OPTIMIZED BEAMFORMING CALIBRATION IN THE PRESENCE OF ARRAY IMPERFECTIONS

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ABSTRACT

For arrays with position and channel errors the calibration becomes very crucial. In the traditional calibration methods one can choose between an optimal SNR and a beam pattern with low side lobes. In this paper we formulate a beam pattern synthesis method which optimize the trade-off between the two criteria.

A classical problem with position errors in an array, is that it is difficult to get low side lobes over the whole side lobe region, since the position errors give rise to direction dependent errors. In this paper this problem is solved by using local (direction dependent) correction matrices in the beam pattern optimization. The new way of using local correction matrices leads to the lowest possible uniform side lobe level, for the chosen SNR, beamwidth and beam pointing direction.

Index Terms— Antenna arrays, calibration, array signal processing, robustness.

1. INTRODUCTION

Traditionally array antennas have been manufactured with a high mechanical and electrical accuracy. But in some applications it can still be difficult to keep a perfect mechanical shape (such as in space applications with inflatable large arrays), and it is also possible that lower manufacturing cost could be achieved if the tolerances in the manufacturing process was not so tough. But increased errors in the array of course has its price. Especially errors which give rise to direction dependent errors, such as position errors, require some special treatment.

In a previous paper [1], array calibration methods for arrays with position and channel errors were evaluated using MUSIC for Direction Of Arrival (DOA) estimation. It was found that if a local (direction dependent) calibration was used instead of a global (direction independent) calibration [2], the DOA estimation performance in the presence of direction dependent errors was improved substantially.

In the DOA estimation with calibration using local models, one correction matrix was used for each direction of interest. But to receive a signal from a certain direction using beamforming, we only have one set of weights which should fulfill our requirements to both receive the interesting signal and suppress the external noise from e.g. ground clutter without amplifying the internal noise in the array. If the array has both direction dependent and direcion independent errors, we can choose between an optimal SNR (Signal to Noise Ratio) for internal noise or a good beam pattern. In this paper an alternative beamforming method is derived which offers a possibility to choose in the trade-off between a high SNR or low side lobe level.

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A well-known problem with position errors is, that it is difficult to achieve low side lobes over the whole side lobe region. Therefore we introduce a new way of using local correction matrices in the beam pattern synthesis, which makes it possible to get low side lobes over the whole side lobe region.

Several papers have been written on array calibration, and two main approaches can be discriminated between; calibration using sources of known locations, [2] and [3], and auto-calibration using unknown locations [4]. Auto-calibration methods are generally limited to small errors and require a known error model. If there are no position errors (or any other errors which are direction dependent) a global (direction independent) calibration can be used [2]. Parametric methods are often used to be able to handle the position errors [3] and [4]. Our local calibration method is a non-parametric method, which does not require any a priori knowledge about the array errors. Furthermore, large errors can be handled and the calibration grid can be made sparse, which is discussed in a companion paper [5]. In this paper, the local calibration is included in the beam pattern synthesis. For a good reference on array pattern synthesis using convex optimization, see [6]. Our way to apply local correction matrices in the pattern synthesis could also have been used on their synthesis method, but our chosen method gives better control of the white noise gain.

In this paper vectors are written in bold lower case letters, and matrices in bold upper case letters. The transpose of a vector $\mathbf{a}$ is marked $\mathbf{a}^T$ while the complex conjugate transpose is marked $\mathbf{a}^*$. The complex conjugate transpose of the inverse of a matrix $\mathbf{A}$ is written $\mathbf{A}^{-*}$.

2. TWO DIFFERENT CALIBRATION METHODS

If a signal $s(t)$ arrives at an array from direction θ_0, the output from the array $\mathbf{x}(t)$ can be modelled as

$$\mathbf{x}(t) = \mathbf{a}(\theta_0)s(t) + \boldsymbol{\eta}(t), \tag{1}$$

where $\boldsymbol{\eta}(t)$ is noise with covariance matrix $\sigma^2\mathbf{I}$ and $\mathbf{a}(\theta)$ is the steering vector for direction θ_0, which for a uniform linear array is given by

$$\mathbf{a}(\theta_0) = [1, e^{j\Delta k \sin(\theta_0)}, ..., e^{j\Delta k \sin(\theta_0)(m-1)}]^T, \tag{2}$$

where Δ is the distance between the sensors, m is the number of sensors and $k = 2\pi/\lambda$ where λ is the wavelength.

