\theta_1(1-F(c)) + \theta_0(1-c)}$$

The following theorem establishes the first part:

Theorem 2.1 *Let F_N be the concave nominal distribution of observations under positive hypotheses such that $F_N'(0) > \beta$ and F_A be the actual distribution of observations under positive hypotheses such that $F_A'(0) > \beta$. Let c be the solution to $F(x) = \beta x$ and u the solution to $F(x) - \epsilon = \beta x$. If $|F_N(x) - F_A(x)| \leq \epsilon(x)$, then for $\xi = (F_N(c) - F_N(u))/(c-u)$,*

$$FNR_{actual} \leq FNR_{nominal} + \frac{1}{(1-\gamma)^2}\frac{\epsilon}{[1 - F_N'(\xi)/\beta]}$$

where $\epsilon = \sup_x\{\epsilon(x)\}$.

This theorem establishes that a small perturbation on the concavity of the nominal distribution under H_1 does not lead to a significant loss of performance in terms of the FNR. In fact, the loss of performance is directly proportional with the upper bound on the perturbation.

The following theorem presents the effect of $\mathcal{T}$ on the total variation distance:

Theorem 2.2 *Let $\mu_N = \int g_{1N}$ be the measure with respect to the nominal distribution of observations under H_1, and define μ_A in a similar fashion for the actual distribution of observations. If $d_{tv}\{\mu_N, \mu_A\} \leq \epsilon$ then $\sup_x |\hat{G}_{1N}(x) - \hat{G}_{1A}(x)| \leq \epsilon$ where $\hat{G}_{1N}(\cdot)$ and $\hat{G}_{1A}(\cdot)$ are the distribution functions obtained by $\mathcal{T}$.*

Theorem 2.2 has the following implication: If the nominal and the actual distribution of observations are ϵ away from each other in terms of total variation distance, then $\mathcal{T}$ will lead to distributions that are at most ϵ away from each other in terms of Kolmogorov distance.

Connecting this with Theorem 2.1 implies that as long as the family of distributions that are considered under the NISM is such that the total variation distance is small, the distributed detection problem can be addressed within the DTFDR framework. The result can be extended to Prokhorov distance, and is presented in [8]. This extension allows us to consider singular distributions as well as continuous ones for H_1.

