FREQUENCY INVARIANT BEAMFORMING WITHOUT TAPPED DELAY-LINES

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Abstract. Broadband beamforming, including frequency invariant beamforming, is often achieved by processing the received sensor signals through tapped delay-lines. Unlike most of the existing techniques, we propose a novel design method for three-dimensional frequency invariant beamformers without employing tapped delay-lines. The resultant beamformer, which can form a beam steerable along both the elevation angle and the azimuth angle, has a very simple implementation . A design example is provided to show the effectiveness of the proposed method.

Keywords. broadband arrays, frequency invariant beamforming, Fourier transform, tapped delay-lines

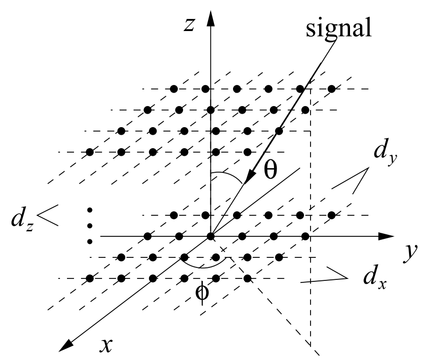

Fig. 1: A three-dimensional sensor array.

1. INTRODUCTION

Beamforming has been studied extensively in the past and recently a lot of interest has been focused on a class of arrays with frequency invariant responses [1, 2, 3, 4, 5, 6, 7, 8], which can achieve a beam pattern independent of frequency, and hence with a constant beamwidth. Traditionally a common feature of broadband beamforming is the use of tapped delay-lines (TDLs), which can form a frequency dependent response for each of the received broadband sensor signals to compensate the phase difference for different frequency components.

As a special class of broadband beamformers, in general the design of a frequency invariant beamformer also requires the use of tapped delay-lines. Recently, a rectangular array with a frequency invariant property was proposed [9] and then further studied in [10]. A special characteristic of this beamformer is that there are no TDLs involved and only one single weight is attached to each sensor. Because of the symmetry of the pattern involved in the design, all of the resultant array coefficients are real, and therefore any phase shift in the array can be avoided, which greatly simplifies the circuits.

However, the problem with this kind of rectangular array is that it can only form a beam along the azimuth angle and basically it works as a linear array, which is due to the ambiguity between frequency and the elevation angle. In order to remove this ambiguity and achieve a beamforming capability along both the azimuth angle and elevation angle, i.e. functioning as a normal rectangular array, we need a three-dimensional (3-D) array system. In this paper, we propose a novel design method for such a frequency invariant 3-D array without TDLs. In Section 2, the beam response of a 3-D array is studied and based on our analysis a novel design method is proposed. A design example is given in Section 3 and finally conclusions are drawn in Section 4.

2. FREQUENCY INVARIANT BEAMFORMING FOR THREE-DIMENSIONAL ARRAYS

Fig. 1 shows an equally spaced three-dimensional array in the (x, y, z) space with a signal coming from a direction of

(θ, ϕ). The spacing of the array elements in the x, y and z directions is d_x, d_y and d_z, respectively.

The array's beam response with respect to temporal frequency $\omega\ rad/s$ and angle of arrival (θ, ϕ) of the impinging signal is given by

$$P(\omega, \theta, \phi) = \sum_{k,l,m=-\infty}^{\infty} D(kd_x, ld_y, md_z) e^{-j\frac{k\omega \sin\theta \cos\phi d_x}{c}} e^{-j\frac{l\omega \sin\theta \sin\phi d_y}{c}} e^{-j\frac{m\omega \cos\theta d_z}{c}} , \quad (1)$$

where $D(kd_x, ld_y, md_z)$ is the response of the sensor at the position (kd_x, ld_y, md_z), $k, l, m = \ldots, -1, 0, 1, \ldots$, and c is the wave propagation speed. Note that $D(kd_x, ld_y, md_z)$ is both a constant and independent of frequency, since there are no tapped delay-lines or any other frequency dependent processing for each received sensor signal.