To be able to use the data model in (1) for a real array with imperfections, the ideal steering vector $\mathbf{a}(\theta_0)$ is replaced by a real steering vector $\mathbf{a_{mod}}(\theta_0)$ including the imperfections. It is given by

$$\mathbf{a_{mod}}(\theta_0) = \mathbf{Q}\mathbf{a}(\theta_0), \tag{3}$$

where $\mathbf{Q}$ is a matrix, called the correction matrix. The imperfections can consist of any type of direction dependent or direction independent errors and also mutual coupling, but in the examples here, only channel errors (errors regarding amplification and phase shift in the receiver channels) and positional errors are included.

Traditionally, the correction matrix has been applied to beamforming in two different ways. In the first method, the SNR regarding the internal noise in the receiver channels is maximized for a signal received from direction θ_0. The weights are chosen as [7, Ch. 3]

$$\mathbf{v_{snr}} = \mathbf{a_{mod}}(\theta_0) = \mathbf{Q}\mathbf{a}(\theta_0). \tag{4}$$

This can be shown by using Schwarz Inequality. Denoting the signal power P, the SNR at the array output is given by

$$SNR = \frac{P}{\sigma^2}\frac{|\mathbf{v}^*\mathbf{a_{mod}}(\theta_0)|^2}{\|\mathbf{v}\|^2} \leq \frac{P}{\sigma^2}\|\mathbf{a_{mod}}(\theta_0)\|^2, \tag{5}$$

where the equal sign is for $\mathbf{v} = \mathbf{a_{mod}}(\theta_0)$. These weights correspond to a matched filter to the distorted array steering vector.

The other method is to use the virtual array approach [8]. Then the received data is filtered so that it appears to come from a ULA without errors by pre-mulitiplying the received signals $\mathbf{x}(t)$ with the inverse of the correction matrix $\mathbf{Q}$

$$\mathbf{x_{corr}}(t) = \mathbf{Q}^{-1}\mathbf{x}(t). \tag{6}$$

Writing the output signal from the array

$$y(t) = \mathbf{a}^*\mathbf{x_{corr}}(t) = \mathbf{a}^*\mathbf{Q}^{-1}\mathbf{x}(t) = \mathbf{v_{virt}}^*\mathbf{x}(t), \tag{7}$$

it is obvious that the pre-multiplication can also be performed by modifying the weights according to

$$\mathbf{v_{virt}} = \mathbf{Q}^{-*}\mathbf{a}(\theta_0). \tag{8}$$

Using the virtual array approach (8), the SNR regarding internal noise is no longer optimal, but in Section 5.1 we will see that the side lobes generally become lower.

If the true correction matrix $\mathbf{Q}$ is used to calculated the weights, the SNR for the two methods are

$$SNR_{snr} = \frac{P}{\sigma^2}\|\mathbf{a_{mod}}(\theta_0)\|^2 \tag{9}$$

and

$$SNR_{virt} = \frac{P}{\sigma^2}\frac{\|\mathbf{a}(\theta_0)\|^4}{|\mathbf{Q}^{-*}\mathbf{a}(\theta_0)|^2}, \tag{10}$$

respectively. In reality, the correction matrix $\mathbf{Q}$ is unknown and the SNR is calculated from (5) using an estimate of the correction matrix. How to estimate the correction matrix from calibration measurements is explained in Section 4.

Finally, the SNR_{gain} is defined as

$$SNR_{gain} = \frac{SNR}{\frac{P}{\sigma^2}|\mathbf{a_{mod}}(\theta_0)|^2}, \tag{11}$$

which results in an SNR_{gain} between 0 and 1.