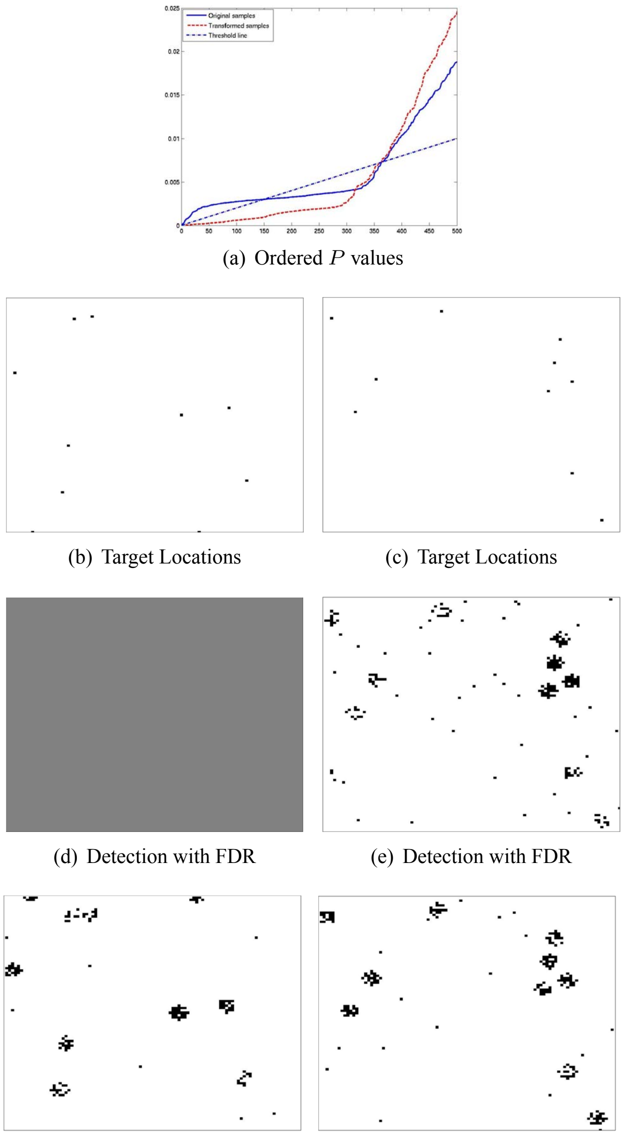
3. SIMULATIONS

Below we present a detection simulation in which we compare FDR and DTFDR procedures with NISM. The sensor field is a grid of size 100x100, where each pixel is assumed to have a sensor. Let the set of targets be $T = \{t_1, t_2, \ldots, t_k\}$ with uniform signal power θ. Define $d(s,t)$ be the Euclidian distance between sensor s and target t. We assume that the target signal is θ within the effective region of the target, and decays rapidly with distance outside this region. H_0 for a sensor is that it is outside the effective region of all targets, and H_1 is that it is inside the effective region of at least one target. Then, with n_s and ν_s being noise at sensor s for H_0 and H_1 respectively, the observation model at sensor s for the NISM is as follows:

$$H_0: \; X_s = \theta_s + n_s, \;\text{ where } \theta_s = \sum_{j=1}^{k} \frac{\theta}{d(s,t_j)}$$
$$H_1: \; X_s = \theta + \nu_s$$

The parameters are: $\gamma = .2$, $\theta = 2.5$, the effective radius of the target $r_{eff} = 2.5$ pixels. $n_s \sim N(0,1)$ and $\nu_s \sim N(0, 0.05)$. There are 10 targets on the field. The communication constraint α is varied and the results are presented for illustrative cases in Fig. 4. For $\alpha \leq 140$, the FDR procedure is unable to detect the significant sensors, whereas the

DTFDR procedure is able to do so. As the communication constraint is loosened, the performance of DTFDR procedure increases proportionally, yet keeping the false alarms at low levels. Although the FDR procedure also detects some significant sensors, the FDR algorithm returns more false alarms. Finally, observe that the FDR procedure has more misses than the DTFDR procedure. This was the expected result throughout the development and analysis.



(a) Ordered P values

(b) Target Locations

(c) Target Locations

(d) Detection with FDR

(e) Detection with FDR

(f) Detection with DTFDR

(g) Detection with DTFDR

Fig. 4. Detection performance of FDR and DTFDR for $\alpha = 140$ bits (b,d,f), $\alpha = 280$ bits (c,e,g).

4. CONCLUSION

In this paper we considered distributed multi-target detection problem in sensor networks. We introduced non-ideal sensing models to the problem and pointed out the important differences between ideal and non-ideal sensing models. In ideal sensing models the observation models are known exactly whereas in the non-ideal sensing model the observation models are only partially available. We extended the DTFDR procedure, which was developed to solve the distributed detection problem under ideal sensing assumptions to address the problem under non-ideal sensing assumptions. This involved establishing the robustness of the DTFDR procedure with respect to model uncertainties. The results prove to be very important as the extensions to NISM connect the theoretical framework we had developed with real life problems in which complete model information may not be available.

5. REFERENCES

[1] J. N. Tsitsiklis, “Decentralized detection,” *Advances in Statistical Signal Processing, H. V. Poor and J. B. Thomas Eds*, vol. 2.

[2] P. K. Varshney, *Distributed Detection and Data Fusion*. Springer, 1997.

[3] Saligrama Venkatesh, M. Alanyali, O. Savas, and S. Aeron, “Classification in sensor networks,” *Proceedings of International Symposium on Information Theory*, Chicago, IL, 2004.

[4] J. F. Chamberland and V. V. Veeravalli, “Decentralized detection in sensor networks,” *IEEE Transactions on Signal Processing*, 2003.

[5] E. B. Ermis, M. Alanyali and V. Saligrama, “Search and Discovery in an Uncertain Networked World,” *IEEE Signal Processing Magazine*, July 2006.

[6] E. Ermis, and V. Saligrama, ”Adaptive Sampling Strategies for Distributed Detection of Localized Phenomena” In *IPSN*, Los Angeles, CA, April 2005.

[7] E. Ermis, and V. Saligrama, ”Decentralized Detection and Estimation of Localized Phenomena” In *ICASSP*, Philadelphia, PA, March 2005

[8] E. Ermis, V. Saligrama Distributed Detection of Localized Phenomena *BU Technical Report 2006-02*

[9] Y. Benjamini, Y. Hochberg, “Controlling the false discovery rate: A pratical and powerful approach to multiple testing,” *Journal of the Royal Stat. Soc.* 1995

[10] C. Genovese, L Wasserman, ”Operating characteristics and extensions of the false discovery rate procedure,” *Journal of the Royal Statistical Society*, Series B, vol.64, pp. 449, 2002.