With the following substitutions

$$\begin{aligned} \omega_1 &= \frac{\omega \sin\theta \cos\phi d_x}{c} \\ \omega_2 &= \frac{\omega \sin\theta \sin\phi d_y}{c} \\ \omega_3 &= \frac{\omega \cos\theta d_z}{c} , \end{aligned} \quad (2)$$

we have

$$P(\omega_1, \omega_2, \omega_3) = \sum_{k,l,m=-\infty}^{\infty} D(kd_x, ld_y, md_z)\ e^{-jk\omega_1} \cdot e^{-jl\omega_2} e^{-jm\omega_3} . \quad (3)$$

Obviously, the beam pattern of such a 3-D array can be obtained by first applying a 3-D Fourier transform to the array's coefficients $D(kd_x, ld_y, md_z)$ according to (3) and then using the above substitutions in (2). Since all the three substitutions are functions of ω, the resultant beam pattern in general will also be frequency dependent. Alternatively, from the desired beam pattern $P(\omega, \theta, \phi)$, we can express it in the form of ω_1, ω_2 and ω_3 and then apply a 3-D inverse Fourier transform to get the coefficients $D(kd_x, ld_y, md_z)$.

Likewise, to achieve a frequency invariant beam pattern ($P(\theta, \phi)$), we again express $P(\theta, \phi)$ as a function of ω_1, ω_2 and ω_3 and then inverse transform to get $D(kd_x, ld_y, md_z)$. The problem is, we need to find a way to eliminate ω in the expressions for θ and ϕ, otherwise, the resultant $D(kd_x, ld_y, md_z)$ will still be a function of ω and not a constant as required. So in the following, we will propose a substitution in the desired frequency invariant response for θ and ϕ, in which ω will disappear due to a special arrangement.

From (2), we can have

$$\begin{aligned} \frac{\omega_2 d_x}{\omega_1 d_y} &= \tan\phi \\ \frac{\omega_1^2/d_x^2 + \omega_2^2/d_y^2}{\omega_3^2/d_z^2} &= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta . \end{aligned} \quad (4)$$

Thus, we easily obtain the following pair of substitutions for θ and ϕ:

$$\begin{aligned} \phi &= \arctan\frac{\omega_2 d_x}{\omega_1 d_y} \\ \theta &= \arctan\sqrt{\frac{\omega_1^2/d_x^2 + \omega_2^2/d_y^2}{\omega_3^2/d_z^2}} \quad (\theta \in [0; \frac{\pi}{2}]) . \end{aligned} \quad (5)$$

Now given the desired frequency invariant response $P(\theta, \phi)$, the design of the 3-D uniformly spaced array can be described as follows:

Step 1. Using the substitutions of (5) in $P(\theta,\ \phi)$, we obtain $P(\omega_1, \omega_2, \omega_3)$, defined over one period $\omega_1, \omega_2, \omega_3 \in [-\pi; \pi)$.

Step 2. Applying a 3-D inverse Fourier transform to $P(\omega_1, \omega_2, \omega_3)$ returns the desired coefficients $D(kd_x, ld_y, md_z)$ for the corresponding sensors with infinite support. Suppose the array dimension is $K \times L \times M$. As an approximation, we can employ the 3-D inverse discrete Fourier transform (IDFT) by sampling $P(\omega_1, \omega_2, \omega_3)$ on the $\tilde{K} \times \tilde{L} \times \tilde{M}$ points of $\omega_1 = -\pi + \frac{2\tilde{k}\pi}{\tilde{K}}$, $\tilde{k} = 0, 1, \ldots, \tilde{K}-1$, $\omega_2 = -\pi + \frac{2\tilde{l}\pi}{\tilde{L}}$, $\tilde{l} = 0, 1, \ldots, \tilde{L}-1$, and $\omega_3 = -\pi + \frac{2\tilde{m}\pi}{\tilde{M}}$, $\tilde{m} = 0, 1, \ldots, \tilde{M}-1$, where $\tilde{K} > K$, $\tilde{L} > L$ and $\tilde{M} > M$. To fit the dimensions of the $K \times L \times M$ of the array, we need to truncate the resultant $D(kd_x, ld_y, md_z)$ to the size of $K \times L \times M$. Note that various "windows" could be used.