3. OPTIMIZED BEAMFORMING WITH CALIBRATION

In the section above, the calibration was a choice between good SNR for internal noise or good beam pattern. So the question is, how can we ensure we get both? Further more, for a ULA one can easily achieve a desired side lobe level by using a taper. In an array with position and channel errors, it is possible to use a taper together with the virtual array calibration, but due to the position errors it is difficult to get the same low side lobe level over the whole side lobe region.

One solution is to optimize the weights such that all our requirements are fulfilled. The aim is to achieve a good beam pattern with a uniform side lobe level and at the same time be able to control the SNR for internal noise. The optimization problem is formulated as a numerical minimization problem with constraints. The maximum side lobe level over the side lobe region specified by $\boldsymbol{\theta}_{sidelobes}$ is minimized, subject to having maximum sensitivity in the main beam direction θ_0 and an SNR_{gain} larger than SNR_{gain}^{min}, according to

$$\begin{aligned} \mathbf{v} = \arg\min_{\mathbf{v}} & [\max\{|\hat{G}(\theta)|^2\}\ \forall\theta \in \boldsymbol{\theta_{sidelobes}}] \\ & \text{s.t. } \|\mathbf{v}\|^2 < \frac{1}{SNR_{gain}^{min}} \\ & \text{and } \frac{\hat{G}(\theta_0)}{\|\hat{\mathbf{a}}_{\mathbf{mod}}(\theta_0)\|} = 1, \end{aligned} \tag{12}$$

where $\hat{G}(\theta) = \mathbf{v}^*\hat{\mathbf{a}}_{\mathbf{mod}}(\theta)$, $\hat{\mathbf{a}}_{\mathbf{mod}}(\theta) = \hat{\mathbf{Q}}(\theta)\mathbf{a}(\theta)$ and θ is the variable angle of the beam pattern. Inserting the first part of (5) and the second constraint in (12) into (11), leads to $SNR_{gain} = 1/\|\mathbf{v}\|^2$. This explains how the constraint put on the norm squared of the weights sets a lower limit of the SNR_{gain}.

As will be shown below, it is important that the local $\hat{\mathbf{Q}}(\theta)$ and not the global correction matrix $\hat{\mathbf{Q}}$ is used in (12) to ensure that for each direction we have the best description of how the array receives signals. Local and global correction matrices are further explained in the section below. The examples in the end of the paper show that if local correction matrices are used, the minimization problem in (12) leads to equi-ripple side lobes which are as low as possible for the selected minimum allowed SNR_{gain}. The region over which the side lobes are given, specifies the width of the main beam. The optimization method is quite convenient, since there are only two parameters to be varied, the main lobe width and the SNR_{gain}.

The implementation was made using *fminimax* from MATLAB's optimization toolbox. This function cannot handle a continuous variable, so the side lobe region was defined by a vector $\boldsymbol{\theta_{sidelobes}}$ of discretized directions. If the grid is chosen too sparse, uncontrolled side lobes may appear between the grid points. User experience has shown that this is avoided by choosing the angular separation between the grid points less or equal to half the half-power beamwidth.

Finally, it should be mentioned that the proposed optimization has similarities with adaptive algorithms such as Capon's algorithm in the sense that it maximizes the SNR while preserving the desired signal. Using Capon's method, however, the side lobe topology is not set on before hand, but given by the statistics of the received array data. A beam pattern with a general low side lobe level is of interest, for example, in situation where there might be one or more unknown interferers.

4. LOCAL OR GLOBAL CORRECTION MATRIX

Two ways of estimating the correction matrix $\mathbf{Q}$ from calibration measurements will be demonstrated. The calibration is performed in the following way. First calibration data is collected. One transmitter is used and moved in a grid of calibration angles $\boldsymbol{\theta_{cal}} = [\theta_{cal1}, \ldots, \theta_{calj}, \ldots, \theta_{calJ}]$, where J is the number of calibration angles. For each angle a steering vector $\hat{\mathbf{a}}_{\mathbf{meas}}(\theta_{calj})$ is estimated from the measurement data $\mathbf{x}(\theta_{calj})$ by picking out the principal eigenvector of the covariance matrix of $\mathbf{x}(\theta_{calj})$. These steering vectors characterize the array. The covariance matrix is estimated using time averaging.