Now there is an effective region for the array's response in the $(\omega_1, \omega_2, \omega_3)$ domain for a given frequency range of interest. From (2), we can have

$$\frac{c^2\omega_1^2}{d_x^2} + \frac{c^2\omega_2^2}{d_y^2} + \frac{c^2\omega_3^2}{d_z^2} = \omega^2 . \quad (6)$$

Suppose the range of the frequency of interest is $\omega \in [\omega_{\min}; \omega_{\max}]$, then we have

$$\frac{c^2\omega_1^2}{d_x^2} + \frac{c^2\omega_2^2}{d_y^2} + \frac{c^2\omega_3^2}{d_z^2} \in [\omega_{\min}^2; \omega_{\max}^2] . \quad (7)$$

Therefore, $P(\omega_1, \omega_2, \omega_3)$ can take any value without affecting the array's response to the frequency range of interest for the following two regions

$$0 < \frac{c^2\omega_1^2}{d_x^2} + \frac{c^2\omega_2^2}{d_y^2} + \frac{c^2\omega_3^2}{d_z^2} < \omega_{\min}^2 \quad (8)$$

and

$$\frac{c^2\omega_1^2}{d_x^2} + \frac{c^2\omega_2^2}{d_y^2} + \frac{c^2\omega_3^2}{d_z^2} > \omega_{\max}^2 . \quad (9)$$

One easy choice is to assign a constant value to it for those two regions. Intuitively, with this approach, the modified

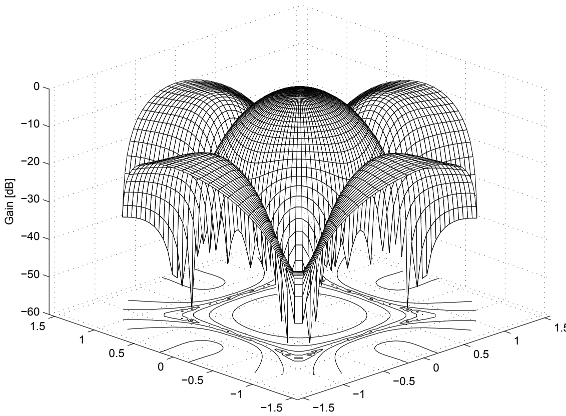

Fig. 2: The resultant beam pattern of the 3-D array at $f = 500Hz$, with respect to azimuth (angle coordinate) and elevation (radial coordinate).

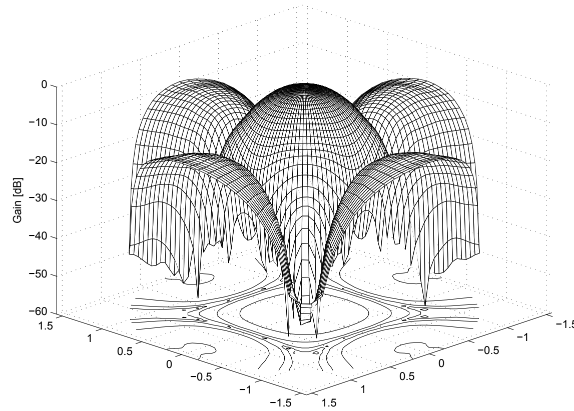

Fig. 3: The resultant beam pattern of the 3-D array at $f = 1500Hz$, with respect to azimuth (angle coordinate) and elevation (radial coordinate).

$P(\omega_1, \omega_2, \omega_3)$ will become smoother than the original one and so the 3-D IDFT can lead to an improved design result.

In the above design, we first need to have the desired frequency invariant response, which can be obtained by a design method for narrowband beamformers and in this case, a narrowband rectangular array. Normally, the $P(\theta, \phi)$ obtained will be in the form of $F(\sin\theta\cos\phi, \sin\theta\sin\phi)$ — see the next section for an example of such a desired response. To avoid aliasing, $d_x, d_y, d_z \leq \lambda_{min}/2$, where λ_{min} is the wavelength of the maximum frequency of interest $\omega_{\max}$, and we set $d_x = d_y = d_z = \lambda_{min}/2$. Then for $\omega > 0$, from (2) and (6), we can easily derive the following substitutions:

$$\begin{aligned} \sin\theta\cos\phi &= \frac{\omega_1}{\sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}} \\ \sin\theta\sin\phi &= \frac{\omega_2}{\sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}} \,. \end{aligned} \tag{10}$$

So $P(\omega_1, \omega_2, \omega_3)$ can be obtained as

$$P(\omega_1, \omega_2, \omega_3) = F\left(\frac{\omega_1}{\sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}}, \frac{\omega_2}{\sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}}\right) . \tag{11}$$

3. DESIGN EXAMPLE

In this section we provide an illustrative design example for acoustic arrays. The frequency range of interest is between 500 Hz and 1500 Hz and the propagation speed is $340m/s$. The dimensions of the 3-D equally spaced array are $15 \times 15 \times 15$ and the adjacent sensor spacing is set to be $34000/(2 \times 1500) = 11.33$ cm. The desired beam pattern

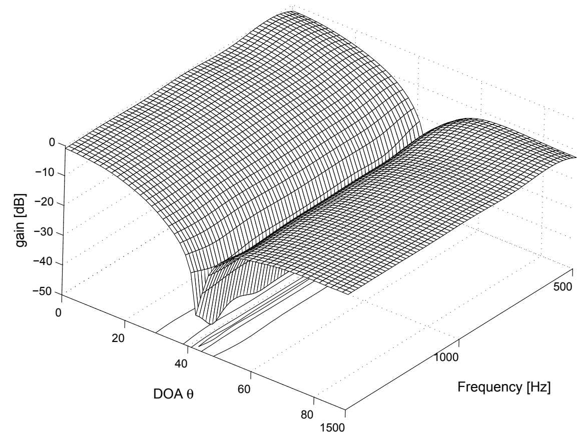

Fig. 4: A slice of the beam pattern (θ, f) at azimuth angle $\phi = 100°$.

is given by

$$F(\sin\theta\cos\phi, \sin\theta\sin\phi) = \frac{1}{9}\sum_{l,m=-1}^{1} e^{-jl\pi\sin\theta\cos\phi} e^{-jm\pi\sin\theta\sin\phi} \,. \tag{12}$$

which forms a main beam towards the broadside $\theta = 0$ and $\phi = 0$. This desired frequency invariant response is obtained by a narrowband rectangular array with uniform weighting. Note that after the substitutions in (10) and (11), the resultant $P(\omega_1, \omega_2, \omega_3)$ is symmetric with $P(\omega_1, \omega_2, \omega_3) = P(-\omega_1, -\omega_2, -\omega_3)$. Hence the coefficients $D(kd_x, ld_y, md_z)$ we obtain after the inverse Fourier transform will be real-valued and the resultant frequency invariant beamformer is not only without tapped delay-lines, but also without phase-shifters, which significantly simplify implementation.

We employed a $32 \times 32 \times 32$-point 3-D IDFT on the

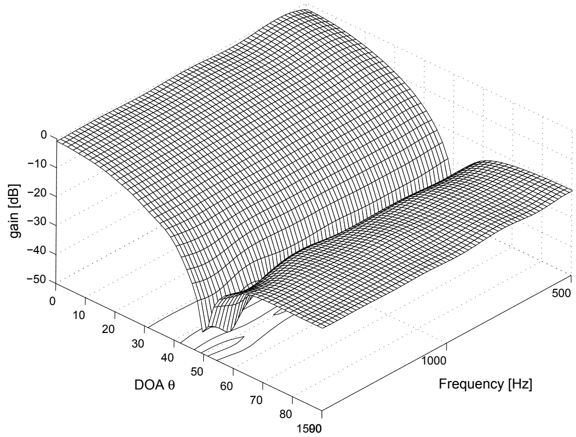

Fig. 5: A slice of the beam pattern (θ, f) at azimuth angle $\phi = 300°$.

resultant function $P(\omega_1, \omega_2, \omega_3)$ and set $P(\omega_1, \omega_2, \omega_3) = 0$ for the area outside the frequency range $[400Hz; 1700Hz]$ according to equations (6) to (9). We left the region between 400 Hz and 500 Hz and the region between 1500 Hz and 1700 Hz as the transition bands. Since the beam pattern is four-dimensional, we can only provide some exemplary snapshots. Figs. 2 and 3 give the array's response in cylindrical coordinates to the frequencies $f = 500$ Hz and 1500 Hz, respectively. The height axis is the magnitude response of the beam, the radial coordinate is for the elevation angle θ and the angle coordinate is for the azimuth angle ϕ. The frequency invariant property can be verified by the clear similarity of these two figures. In addition, the response with respect to θ and f for two different values of ϕ are given in Figs. 4 and 5, with $\phi = 100°$ and $300°$, respectively. The frequency invariant property is clearly visible.

4. CONCLUSIONS

We have proposed a novel frequency invariant beamforming design method for three-dimensional broadband arrays without tapped delay-line processing. Different from the previously proposed rectangular arrays without tapped delay-lines, this 3-D array can form a beam steerable along both the azimuth angle and the elevation angle and has a very simple implementation. A design example has been provided, yielding satisfactory frequency invariant characteristics over the range of frequencies of interest.

5. ACKNOWLEDGEMENTS

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