In this paper both local [1] and global calibration [2] are used. In both cases a correction matrix is calculated as the optimal matrix in a (weighted) least-square sense according to

$$\hat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} \|(\hat{\mathbf{A}}_{\mathbf{meas}}(\boldsymbol{\theta_{cal}}) - \mathbf{Q}\mathbf{A}(\boldsymbol{\theta_{cal}}))\mathbf{W}^{\mathbf{1/2}}\|_F, \quad (13)$$

where the subscript F means Frobenius norm, $\mathbf{W}$ is a weight matrix, $\boldsymbol{\theta_{cal}}$ is the vector of calibration angles, $\hat{\mathbf{A}}_{\mathbf{meas}}(\boldsymbol{\theta_{cal}})$ is a matrix with all the estimated steering vectors and $\mathbf{A}(\boldsymbol{\theta_{cal}})$ is a matrix with the corresponding ideal steering vectors. In the global case, the matrix $\mathbf{W}$ is the identity matrix and $\hat{\mathbf{Q}}$ is direction independent. Only one, full, matrix is calculated for all values of θ. In the local case, the weight matrix $\mathbf{W}(\theta)$ is direction dependent, and one $\hat{\mathbf{Q}}(\theta)$ is calculated for each θ of interest. Since each matrix $\hat{\mathbf{Q}}(\theta)$ is calculated for a single direction θ, only a diagonal local correction matrix $\hat{\mathbf{Q}}(\theta)$ is needed.

The weight matrix $\mathbf{W}(\theta)$ should have the property that calibration data for angles θ_{calj} close to θ are given high weight. In this paper, $\mathbf{W}(\theta)$ is a diagonal matrix with the diagonal elements, $w_j(\theta) = \exp(-hD_j)$ and the distance function $D_j = |\theta - \theta_{calj}|^2$. The parameter h determines the width of the weight function. In this paper it is set by trial and error, but it can also be calculated from the calibration angles using a leave-one-out approach, see [9]. Also other distance functions can be used, but the choice is not so crucial as long as the parameter h is large.

5. EXAMPLES

Array data containing channel and position errors has been simulated using the function

$$\mathbf{x}(t) = \mathbf{D}\mathbf{a}(\theta_0)s(t) + \boldsymbol{\eta}(t), \quad (14)$$

where $\mathbf{D}$ is a diagonal matrix modelling channel errors as complex white gaussian additive noise with standard deviation 0.5, $\boldsymbol{\eta}$ is additive white gaussian noise in the receiver channels having unit standard deviation and the signal $s(t)$ is 20dB above the noise $\boldsymbol{\eta}$. The array is assumed to be an $m = 16$ element array of isotropic elements. The steering vector $\mathbf{a}(\theta_0)$ has been modified to include random positional errors evenly distributed between -0.125λ and 0.125λ in the direction along the array. The nominal element separation is 0.5λ.

In the examples below, both calibration and validation data are simulated using the data model in (14). All beam patterns in this paper are calculated for the same realization of the position and channel errors. The beam patterns are calculated using $|G(\theta)|^2 = |\mathbf{v}^*\mathbf{a}_{\mathbf{mod}}(\theta)|^2/\|\mathbf{v}\|^2$, and presented as 2D directivity according to $dir = 2\pi|G(\theta)|^2/\int_0^{2\pi}|G(\theta')|^2 d\theta'$.

5.1. Evaluating the two classical calibration methods

We will now evaluate the SNR_{gain} and the beam patterns for an array with position and channel errors using calibration for optimal SNR (4) and the virtual array calibration (8). We differ between four different cases. Using the true correction matrix $\mathbf{Q}$, estimating the correction matrix $\hat{\mathbf{Q}}$ using local or global calibration or using no correction matrix, that is using $\mathbf{v} = \mathbf{a}(\theta_0)$. For the local calibration, we use the correction matrix which is optimized for the beam pointing direction, in this case $\theta_0 = 20°$.

Calibration data was created over the region $\pm 90°$ every 5th degrees and 1000 snapshots were used to estimate the calibration steering vectors. For the local calibration method the angular bandwidth

Table 1. Optimal SNR calibration with different correction matrices.

Method	SNR_{gain}	Cond ($\mathbf{Q}$)
$\mathbf{v_{snr}} = \mathbf{Q}\mathbf{a}(\theta_0)$	1.00	0.29
$\mathbf{v_{snr}} = \hat{\mathbf{Q}}_{\mathbf{global}}\mathbf{a}(\theta_0)$	0.997	0.14
$\mathbf{v_{snr}} = \hat{\mathbf{Q}}_{\mathbf{local}}(\theta_0)\mathbf{a}(\theta_0)$	1.00	0.29

Table 2. Virtual array calibration with different correction matrices.

Method	SNR_{gain}	Cond ($\mathbf{Q}^{-*}$)
$\mathbf{v_{virt}} = \mathbf{Q}^{-*}\mathbf{a}(\theta_0)$	0.611	0.29
$\mathbf{v_{virt}} = \hat{\mathbf{Q}}_{\mathbf{global}}^{-*}\mathbf{a}(\theta_0)$	0.559	0.12
$\mathbf{v_{virt}} = \hat{\mathbf{Q}}_{\mathbf{local}}^{-*}(\theta_0)\mathbf{a}(\theta_0)$	0.611	0.29

parameter h was set to 500. The resulting SNR_{gain} was calculated using 10000 simulations with different position and channel errors. The results calculated using weights for the optimal SNR method is shown in Table 1 and the results for the virtual array approach in Table 2. Without calibration, the resulting SNR_{gain} is 0.794. The SNR is apparently better for the optimal SNR method. The virtual array method gives worse results, especially for the global calibration, which also has the lowest condition number of the correction matrix (marked with Cond $\mathbf{Q}$ and Cond $\mathbf{Q}^{-*}$ in Table 1 and 2, respectively).

The beam patterns calculated using weights for optimal SNR (4) are shown in Figure 1 and for the virtual array approach (8) in Figure 2. A Taylor taper (three side lobes 30 dB below main lobe maximum) is applied to the weights. The results depend on the specific position and channel errors, but still some general conclusions can be drawn. The beam patterns for the optimal SNR method (Figure 1) are in general poor (high side lobes, and in this case also shoulders on the main beam). The virtual array method (Figure 2) with a local correction matrix gives a good beam pattern only close to the main beam, since the local correction matrix only tries to equal a ULA in the beam direction. The global correction matrix tries to achieve a ULA over the whole calibration region and gives a good beam pattern over a wider angular sector. Still, for far-out side lobes it fails, and high side lobes appear.

If there are only channel errors (no position errors), the side lobe level becomes better. This is due to that channel errors can be compensated for in the same way for all directions, while the compensation for position errors is direction dependent.

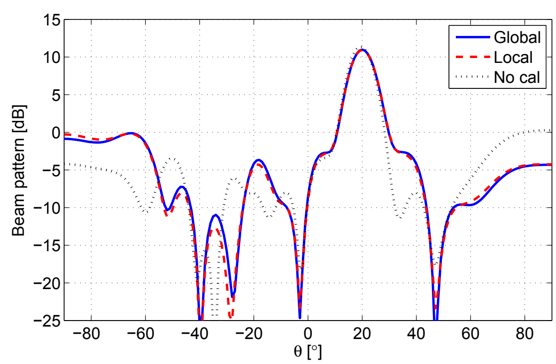

Fig. 1. Beam pattern using a Taylor taper (three side lobes 30 dB below main lobe maximum) and optimal SNR calibration with different correction matrices.

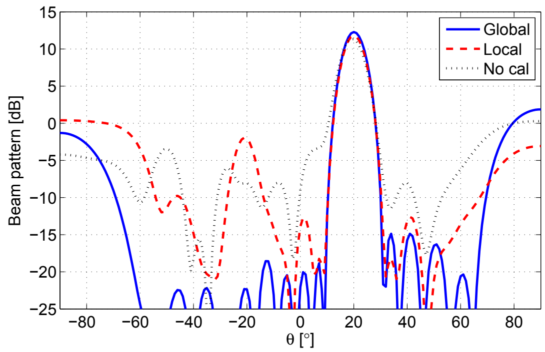

Fig. 2. Beam pattern using a Taylor taper (three side lobes 30 dB below main lobe maximum) and virtual array calibration with different correction matrices.

5.2. Evaluating the optimized beamforming

This section shows some results from using the beam pattern optimization, and the importance of using local correction matrices in the optimization.

First, let us study an array without position and channel errors, but otherwise the array is given by the parameters specified in Section 5. The side lobe region was $\pm 90^\circ$ (sampled every 0.2°) except for the region $\pm 9^\circ$ around the main lobe direction. Figure 3 shows the resulting beam patterns using a global correction matrix. Setting $SNR_{gain}^{min} = 1$ (the maximum possible) leads to beamforming weights $\mathbf{v}$ with a uniform distribution, while allowing a slightly lower SNR_{gain}^{min} gives an equi-ripple side lobe pattern with a slightly wider main beam. Adding channel errors, almost as good beam patterns can be achieved, but at the cost of a lower SNR_{gain}.

Next, both position and channel errors are added. Figure 4 shows the beam patterns for the global and local correction matrices with different SNR_{gain}. (The result depends on the specific errors.) The global correction matrix gives rise to high side lobes far from broadside, due to an insufficient description the position errors (also obvious in Figure 2, global correction matrix). Using the local correction matrices, equi-ripple side lobes are achieved and the trade-off between low side lobes and high SNR is obvious. We can also conclude that the position errors lead to an increased side lobe level.

In the example, only position errors along the direction of the array was studied, but also other types of position errors can be handled. Furthermore, if other types of side lobe topographies are desired, these can be included by rewriting the side lobe requirement.

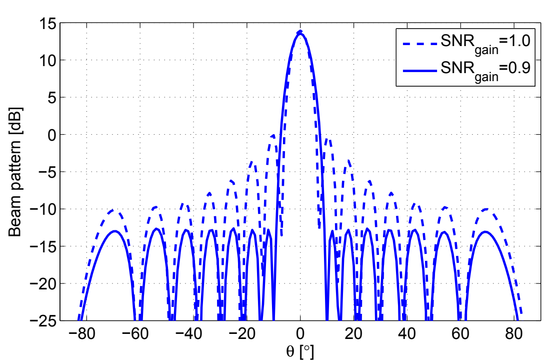

Fig. 3. Beam pattern from optimization. No errors.

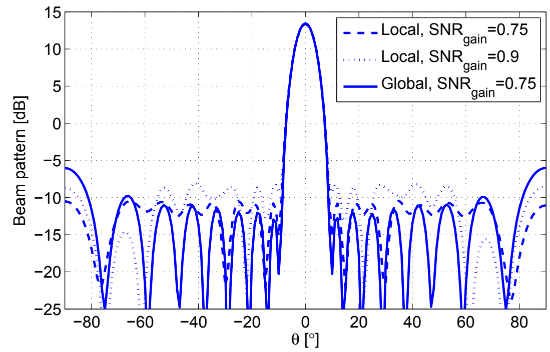

Fig. 4. Beam pattern from optimization. Position and channel errors.

6. CONCLUSIONS

In an array with position and channel errors, the choice of how to use the correction matrix to calibrate the array becomes a trade-off between a high SNR for internal noise and a beam pattern with low side lobes. To combine the requirements, a new beamforming optimization method including a direction dependent array description was proposed. Using the method, a beam pattern with uniform side lobe level can be achieved also for arrays with large position errors. Furthermore, an array with position errors and channel errors was found to not give the same low side lobe level (for a fixed beamwidth and SNR), as the corresponding array without errors.

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